EE243 Advanced Electromagnetic Theory

Lec # 21 Radiation

Fixed Slides 2, 3, 5, 9

- **Vector Potential Formulation for Finding Fields**
- **Near and Far Field Limits**
- **Multipole Components**
- **Far-Field Applications**

Reading: Jackson Chapter 9

Overview

Charges in motion radiate. The radiation can be found by evaluating the vector potential and then taking the curl to get H and then another curl to get E. The radiating fields have E and H transverse to the outward direction and E/H is proportional to the impedance of free space and decrease as 1/r. Various oscillating charge moments create electric and magnetic dipoles and multipoles and each has a characteristic radiation pattern. These moments help characterize radion from small holes and slots. In the far-field the radiation pattern is the Fourier transform of the current distribution.

Localized Oscillating Source

$$\rho(\bar{x},t) = \rho(\bar{x})e^{-i\omega t}$$

$$\bar{J}(\bar{x},t) = \bar{J}(\bar{x})e^{-i\omega t}$$

$$\bar{A}(\bar{x}) = \frac{\mu_0}{4\pi}\int \bar{J}(\bar{x}')\frac{e^{ik|\bar{x}-\bar{x}'|}}{|\bar{x}-\bar{x}'|}d^3x'$$

$$\bar{H} = \frac{1}{\mu_0}\nabla \times \bar{A}$$

$$\bar{E} = \frac{iZ_0}{k}\nabla \times \bar{H} - \nabla\Phi$$

$$\Phi(\bar{x}) = \int \frac{\rho(\bar{x}')}{|\bar{x}-\bar{x}'|}d^3x'$$

- Electric monopole part of the potential (and fields) of a localized source is of necessity static.
- Hence the vector potential is sufficient to describe the radiating field.

Radiating Zones

- Near (Static) Zone d << r << λ
  - Exponential is unity, => static and no radiation
- Intermediate (Induction) Zone d << r ~ λ
  - General expansion required
- Far (Radiation) Zone d << λ << r
  - Approximate denominator as 1/r
  - Approximate exponential as quadratic => Fresnel
  - Or Approximate exponential as linear => Fraunhoffer

$$|\,\overline{x} - \overline{x}' \approx r - \overline{n}\cdot\overline{x}'$$

$$\overline{A}(\overline{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \overline{J}(\overline{x}')e^{ik(\overline{n}\cdot\overline{x}')}d^3x'$$

Approximated by projection parallel to n In Fourier transform

Electric Dipole Fields and Radiation

$$\overline{A}(\overline{x}) = \frac{\mu_0}{4\pi}\int \overline{J}(\overline{x}')\frac{e^{ik|\overline{x}-\overline{x}'|}}{|\overline{x}-\overline{x}'|}d^3x' = -\frac{i\mu_0\omega}{4\pi}\,\overline{p}\,\frac{e^{ikr}}{r}$$

$$\overline{p} = \int \overline{x}'\rho(\overline{x}')d^3x'$$

$$\overline{H} \approx \frac{ck^2}{4\pi}(\overline{n}\times\overline{p})\frac{e^{ikr}}{r}$$

$$\overline{E} = Z_0\overline{H}\times\overline{n}$$

- Approximate exponent as constant
- Apply iωρ = Div J
- Integrate by parts
- Fields are perpendicular to n and perpendicular to each other
- Both E and H decrease as 1/r

Poynting Vector for Electric Dipole

$$\frac{dP}{d\Omega} = \frac{1}{2}\mathrm{Re}\left[r^2\overline{n}\cdot\overline{E}\times\overline{H}^*\right]$$

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2}k^4\left|(n\times p)\times n\right|^2$$

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2}k^4\left|\overline{p}^2\right|\sin^2\theta$$

$$P = \frac{c^2 Z_0}{12\pi}k^4\left|\overline{p}^2\right|$$

- Poynting vector gives power density per unit solid angle
- Substitute for fields
- Sin squared polar angle
- Integrate over azimuthal and polar angles to get net power radiated.

Radiation for Short Wire Antenna

$$p = \frac{iI_0 d}{2\omega}$$

$$\frac{dP}{d\Omega} = \frac{Z_0 I_0^2}{128\pi^2}(kd)^2 \sin^2\theta$$

$$P = \frac{Z_0 I_0^2}{48\pi^2}(kd)^2$$

$$R_{RAD} \approx 5(kd)^2 \, ohms$$

- Assume current is triangular –d/2 to d/2
- Evaluate dipole moment
- Evaluate Power
- Interpret coefficient of maximum current squared over 2 as resistance
- Cell phone at 2 GHz and 5 cm high has impedance of about 15 ohms

Magnetic Dipole and Electric Quadrapole Fields

- Approximate exponential phase in integral for A over source by more terms in a Taylor series
- Constant => electric dipole **p**
- First => magnetic dipole **m** (circulating current) plus quadrapole $Q_{\alpha\beta}$
- Second = further monopoles

Characterizing Small Sources

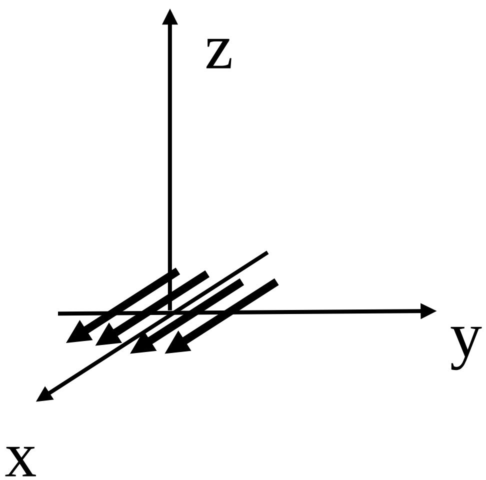
$$\overline{p}_{eff} = \varepsilon \overline{n} \int \left(\overline{x} \cdot \overline{E}_{\tan} \right) da = -\frac{4\varepsilon_0 R^3}{3} \overline{E}_0$$

$$\overline{m}_{eff} = \frac{2}{i\mu\omega} \int \left(\overline{n} \times \overline{E}_{\tan} \right) da = 2\int \overline{x} \left(\hat{n} \cdot H_0 \right) da = \frac{8R^3}{3} \overline{H}_0$$

$$A_\lambda^\pm = i \frac{\omega Z_\lambda}{4} \left[\overline{p}_{eff} \cdot \overline{E}_\lambda^\pm (0) - \overline{m}_{eff} \cdot \overline{B}_\lambda^\pm (0) + \ldots \right]$$

- Many types of sources small sources
  - Probes, current loops, holes in metal screens,
- When sources are smaller than a wavelength they can be approximated by their electric and magnetic dipole moments
- The source contributions to producing fields can be evaluated using the reaction theorem
- Above is an example for waveguide apertures of radius R

Aperture Radiation



$$|\bar{x}-\bar{x}' \approx r-\bar{n}\cdot\bar{x}'$$

$$\bar{A}(\bar{x})=\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \bar{J}(\bar{x}')e^{ik(\bar{n}\cdot\bar{x}')}d^3x'$$

$$\bar{A}(\bar{x})=\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int_{-a/2}^{a/2}e^{ikx'}dx'\int_{-b/2}^{b/2}e^{iky'}dy'$$

$$\bar{A}(\bar{x})=\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}ab\left[\frac{\sin(kax/2r)}{kax/2r}\frac{\sin(kay/2r)}{kay/2r}\right]$$

- Rectangular current patch flowing in x direction over
  - $-a/2 < x < a/2$
  - $-b/2 < y < b/2$
- Plug in Fraunhoffer approximation for A
- Factor to F(x)G(y)
- View as product of two Fourier Transforms

Aperture Radiation Beamwidth

$$\overline{A}(\overline{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}ab\left[\frac{\sin(kax/2r)}{kax/2r}\frac{\sin(kay/2r)}{kay/2r}\right]$$

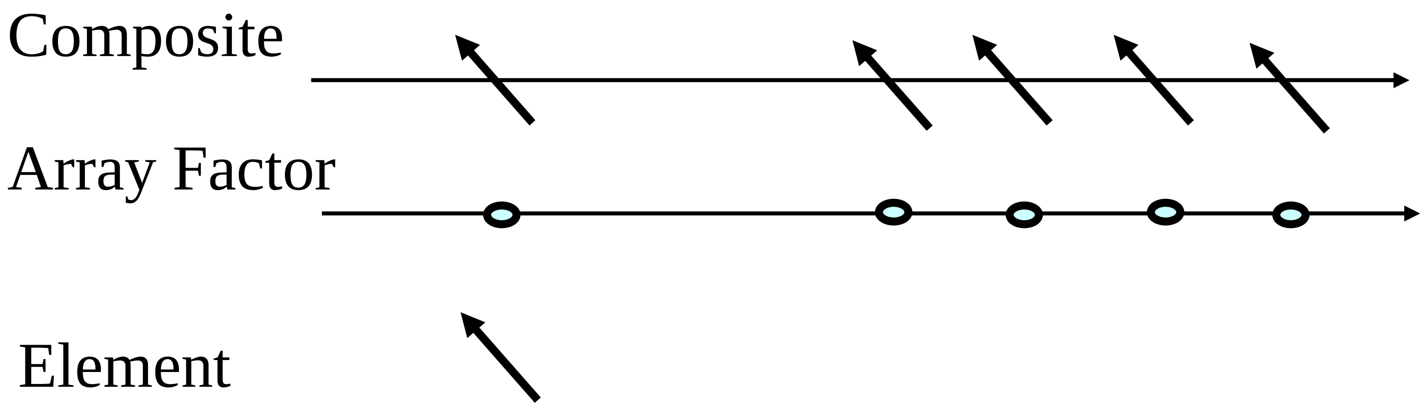
$$\frac{\sin(kax/2r)}{kax/2r} = 0$$

$$kax/2r = \pi/2$$

$$x/r = \lambda/2a$$

- Look for first null
- Occurs when aperture is one wavelength wide and ful cycle integrates to zero
- FWHM = 60 degrees/size in wavelengths

Antenna Array Patterns



- Composite Array id built from a element instantiated at array positions (convolution of element with space array factor)
- FT of convolution is product of FT's
- Composite pattern is the array pattern times element pattern.