EE243 Advanced Electromagnetic Theory

Lec # 15 Plasmons

Fixed Slides 6, 12

- **(Group and Phase Velocity for Guided Waves)**
- **(Sign Reversals for Propagation Direction Reversal)**
- **Plasmon Fields, B.C. and Dispersion Equation**
- **Physical Interpretation**
- **Typical Values for Silver**
- $\omega-\beta$ **diagram and fields at various points**
- **Assistance from topographical features**

Reading: These PPT notes and Harrington on corrugated surface 4.8.

Midterm Exam

Review Th, Oct 19

- In Class Tuesday October 24th
- Covers material through today (Chapter 7)
- Open Book, Open Notes, Bring Calculator, Paper Provided
- Topics
  - Green's functions free space and use in Theorems and concepts with emphasis on statics
  - Separation of variables in rectangular coordinates using N-1 and N method
  - Time-Harmonic ME, planewaves, boundary conditions, and dispersion

Overview

- (First touch up
  - product of group and phase = c^2 for guided waves
  - Reverse direction reverse sign E_z and H_t).
- Surface Plasmons on metals
  - are TM guided waves
  - are made possible by free electrons oscillating in the propagating field just below their plasma frequency
  - this makes the permittivity both negative and larger in magnitude than the permittivity of air or a coating dielectric.
- Plasmons are described by Maxwell Equations
  - Their dispersion relationship is found by matching B.C.
  - They propagate a distance of up to 30 μm
  - Near the plasma frequency their wavelength along the surface can be 5X smaller that the free space wavelength
  - The can be used to form high resolution (λ/6) intensity probes but the probe height is roughly equal to the resolution and thus decreases.

Clarification on Phase and Group Velocity

$$\gamma^2 = \mu\varepsilon\omega^2 - k^2$$

$$\omega = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\lambda^2 + k^2}$$

$$v_p = f\lambda = \frac{2\pi f}{\frac{2\pi}{\lambda}} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\frac{\lambda^2}{k^2}+1} > c$$

$$v_g = \frac{\partial\omega}{\partial k} = \frac{\frac{1}{2}(2k)}{\sqrt{\mu\varepsilon}\sqrt{\lambda^2+k^2}} = \frac{1}{\sqrt{\mu\varepsilon}}\frac{1}{\sqrt{\frac{\lambda^2}{k^2}+1}} < c$$

$$v_p v_g = c^2$$

- The fact that the product of the phase and group velocity is equal to the square of the speed of light holds for any (propagating) mode in a lossless sytemdue to its eigenfunction and eigenvalue γ^2.

Sign Reversal with Direction Reversal

$$\nabla_t \times \overline{E}_{t\lambda} = i\omega\mu\overline{H}_{z\lambda}$$

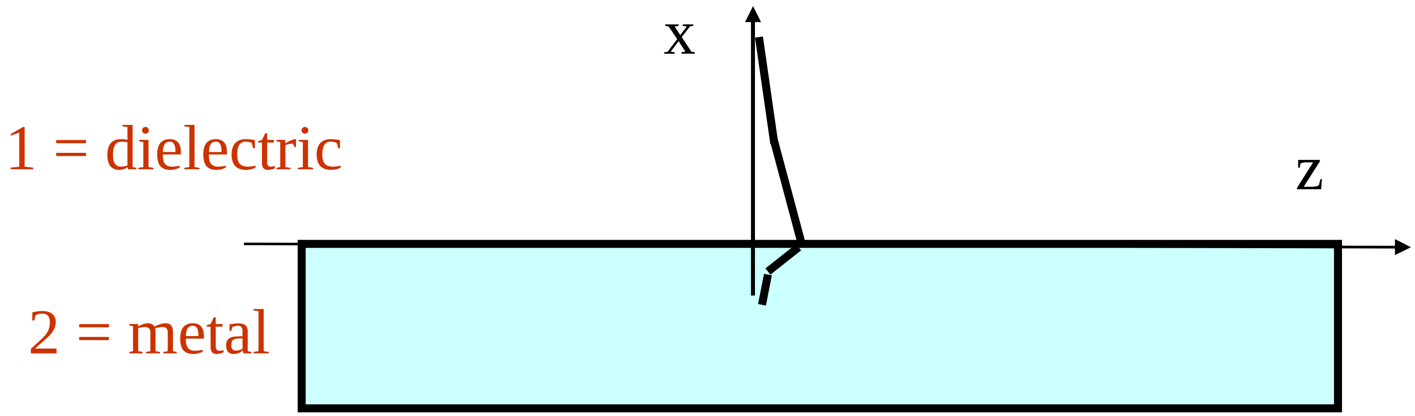
$$\nabla_t \times \overline{H}_{t\lambda} = -i\omega\varepsilon\overline{E}_{z\lambda}$$

$$\nabla_t \times \overline{E}_{z\lambda} + i\beta_\lambda \hat{z} \times \overline{E}_{t\lambda} = i\omega\mu\overline{H}_{t\lambda}$$

$$\nabla_t \times \overline{H}_{z\lambda} i + i\beta_\lambda \hat{z} \times \overline{H}_{t\lambda} = -i\omega\varepsilon\overline{E}_{t\lambda}$$

- Take longitudinal and transverse components of curl equations with z phase variation.
- With propagation direction reversal the sign of β_λ changes
- But if sign of E_z and H_t are also reversed the original equations result
- Thus the reverse guided wave solution can be nothing more than the forward guided wave solution with these changes

Are There Waves on Material Surfaces?



$$H_{1y} = H_1 \hat{y} e^{-v_1 x} e^{ik_z z}$$

$$H_{2y} = H_2 \hat{y} e^{+v_2 x} e^{ik_z z}$$

$$v_1 = \sqrt{k_z^2 - \omega^2 \mu_0 \varepsilon_1}$$

$$v_2 = \sqrt{k_z^2 - \omega^2 \mu_0 \varepsilon_2}$$

- Consider TM w/r z case with Hy given and same z phase variation
- Will have Hy, Ez and Ex (but Ey = Hx = Hz = 0)

Boundary Conditions

$$E_{1z} = \frac{-v_1}{-i\omega\varepsilon_1} H_{1y}$$

$$E_{2z} = \frac{+v_2}{-i\omega\varepsilon_2} H_{2y}$$

$$\frac{-iv_1}{\varepsilon_1} = \frac{+iv_2}{\varepsilon_2}$$

$$\frac{E_{1z}}{H_{1y}} = \frac{E_{2z}}{H_{2y}}$$

$$-Z_{+x} = \frac{-v_1}{-i\omega\varepsilon_1} = \frac{+v_2}{-i\omega\varepsilon_2} = Z_{-x}$$

- Hy continuous (or) D normal continuous gives $H_{10} = H_{20}$.
- Ez continuous gives final constraint to find k_z.
- This constraint is the same as setting the impedance looking upward equal to the negative of the impedance looking downward.
- Impedance looking upward is capacitive (neg imy).
- Impedance looking downward thus need to be inductive.

Solving for Surface Wave Conditions

$$\frac{v_1}{\varepsilon_1} = \frac{-v_2}{\varepsilon_2}$$

$$v_1 = \sqrt{k_z^2 - \omega^2 \mu_0 \varepsilon_1}$$

$$v_2 = \sqrt{k_z^2 - \omega^2 \mu_0 \varepsilon_2}$$

$$k_z = k_1 \sqrt{\frac{\varepsilon_2}{(\varepsilon_2 + \varepsilon_1)}}$$

$$v_1 = k_1 \sqrt{\frac{-\varepsilon_1}{(\varepsilon_2 + \varepsilon_1)}}$$

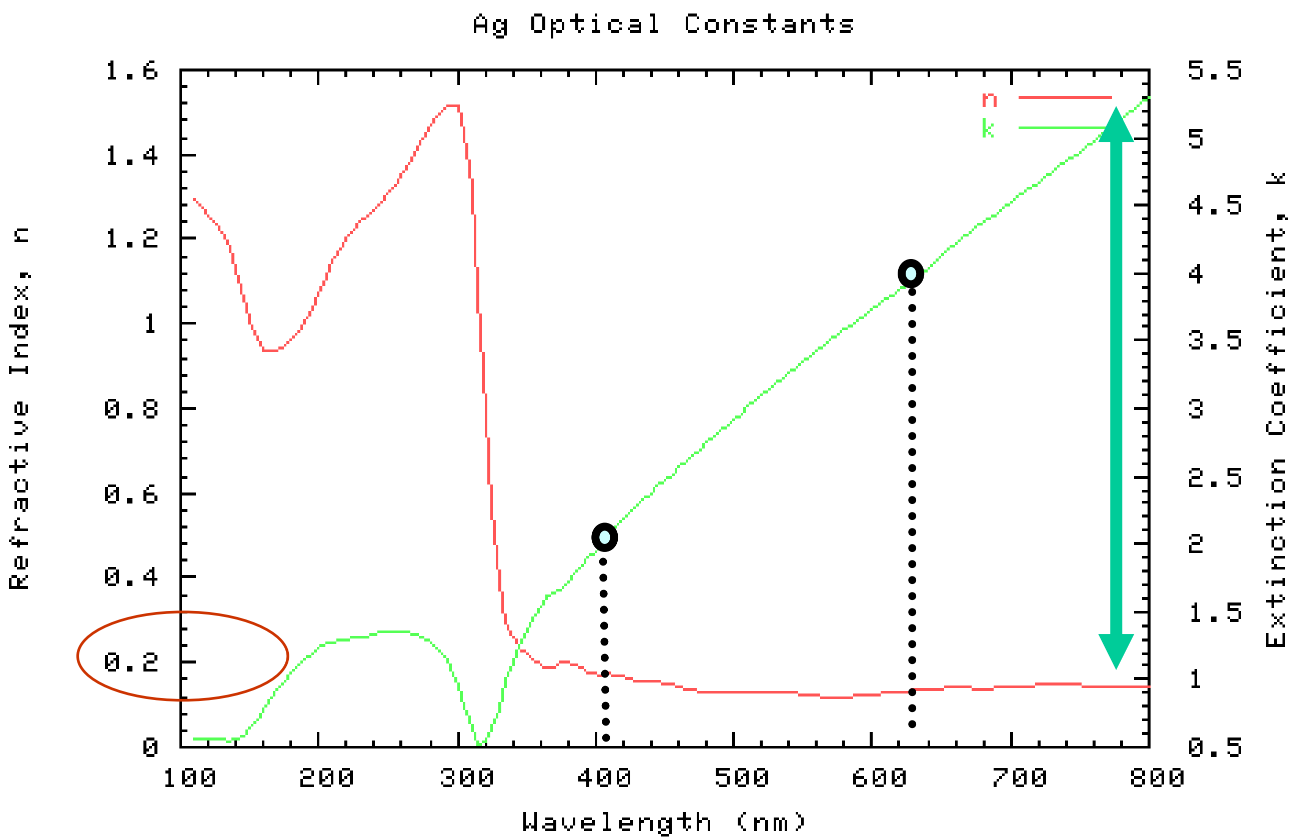
- Constraint
- Substitute definition of v_1 and v_2 to solve for k_z.
- Substitute solution for k_z to find other properties
  - v_1 and v_2 (localization in x)
  - Resolution in z with large k_z
  - Probe height in x

Physical Interpretation

$$v_1 = k_1 \sqrt{\frac{-\varepsilon_1}{(\varepsilon_2 + \varepsilon_1)}} \qquad k_z = k_1 \sqrt{\frac{\varepsilon_2}{(\varepsilon_2 + \varepsilon_1)}} \qquad \frac{v_2}{v_1} = \frac{-\varepsilon_2}{\varepsilon 1}$$

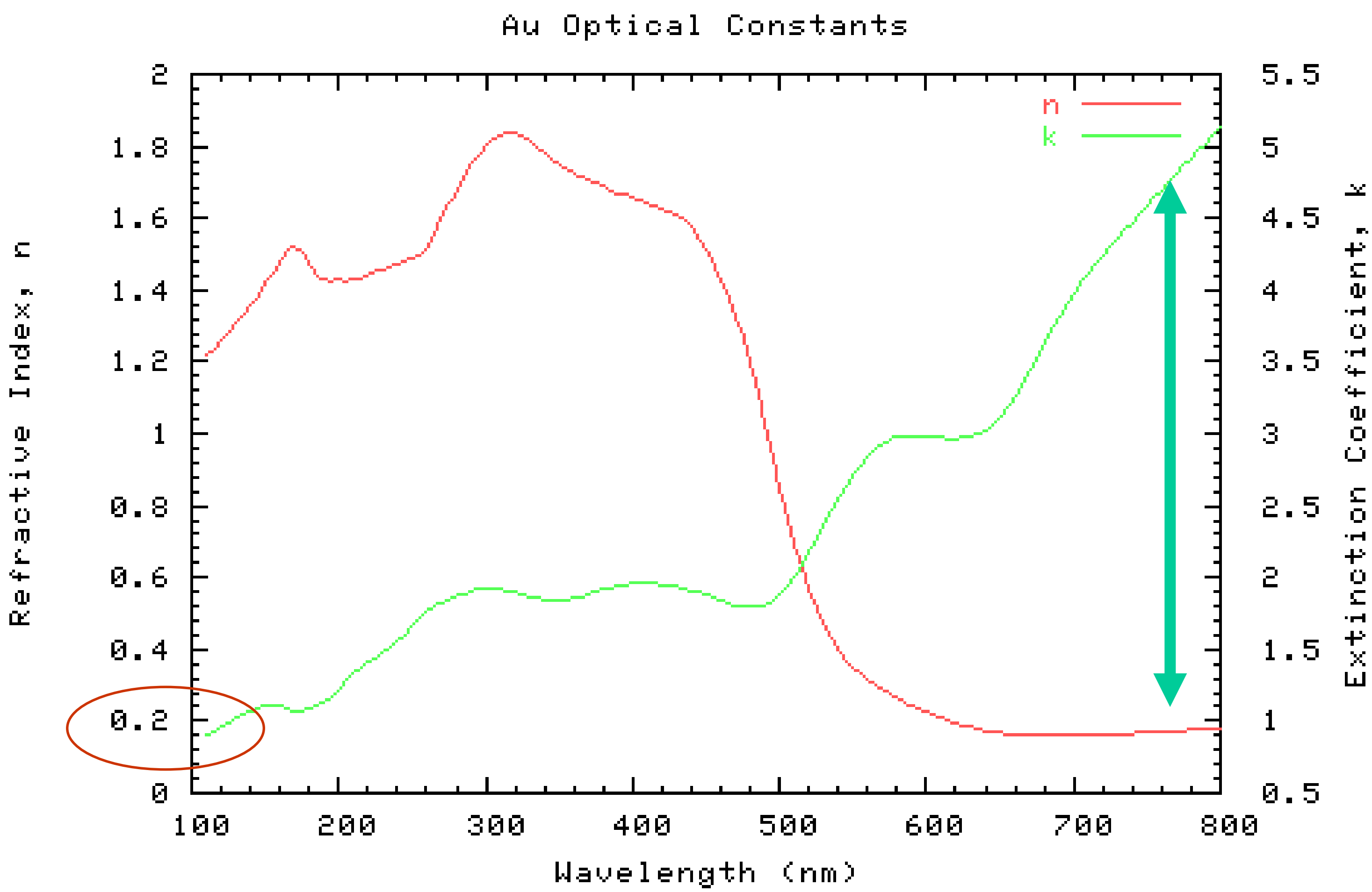
- Want v_1 real so exponential decay away from boundary
- Since ε_1 is real and positive this means that the denominator must go negative.
- This occurs when $\varepsilon_2 < -\varepsilon_1$. (plasmon condition for a single surface to support a guided wave)
- In metals just below their plasma resonance frequency a negative permittivity can occur (Ag, Au, Cu, Al)
- In general k_z is complex and the waves die out in a few microns

Refractive Index of Silver



- http://www.microe.rit.edu/research/lithography/utilities.htm

Refractive Index of Gold



- http://www.microe.rit.edu/research/lithography/utilities.htm

Physical Estimates for Air and Silver

$$\frac{v_2}{v_1} = \frac{-\varepsilon_2}{\varepsilon_1} \approx \frac{K^2}{1}$$

$$n_2 = n_r + iK$$

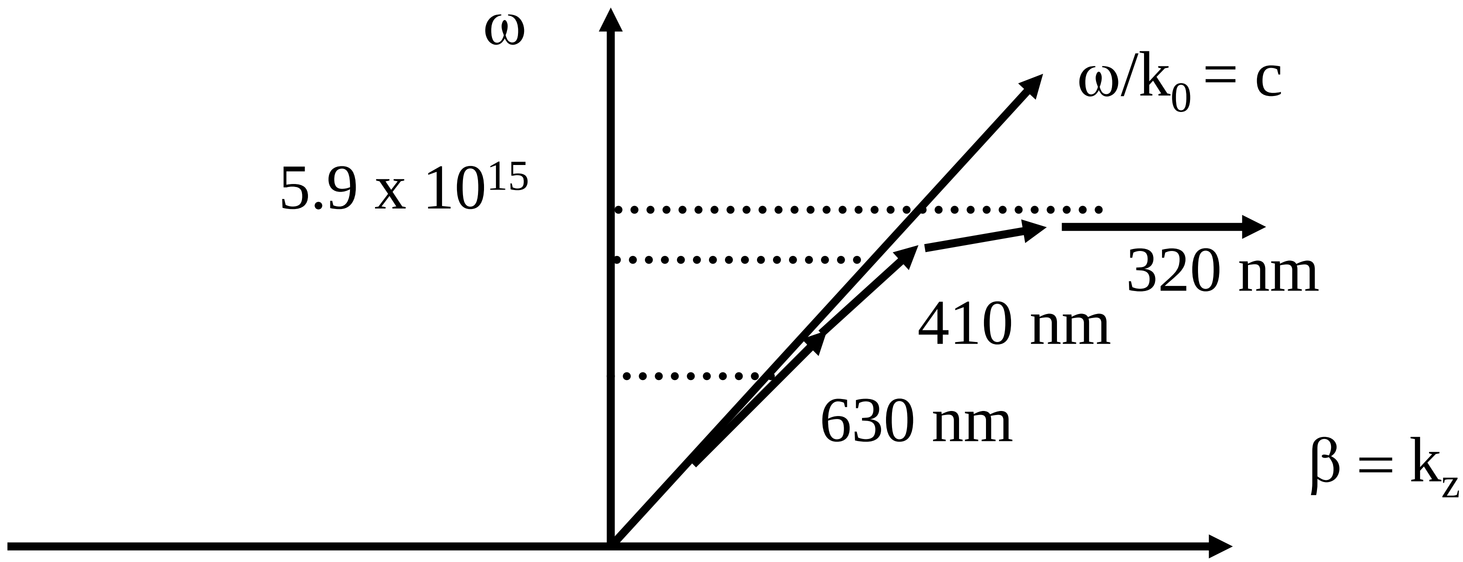
$$n_1 = 1.0$$

$$k_z = k_0\sqrt{\frac{\varepsilon_2}{(\varepsilon_2 + \varepsilon_1)}} \approx k_0\left[\frac{K}{\sqrt{K^2 - 1}} + \frac{in_r}{2(K^2 - 1)^{3/2}}\right]$$

$$l_z/\lambda = \frac{2(K^2 - 1)^{3/2}}{n_r}$$

- Approximately for silver in the visible
  - n_r =0.2, and K = 2 (410 nm) and 4 (630 nm)
  - the velocity is 0.87c and 0.97c,
  - □ $\lambda_{surface}$ = ($2\pi/k_z$) can be smaller than λ_{air} by over 3X => 3X resolution improve
  - the 1/e length is 4λ (1.8 μm) and 295λ (2.9 μm)
  - The 1/e penetration into the metal is about λsurface/5
  - The 1/e penetration into air is 4 to 16 times larger

ω–β Diagram for Plasmon

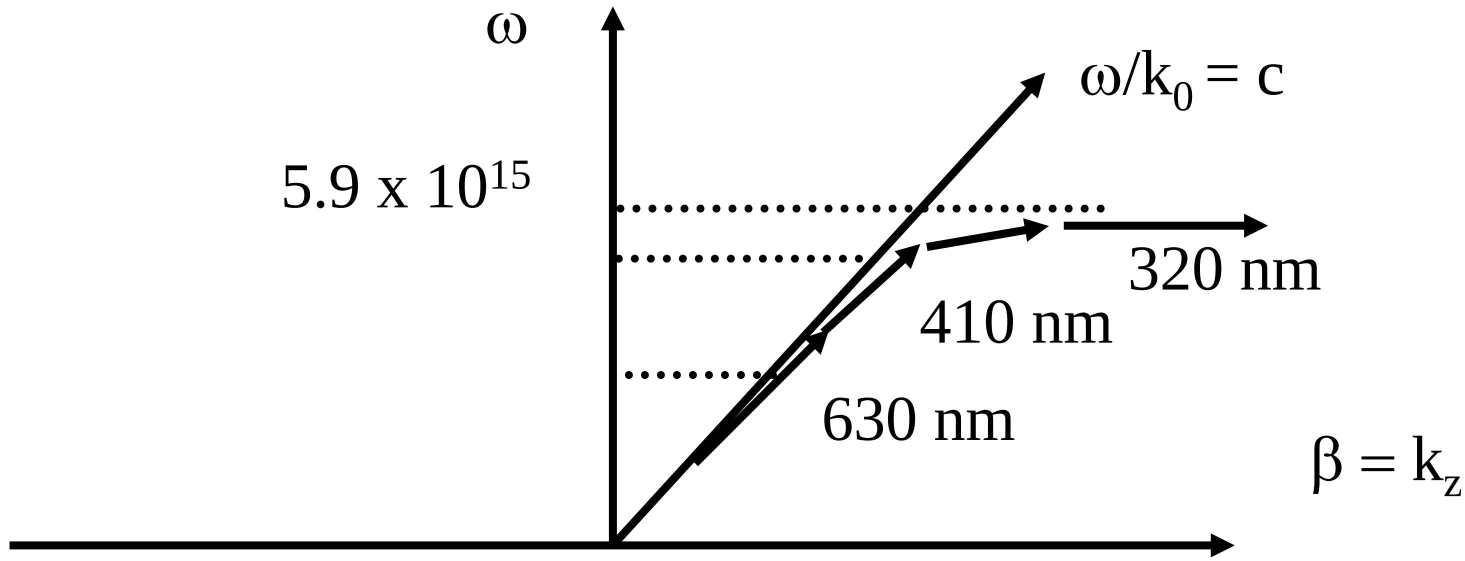


$$f = 2\pi\left(3 \cdot 10^{17}\, nm/s\right)/\lambda_{nm}$$

$$f = \left(1.89 \cdot 10^{18}\, nm/s\right)/320_{nm} = 5.9 \cdot 10^{15}$$

- The plasmons start as frequency is increased
  - close to the speed of light line,
  - become slightly slower, and
  - turn into a very slow wave (horizontal line) at the plasma frequency.

Plasmon Characteristics



- 630 nm: 0.97c, 1/e 46.3λ (29 μm), 16X air
- 410 nm: 0.86c, 1/e 4.1λ (1.8 μm), 4X air
- 320 nm:
  - $k_z = 10k_0$ => 10X resolution
  - But evanescent near fields are 10X faster decay

Near Field Probe Limits

$$R = \frac{\lambda_{surface}}{2\pi} = \frac{1}{k_z} = \frac{1}{k_1\sqrt{\frac{\varepsilon_2}{(\varepsilon_1 + \varepsilon_2)}}}$$

$$h_{1/e} = \frac{1}{v_1} = \frac{1}{k_1\sqrt{\frac{-\varepsilon_1}{(\varepsilon_1 + \varepsilon_2)}}}$$

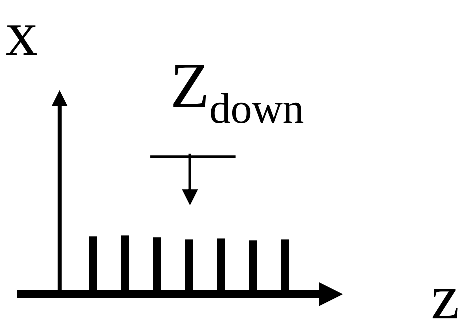
$$h_{1/e} = \sqrt{\frac{-\varepsilon_1}{\varepsilon_2}} R \approx R$$

- Work near the plasmon frequency to make k_z large and resolution smaller than normal by 3 to 10X.
- But the 1/e height reduces by the same amount that the resolution is reduced

Surface Topography Can Aid Guided Waves

Harrington 4.8

Corrugated Surface



Impedance looking down into the corrugations is inductive.

$$Z_{down} = -i\sqrt{\frac{\mu_1}{\varepsilon_1}}\tan k_0 d = -i377\tan k_0 d$$

$$Z_{up} = \frac{-v_1}{i\omega\varepsilon_1}$$

$$k_z = k_0\sqrt{1+\tan^2 k_0 d}$$

- Impedance looking into slot is that of a parallel plate waveguide terminated in a short.
- Slsot must be narrow compared to a wavelength
- Depth must be >5% of wavelength to contribute.
- For plasmons
  - effects might add
  - which wavelength should be used