EE243 Advanced Electromagnetic Theory

Lec # 18 Coupled Mode Theory

- **Application Examples**
  - **Directional Couplers**
  - **Contra-Directional Couplers**
  - **Distributed Bragg Reflectors**
  - **Beam - Guide Couplers**
  - **Grating Mirrors**
- **Mode by Mode Formulation**
- **Expansion of Coupling Polarization (Et, Ez)**
- **Solutions for Uniform Couplers**

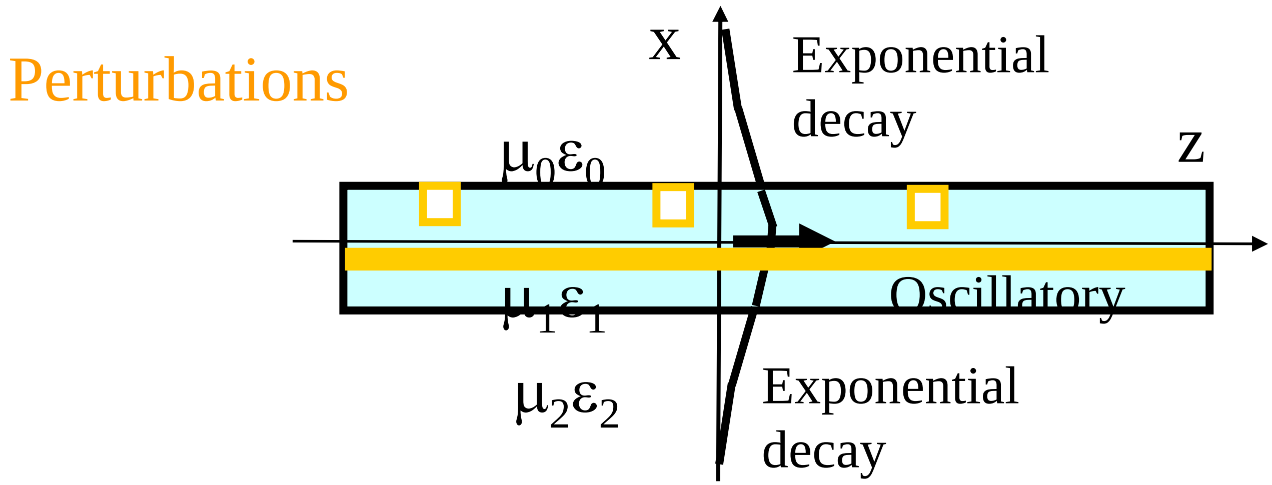
Reading: Kogelnik 2.6

Overview

Coupling between modes due to uniform and periodic geometry induced changes in polarization lead to slowly changing amplitudes of discrete and radiation modes that can be modeled by coupled first order differential equations known as coupled mode theory.

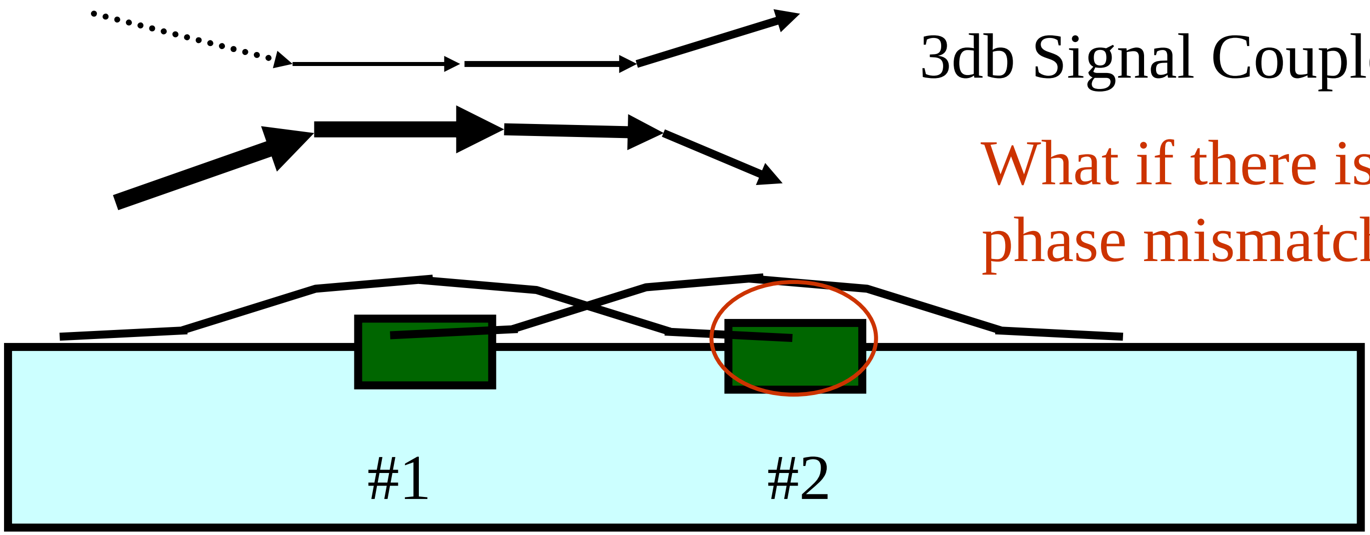
- Modes and Reciprocity give a general formulation
- Synchronism is in terms of group velocity of energy flow
- Coupling can be factored to Et and Ez and misalignment of k-vectors can be included
- Leads to
  - Real propagation constants (growth or decay)
  - Imaginary propagation constants (stop band)

Coupled-Mode Concept



- Consider a geometry or material change for which there is an additional source of excitation with complex polarization amplitude P
- This polarization can be due to the E field from a strong mode hitting a region of missing or added dielectric.
- This polarization source then drives other modes.
- This sourcing of other modes can occur simultaneously among modes and is know as coupled modes.
- The distribution of the polarization can also be made periodic in distance along the guide to couple in our out planewaves.

Directional Coupler



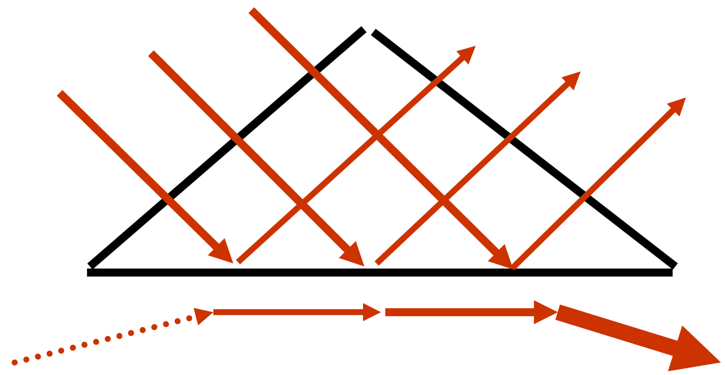
3db Signal Coupler

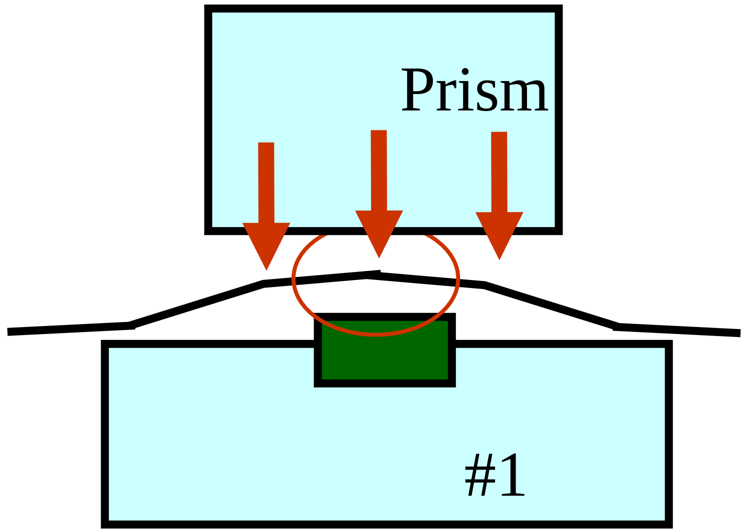
What if there is a phase mismatch?

- Waves on adjacent guides spillover laterally and induce additional polarization when they encounter dielectric material not in included in their normal structure definition.
- This additional polarization acts a source that excites the modes on the second structure.
- This can be used to design 3db directional couplers.
- Modes on the composite structure (super modes) can also be used.

Distributed Beam to Guide Coupler

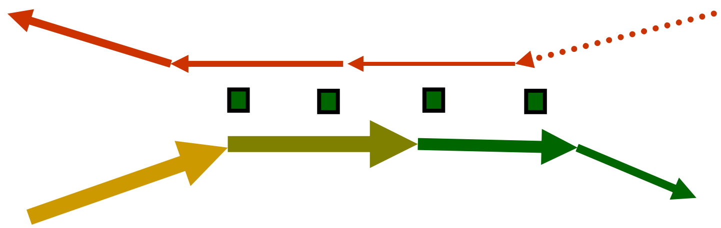
Prism Tunneling Coupler



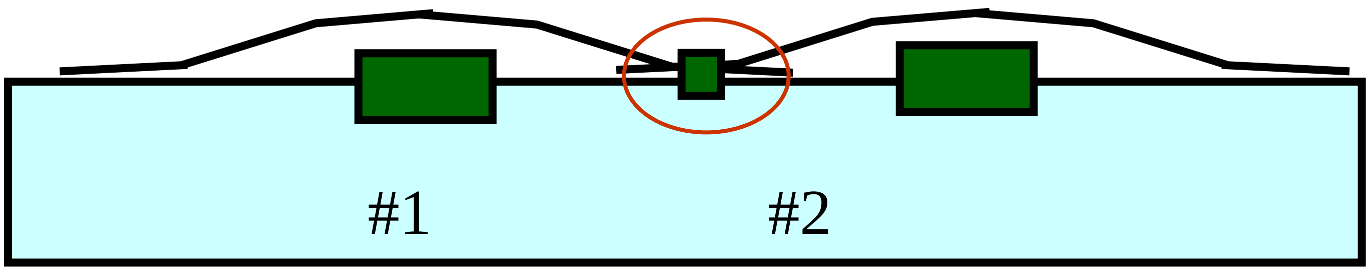


- Prism k-vector matches incident plane-wave to the guide wave .
- The tunneling fields excite polarization in the dielectric wave guide.
- This couples the light into the surface guided mode for signal processing
- This tunneling process causes the guided wave to leak back into air (Leaky Wave Phenomena).

Channel Drop Filter

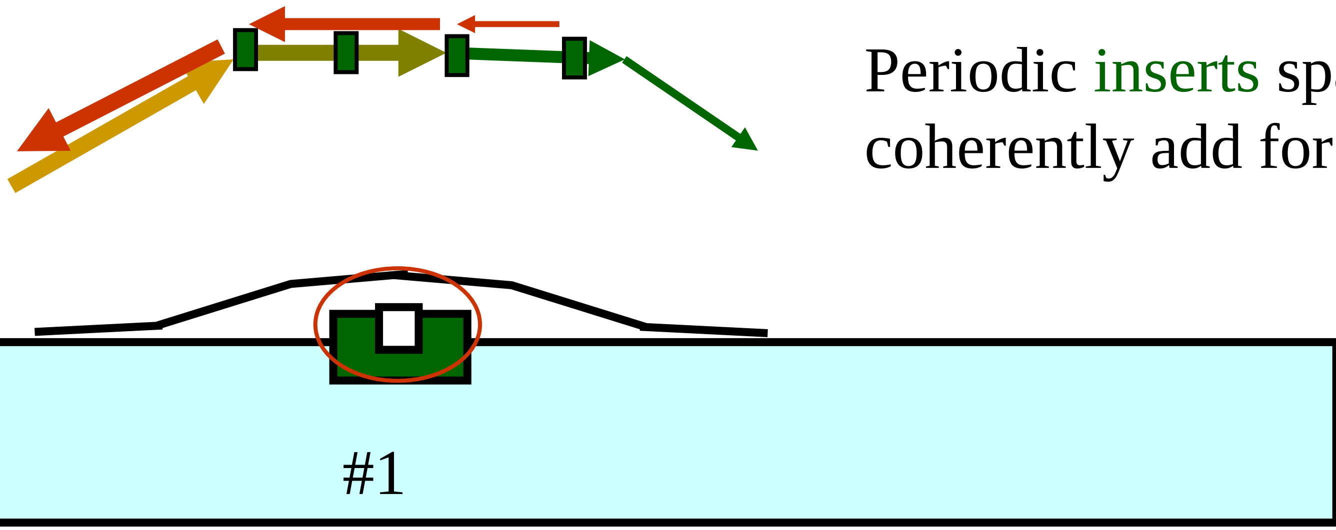


Periodic inserts spaced to coherently add for red



- Waves on adjacent guides spillover laterally to programmed dielectric inserts not in included in their normal structure definition.
- This additional polarization acts a source that excites reverse modes in second structure structures
- This can be used to design frequency (color) dependent couplers to separate out frequency components (colors).

Stop Band (Photonic Crystal)

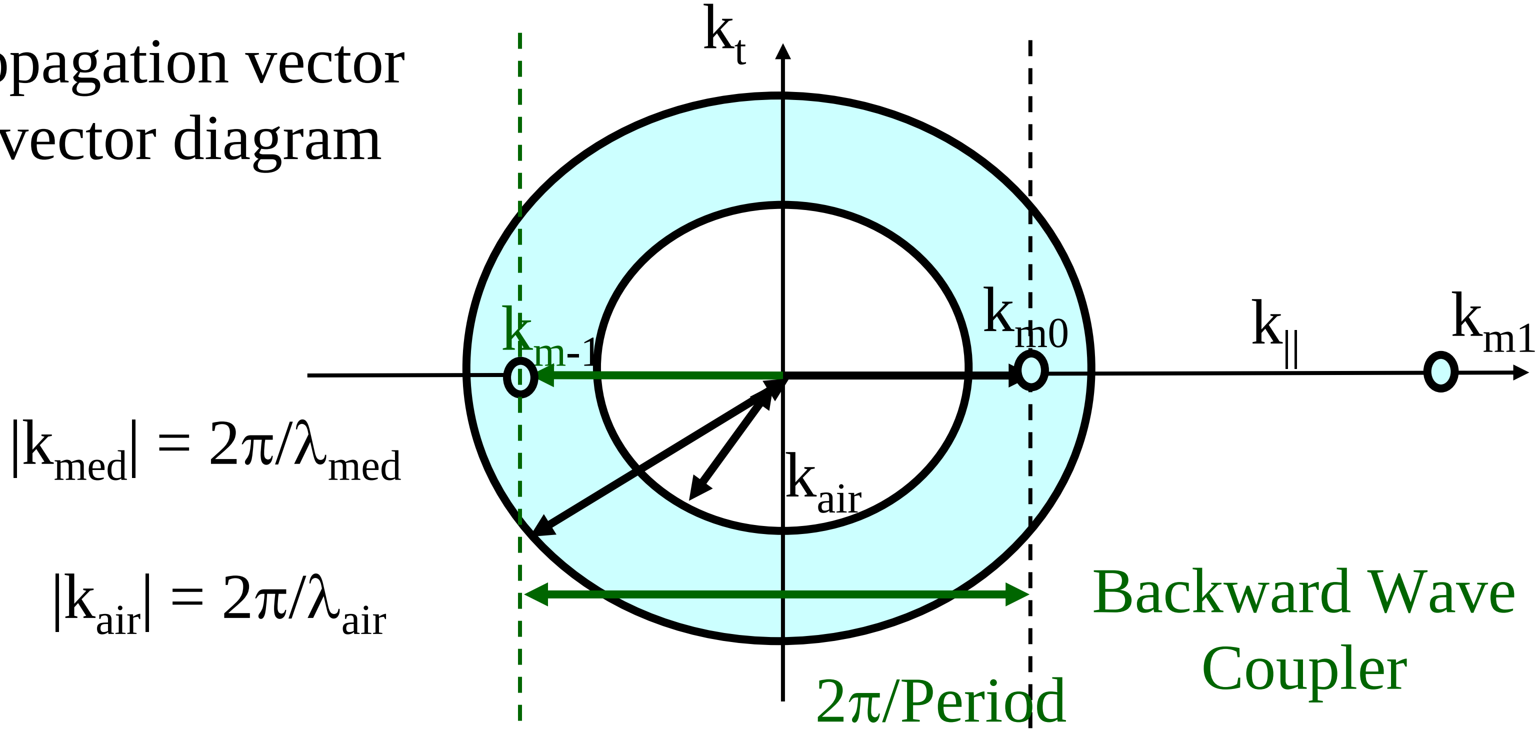


Periodic inserts spaced to coherently add for red

- Wave on guide encounters periodic insert not in included in their normal structure definition.
- This additional polarization acts a source that excites reverse modes in the same structure
- This can be used to design a frequency (color) dependent stop band.
- This is a 1-D form of a photonic crystal

Periodic Wave Vectors: Stop Band

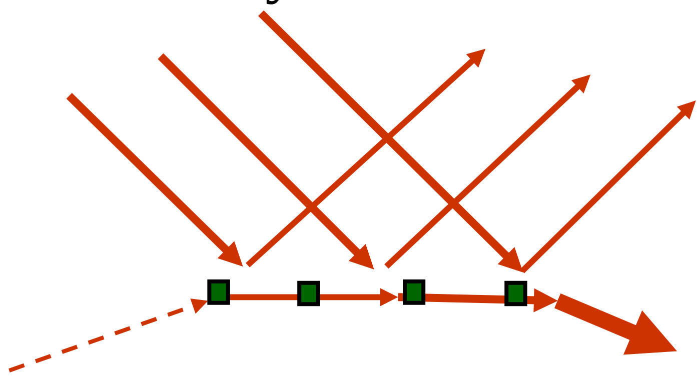
Propagation vector
k-vector diagram



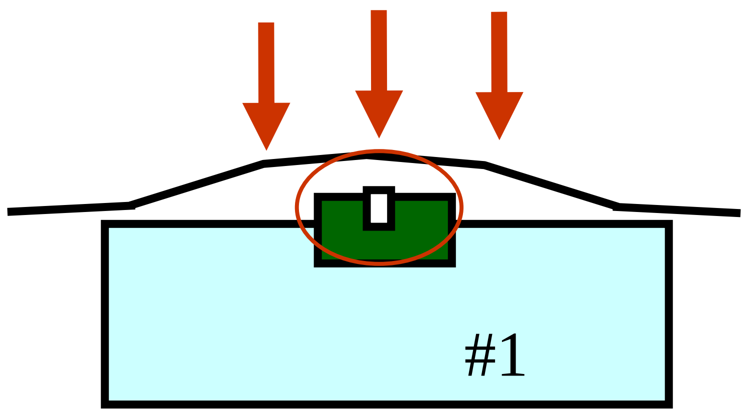
- The mode k-vector is larger than k_O and smaller than k_G
- The periodic coupling creates new k-vectors spaced by 2π/Period
- The new k-vectors match the k-vector of a backward propagating guided wave

Distributed Bragg Beam Coupler

Periodic inserts spaced to coherently add for red

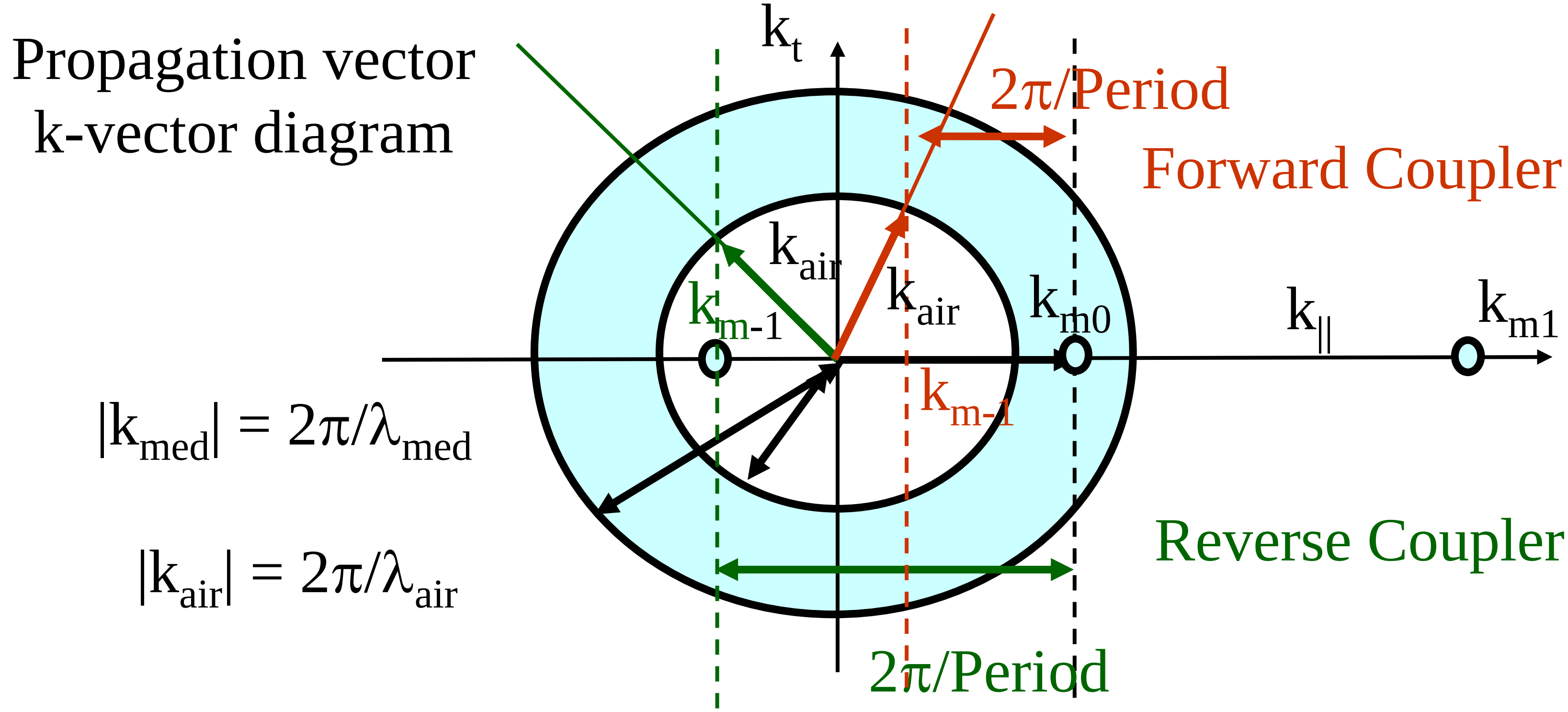


Plane Wave



- Plane wave excites polarization in periodic dielectric inserts
- The period of these inserts k-vector maps an excitation component to match the k-vector of the mode and thus excite the mode
- This can be used to couple a beam into a surface wave in either the forward or reverse direction.

Periodic Wave Vectors

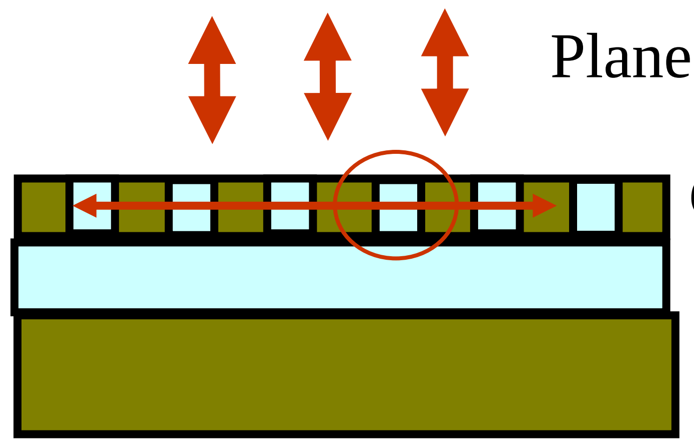


- The mode k-vector is larger than k_O and smaller than k_G
- The periodic coupling creates new k-vectors spaced by 2π/Period
- The new k-vectors within the k_0 circle correspond to radiation waves
- Forward coupling uses a larger period than backward coupling.

Grating Mirror

Reflectivity > 0.99 over 30% bandwidth for TM (> reflectivity of Al)

Periodic inserts spaced to simultaneously produce beam coupling and guided wave stop band.



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Grating Mirror

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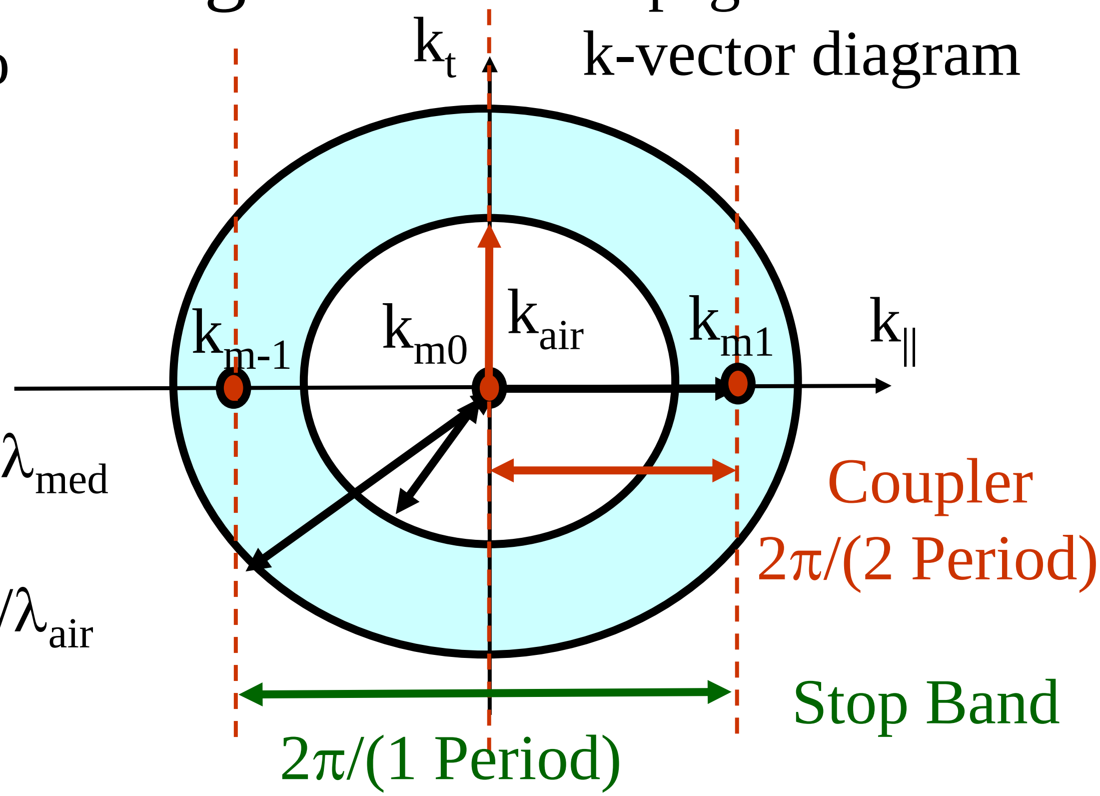
Propagation vector k-vector diagram

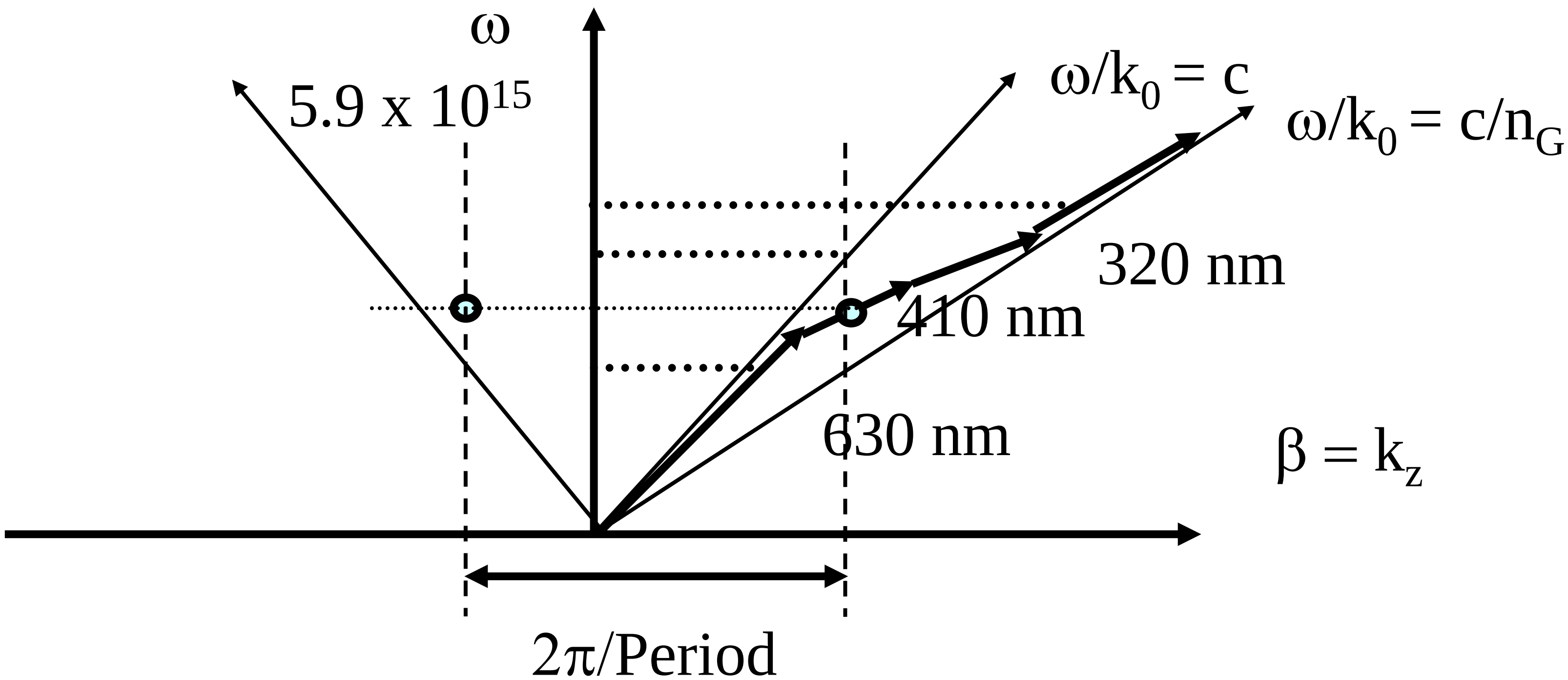


$|k_{med}| = 2\pi/\lambda_{med}$

$|k_{air}| = 2\pi/\lambda_{air}$

- The double period of SiO_2 creates new k-vectors spaced by 2π/(2 Period) that couple to the mode.
- The single period of SiO_2 creates a stop band for the forward and backward modes 2π/(2 Period).
- The double period of SiO_2 couples the energy back into reflection.

Group Velocity is What is Matched



- Energy flow is what is being tracked (as we do not consider evanescent waves and their k-vectors)
- Reciprocity theorem type derivation of cross coupling will need to be averaged along over a band of frequencies to allow the signals start and stop.
- (The Poynting Vector can be written as $v_g W$)

Excitation of Waveguide Modes

Kogelnik 2.6

$$\nabla \cdot \left(\overline{E}_1 \times \overline{H}_2^* + \overline{E}_2^* \times \overline{H}_1 \right) = -j\omega \overline{P}_1 + j\omega \overline{P}_2^*$$

$$\iint_\infty dxdy \frac{\partial}{\partial z} \left(\overline{E}_1 \times \overline{H}_2^* + \overline{E}_2^* \times \overline{H}_1 \right)_z = -j\omega \iint_\infty dxdy \overline{P} \cdot E_2^*$$

- No magnetization and only one polarization source that accounts for the material perturbation of the dielectric guide P1(x,y,z) (This is ΔP and is only the change in P)
- Use a reciprocity type cross product formulation between a full set of modes with the above source and a single test mode.
- Carry out the integral over the cross section and use divergence theorem.

Excitation of Waveguide Modes (Cont.)

Kogelnik 2.6

$$\overline{E}_{1t} = \sum (a_v + b_v)\overline{E}_{tv}$$

$$\overline{H}_{1t} = \sum (a_v - b_v)\overline{H}_{1v}$$

$$\overline{E}_2 = \overline{E}_u e^{-j\beta_u z}$$

$$\overline{H}_2 = \overline{H}_u e^{-j\beta_u z}$$

$$a_u' + j\beta_u a_u = -j\omega \iint_\infty dxdy\overline{P} \cdot E_u^*$$

$$\overline{E}_2 = \overline{E}_{-u} e^{+j\beta_u z}$$

$$\overline{H}_2 = \overline{H}_{-u} e^{+j\beta_u z}$$

$$b_u' - j\beta_u b_u = j\omega \iint_\infty dxdy\overline{P} \cdot E_{-u}^*$$

- Z component of cross product only contains transverse fields
- Plug in expansion for all modes and transverse fields.
- Plug in z variation of the test mode going in +z
- Plug in z variation of the test mode going in -z

Excitation of Waveguide Modes (Cont. 2)

Kogelnik 2.6

$$a_u = A_u e^{-j\beta_u z}$$

$$b_u = B_u e^{+j\beta_u z}$$

$$A_u' = -j\omega \iint_\infty dxdy \overline{P} \cdot E_u^* e^{+j\beta_u z}$$

$$B_u' = j\omega \iint_\infty dxdy \overline{P} \cdot E_{-u}^* e^{-j\beta_u z}$$

$$\overline{P} = \Delta\varepsilon \overline{E}$$

$$\overline{P}_i = \Delta\varepsilon_{ij} \overline{E}_j$$

$$\Delta\varepsilon_{ij} = \varepsilon_0 X_{ijk} \overline{E}_k$$

- Separate slowly varying and normal phase variation in z
- Consider polarization as caused by Δε as weighted by influence of field strength of all other modes specific to that location.
- Can also include off-axis terms due to electro-optic (and other nonlinear effects).