EE243 Advanced Electromagnetic Theory

Lec # 17 Dielectric Waveguides - Kogelnik

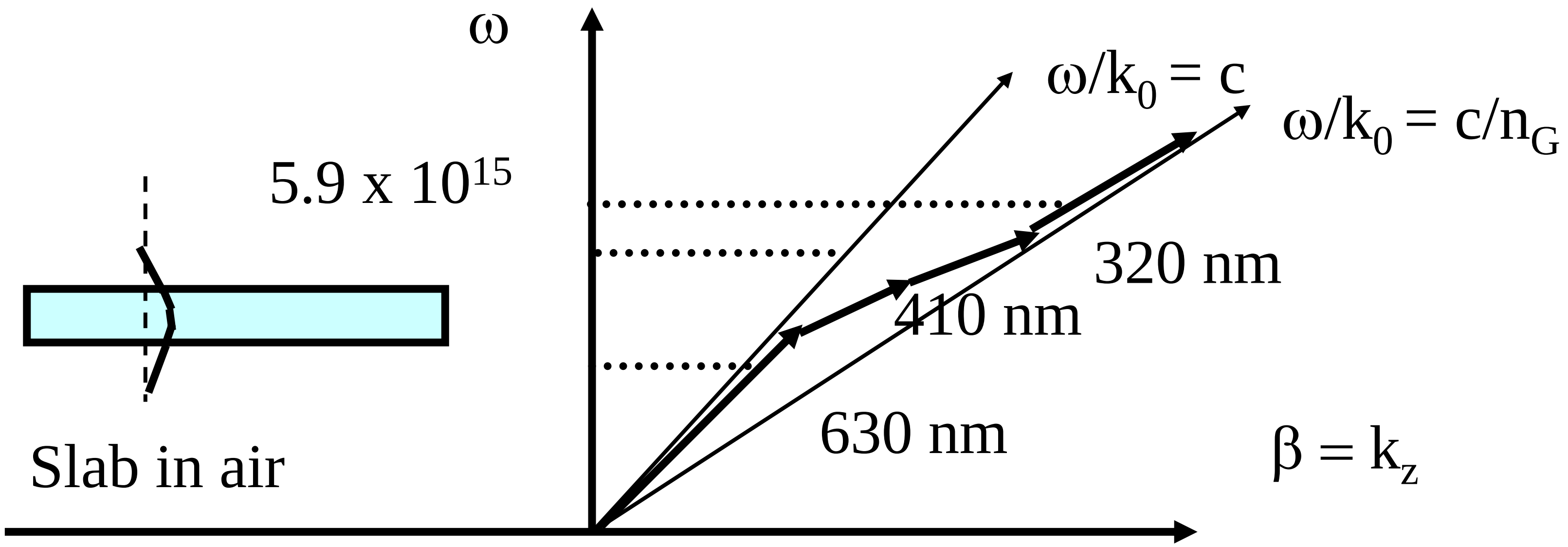
- **Symmetries for time-reversal and direction reversal**
- **Orthogonality of modes with different β's**
- **Mode Expansion including Radiation Modes**
- **Coupled Modes (via Polarization)**

Reading: Kogelnik 34-44, 66-78

Overview

- The very general set of principles developed by Jackson for metallic waveguides also apply to dielectric waveguides of arbitrary cross sections.
  - Propagating modes have real transverse fields and imaginary longitudinal fields
  - Evanescent modes have real valued electric fields and imaginary valued magnetic fields and are 90 out of phase
  - Mode Orthogonality (when propagation constant due to eigenvalues differ)
  - Expansion in Modes including Radiation Modes
  - Coupling between modes due to geometry induced changes in polarization

ω–β Diagram for Dielectric Guide

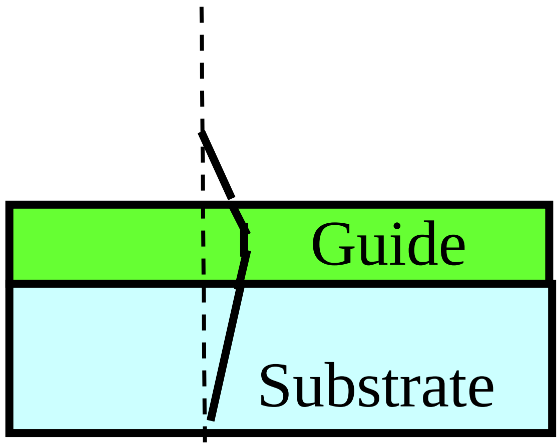


$$f = 2\pi\left(3\cdot 10^{17}\,nm/s\right)/\lambda_{nm}$$

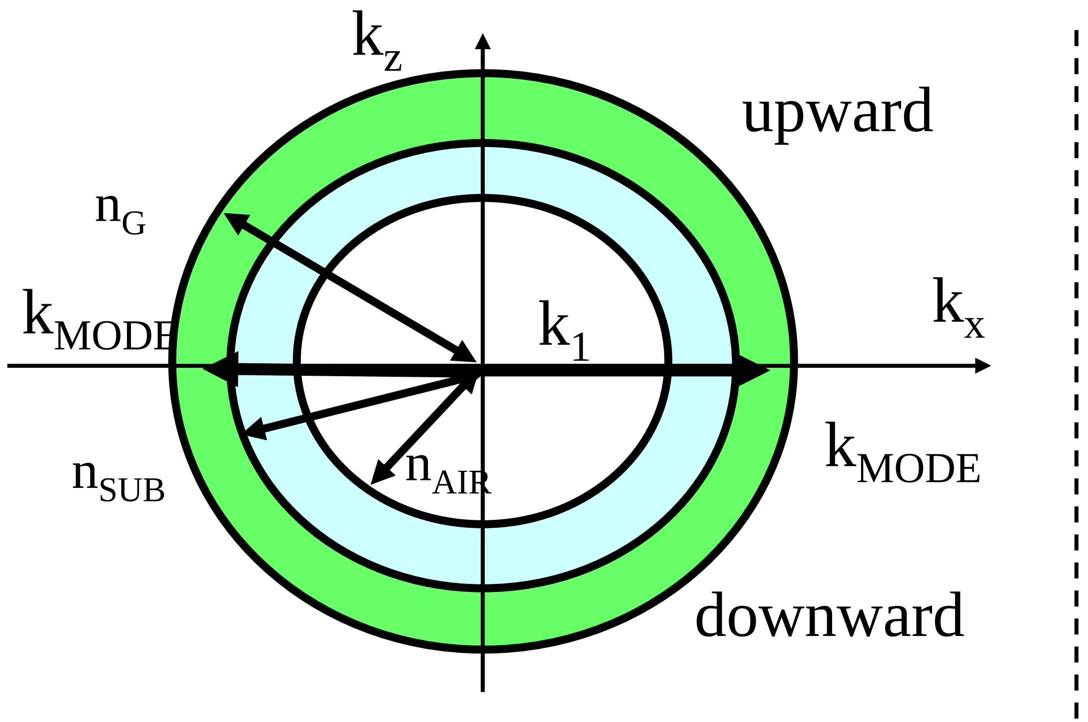
$$f = \left(1.89\cdot 10^{18}\,nm/s\right)/320_{nm} = 5.9\cdot 10^{15}$$

- The mode may starts along n_{AIR} at low frequency
- Then transitions toward the n_{GUIDE}
- And asymptotes to N_{GUIDE}

Dielectric Layer with Substrate



$n_{GUIDE} > N_{SUB} > N_{AIR}$



Mode Properties Follow from Symmetry

$$\overline{E}_2(r,t) = \overline{E}_1(r,-t)$$
$$\overline{H}_2(r,t) = -\overline{H}_1(r,-t)$$
$$\overline{E}_2(r) = \overline{E}_1^*(r,)$$
$$\overline{H}_2(r) = -\overline{H}_1^*(r)$$

$$\overline{E}_{t2}(z) = \overline{E}_{t1}(-z)$$
$$\overline{H}_{t2}(z) = -\overline{H}_{t1}(z)$$
$$\overline{E}_{2z}(z) = -\overline{E}_{z1}^*(z)$$
$$\overline{H}_{z2}(z) = \overline{H}_{z1}^*(z)$$

- Two Symmetries
  - Time-reversal in Maxwell's Equation reverses H but not E
  - Propagation direction reversal with e^{jwt} reverses E longitudinal and H transverse
- Result
  - Propagation real valued transvers, imy valued longitudinal
  - Evanescent real valued E and imaginary valued H

Orthogonality of the Modes

Kogelnik 2.25:

Derivation similar to that for Lorentz reciprocity

- use E x H cross product for two modes
- reverse order take complex conjugate and add,
- Apply divergence theorem to cross section, argue integral over contour at infinity is zero, and
- finally apply to mode in reverse direction and add

Result the transverse E crossed Transverse H integrated over the cross section is zero when the propagation constant of the two modes differs.

Orthogonality of Modes (Cont.)

$$\nabla_t\left(\overline{E}_\nu \times \overline{H}_\mu^* + \overline{E}_\mu^* \times \overline{H}_\nu\right) - j\left(\beta_\nu - \beta_\mu\right)\left(\overline{E}_{t\nu} \times \overline{H}_{t\mu}^* + \overline{E}_{t\mu}^* \times \overline{H}_{t\nu}\right) = 0$$

- Apply divergence theorem to cross section, argue integral over contour at infinity is zero, and
- finally apply to mode in reverse direction and add
- Result the transverse E crossed Transverse H integrated over the cross section is zero when the propagation constant of the two modes differs.
- Apply to find mode amplitudes produced by E_{TAN} and H_{TAN} on a cross sectional plane

$$a_\nu = \iint_\infty dxdy\left(\overline{E}_t \times \overline{H}_\nu^* + \overline{E}_\nu^* \times \overline{H}_t\right)$$

$$b_\nu = \iint_\infty dxdy\left(\overline{E}_t \times \overline{H}_\nu^* - \overline{E}_\nu^* \times \overline{H}_t\right)$$

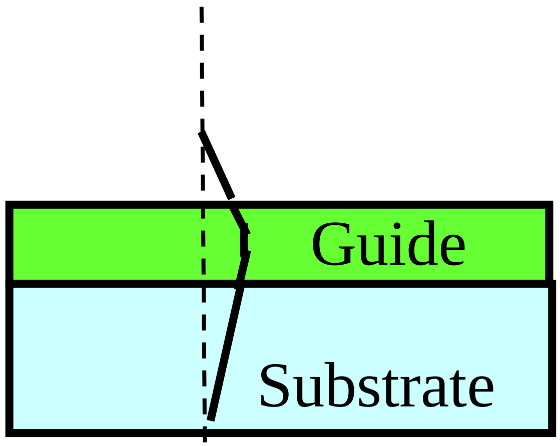
Modal Representation

$$\overline{E}_t(x,y) = \sum_{\nu}\sum_{\mu} \alpha_{\nu\mu}\overline{E}_{\nu\mu}(x,y) + \int\int_0^{\infty} d\mu d\nu a(\nu,\mu)\overline{E}(\mu,\nu;x,y)$$

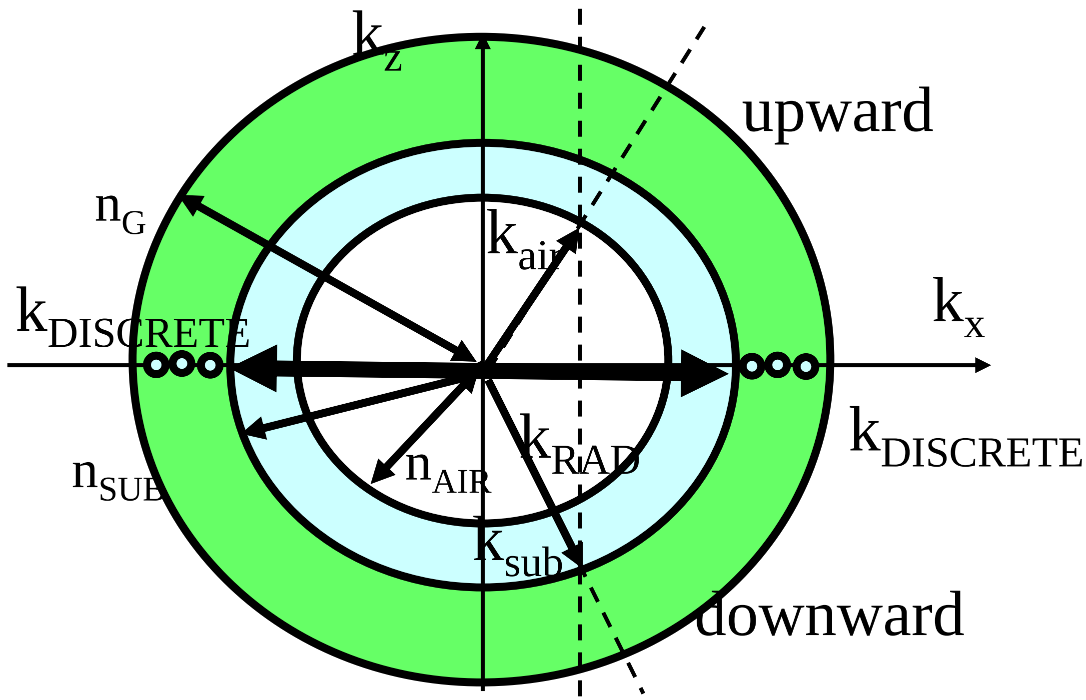
$$\overline{H}_t(x,y) = \sum_{\nu}\sum_{\mu} \alpha_{\nu\mu}\overline{H}_{\nu\mu}(x,y) + \int\int_0^{\infty} d\mu d\nu a(\nu,\mu)\overline{H}(\mu,\nu;x,y)$$

The summation is over the discrete set of guided modes.
The integral is over the continuous set of plane wave like radiation modes.
Note that the sum has a double index because the cross sections has n-1 = 2 directions.
In addition the sume over TE and TM is also implicit.

Dielectric Layer Modes



$n_{GUIDE} > N_{SUB} > N_{AIR}$



- Discrete guided modes
- Continuum of radiating modes in air and substrate
- TE and TM cases not separated

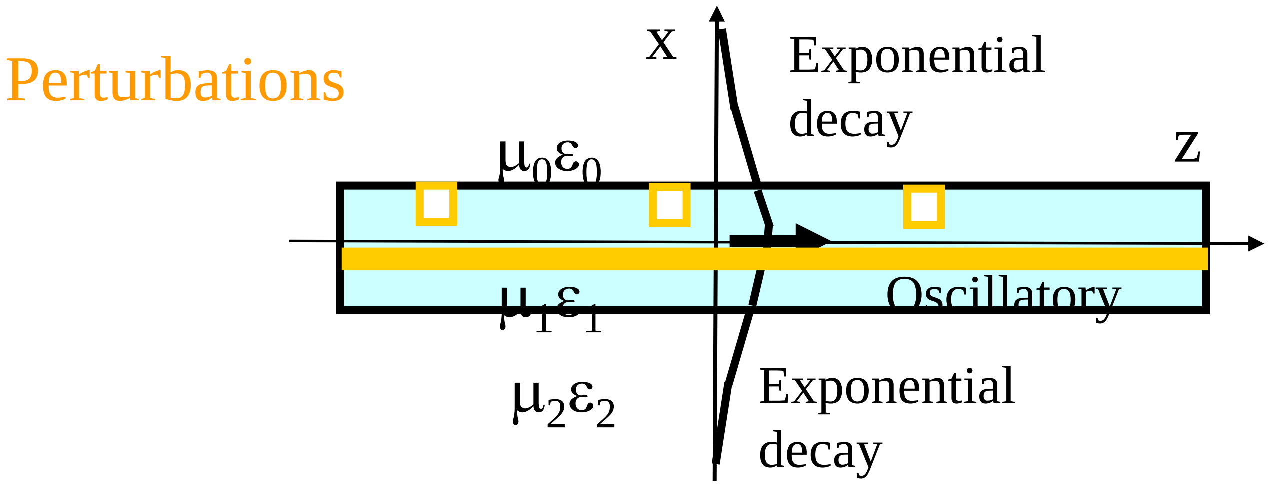
Modal Expansion Coefficients

- Both the discrete and continuous modes can be normalized to give the Kronecker delta and delta function.
- The modes independently carry power
- The amplitude of a given mode can be found by multiplying by the model spatial variation and integrating over the cross section.
- The transverse E and H over a given cross-section can be broken down in to forward and reverse modes by integrating combinations such as $E_F x H_M^* + E_M^* x H_F$ over the cross section. Here M is a mode and F is an arbitrary field on the cross section.

Power Flow and Stored Energy

- The poynting vector is integrated over the cross-section
- The definition of Stored energy and group velocity are used Power = $v_g W$
- The situation is specialized to lossless and $v_p/v_g = (W_t+W_z)(W_t-W_z)$

Coupled-Mode Concept



- Consider a geometry or material change for which there is an additional source of excitation with complex polarization amplitude P
- This polarization can be due to the E field from a strong mode hitting a region of missing or added dielectric.
- This polarization source then drives other modes.
- This sourcing of other modes can occur simultaneously among modes and is know as coupled modes.
- The distribution of the polarization can also be made periodic in distance along the guide to couple in our out planewaves.

Coupled Mode Formalism

- Start a time-average Lorentz Reciprocity like formulation with two sources and two sets of fields
- Integrate over the cross-section
  - treat the left side with the divergence theorem to get a cross-section integral of the rate of change of the added double cross product of the two sets of transverse fields
  - the right hand side is the cross sectional integral of the polarization times the fields
- Substitute the transverse field components assuming a direction for each and that their amplitudes a and b are slowly varying.
- For the same direction

$$a'_\mu + j\,\beta_\mu a_\mu = -j\omega \text{ integral PE}$$
$$b'_\mu + j\,\beta_\mu b_\mu = +\,j\omega \text{ integral PE}$$

Coupled Mode Formalism (Cont.)

$$\nabla\left(\overline{E}_1 \times \overline{H}_2^* + \overline{E}_2^* \times \overline{H}_1\right) = -j\omega \overline{P}_1 \cdot \overline{E}_2^* + j\omega P_2^* \cdot \overline{E}1$$

$$\overline{P} = \Delta\varepsilon \overline{E}$$

$$\overline{P}_i = \Delta\varepsilon_{ij} \overline{E}_j$$

$$\frac{da_\mu}{dz} + j\beta_u a_\mu = -j\omega \iint_\infty dxdy \overline{P}_{TOT} \cdot \overline{E}_\mu^*$$

$$\frac{db_\mu}{dz} - j\beta_u b_\mu = j\omega \iint_\infty dxdy \overline{P}_{TOT} \cdot \overline{E}_{-\mu}^*$$

Coupled Mode Situations

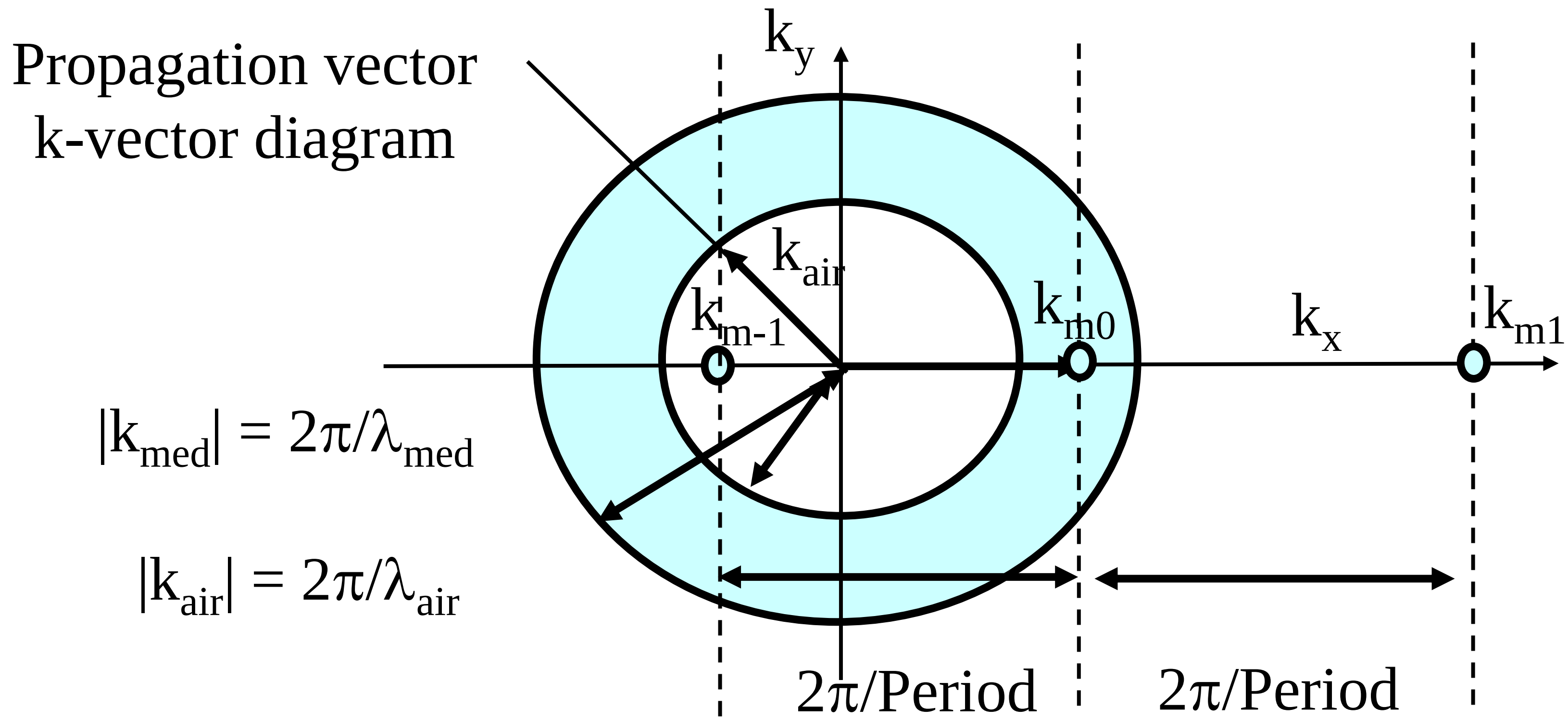
Uniform Guides

- Waves in same or opposite directions
- Propagating or evanesent modes

Periodic Couplers

- Surface height or dielectric material variation
- Coupling becomes a periodic function that introduces new modes with $\beta_n = \beta_o + 2\pi/\text{Period}$
- These k-vectors may be in the range $-k_0$ to $+ k_0$ and give rise to plane waves that propagate away from the structure.

Periodic Wave Vectors



- The mode k-vector is larger than k_O and smaller than k_G
- The periodic coupling creates new k-vectors spaced by 2π/Period
- The new k-vectors within the k_0 circle correspond to radiation waves
- Move upward vertically from k_{m-1} to find the k_y and angle.