EE243 Advanced Electromagnetic Theory

Lec # 25 Imaging as Diffraction (Cont.)

- **Standing Waves in Films on Substrates**
- **Phasor Perturbational Images of Small Features**
- **Aberrations**
- **Interferometric Measurements**
  - **Phase Contrast; Point Spread; Pattern-and-Probe**

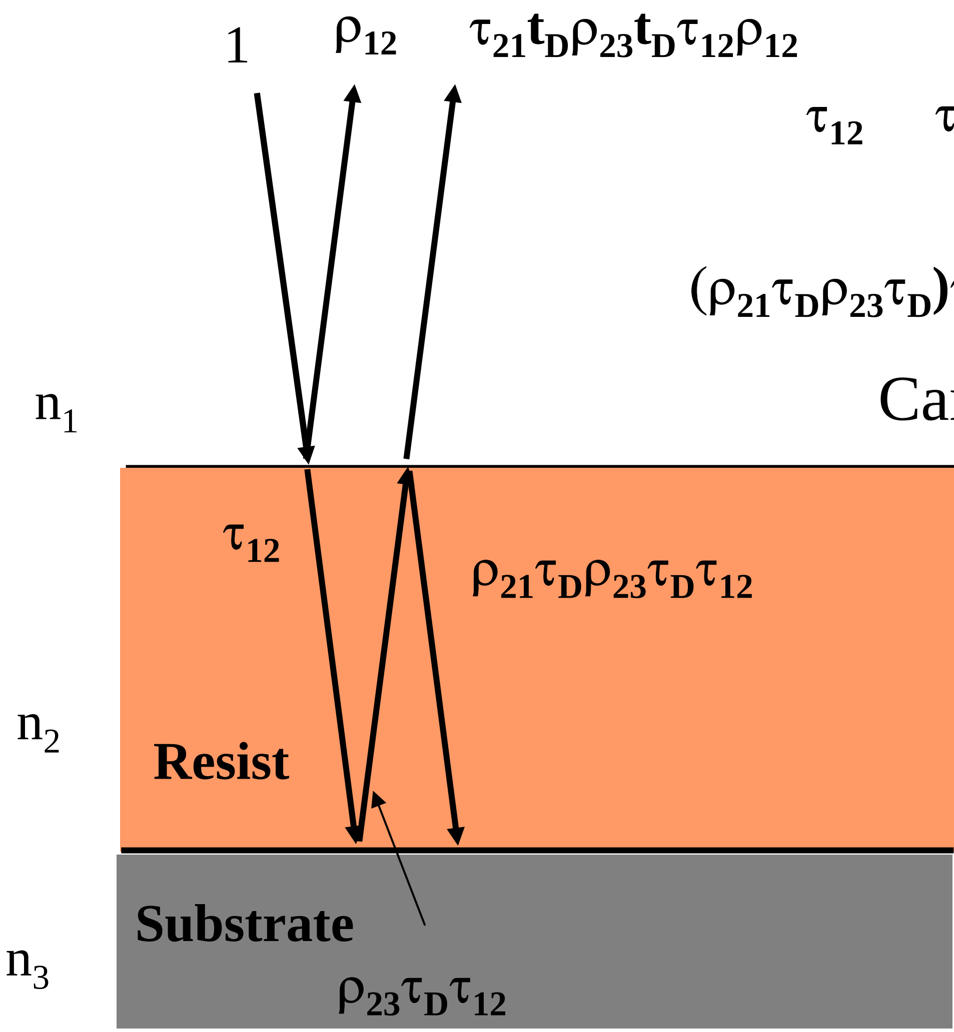
Reading: (This lecture is self contained and is based on excerpts from Born and Wolf Chapters 8 and 9 plus Chapter by Smith in Sheats and Smith)

Overview

Optical imaging also involves many other engineering aspects:

- Standing waves in films on substrates
- Pertubational methods for images of small features
- Aberrations and Strehl Ratio effects
- Phase interferometric measurement concepts
- Partial coherence of the illumination on a mask

Recording Media on a Substrate



τ_{12} $\tau_D \rho_{23} \tau_D \tau_{12}$

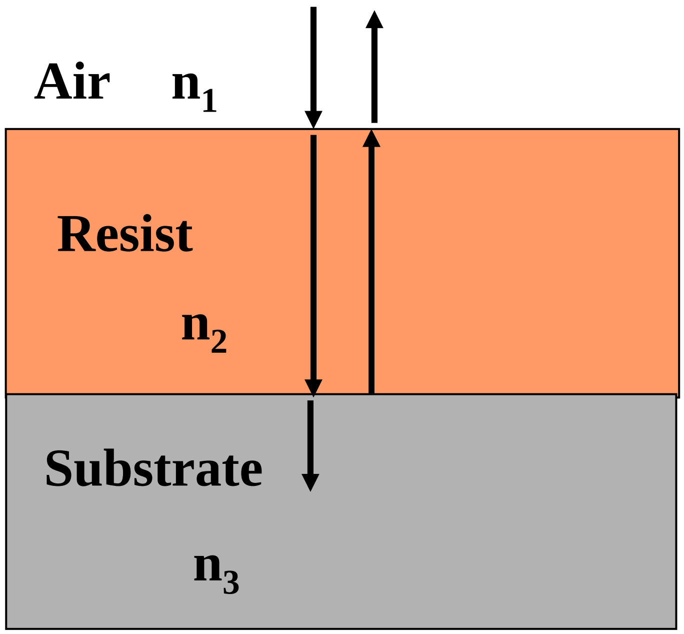
$(\rho_{21}\tau_D\rho_{23}\tau_D)\tau_{12}$ $\tau_D\rho_{23}\tau_D(\rho_{21}\tau_D\rho_{23}\tau_D)\tau_{12}$

Can be summed in closed form.

The extra term has a magnitude of about 0.05 to 0.20 and has a phase that oscillates with thickness D.

This round trip wave gives rise to a change in energy coupling and the effect know as the swing curve.

Electric Field within Resist



5 waves
match boundary conditions
(or use signal flow analysis)
use definition of τ_D

Downward wave

Upward wave

Round trip propagation

$$E_{RESIST}(x,y,z) = E_{AIR\_INC}(x,y)\frac{\tau_{12}\left(e^{-jk_2 z} + \rho_{23}\tau_D^2 e^{+jk_2 z}\right)}{\underline{1+\rho_{12}\rho_{23}\tau_D^2}}$$

Transmission in

Reflection at substrate

Round trip loop gain (loss)

$$\rho_{12} = -\rho_{21}$$

Waves in Recording Media

The wavelength in the media is shorter and waves are attenuated

$$n = n_r + jn_i$$

$$k_m = (n_r + jn_i)k_{air} = (n_r + jn_i)\frac{2\pi}{\lambda_{air}}$$

$$n_r \Rightarrow \lambda_m = \frac{\lambda_{air}}{n_r} \qquad n_i \le 0 \rightarrow attenuation$$

To match the lateral variation across the air resist interface

$$k_{m_x} = k_{air_x} \qquad k_{m_y} = k_{air_y}$$

Snell's Law $\sin\theta_m = \dfrac{\sin\theta_{air}}{n_r}$

Reflection and Transmission

Reflection and transmission coefficients in going from media i to media j

$$\rho_{ij} = \frac{n_i - n_j}{n_i + n_j} \qquad \tau_{ij} = \frac{2n_i}{n_i + n_j}$$

Note: 1 + ρ = τ

Phase change and attenuation with distance z

Example: air to quartz (n_{qz} = 1.5); ρ = -0.2 and τ = 0.8

$$\tau(z) = e^{-j(n_r + jn_i)\frac{2\pi}{\lambda_{air}}z}$$

Example: complex propagation factor in going from z=0 to z=D is

$$\tau_D = e^{-j(n_r + jn_i)\frac{2\pi}{\lambda_{air}}D}$$

The same net complex factor occurs for the upward wave in going from z=D to z=0.

Standing Wave Models (365 nm Example) Shipley 511A (neglecting n_i)

$$n_{RESIST} = 1.7 \qquad n_{SUB} = 6.52 - j2.71$$

$$\rho_{RESIST} = \frac{n_{RESIST} - n_{SUB}}{n_{RESIST} + n_{SUB}} = 0.64\angle 179.5^{o}$$

$$E_{MAX} = 1 + |\rho| = 1.64 \qquad E_{MIN} = 1 - |\rho| = 0.36$$

$$I_{MAX} = \left(1 + |\rho|\right)^2 = 2.87 \qquad I_{MIN} = \left(1 - |\rho|\right)^2 = 0.13$$

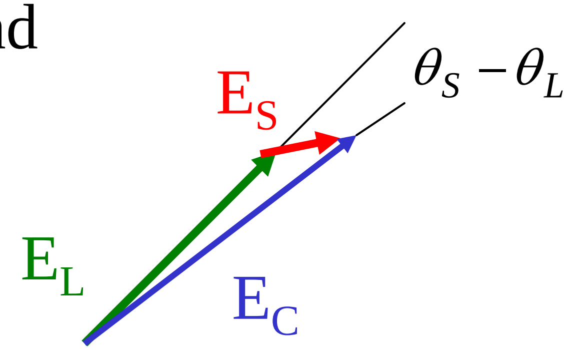
$$C_{VERTICAL} = \frac{I_{MAX} - I_{MIN}}{I_{MAX} + I_{MIN}} = \frac{2.87 - 0.13}{2.87 + 0.13} = 0.91$$

Perturbation Model of Image Contributions

$$(1+x)^2 = 1 + 2x + x^2 \qquad (1+0.1)^2 = 1 + 0.2 + 0.01$$

Consider a composite electric field made up of a large electric field and a small electric field that are time- harmonic, possibly out of phase and oriented in a co-linear direction.

$$E_C = E_L + E_S = E_L e^{j\theta_L} + E_S e^{j\theta_S}$$



Phasor Diagram in complex plane

The intensity is proportional to the electric field times its conjugate

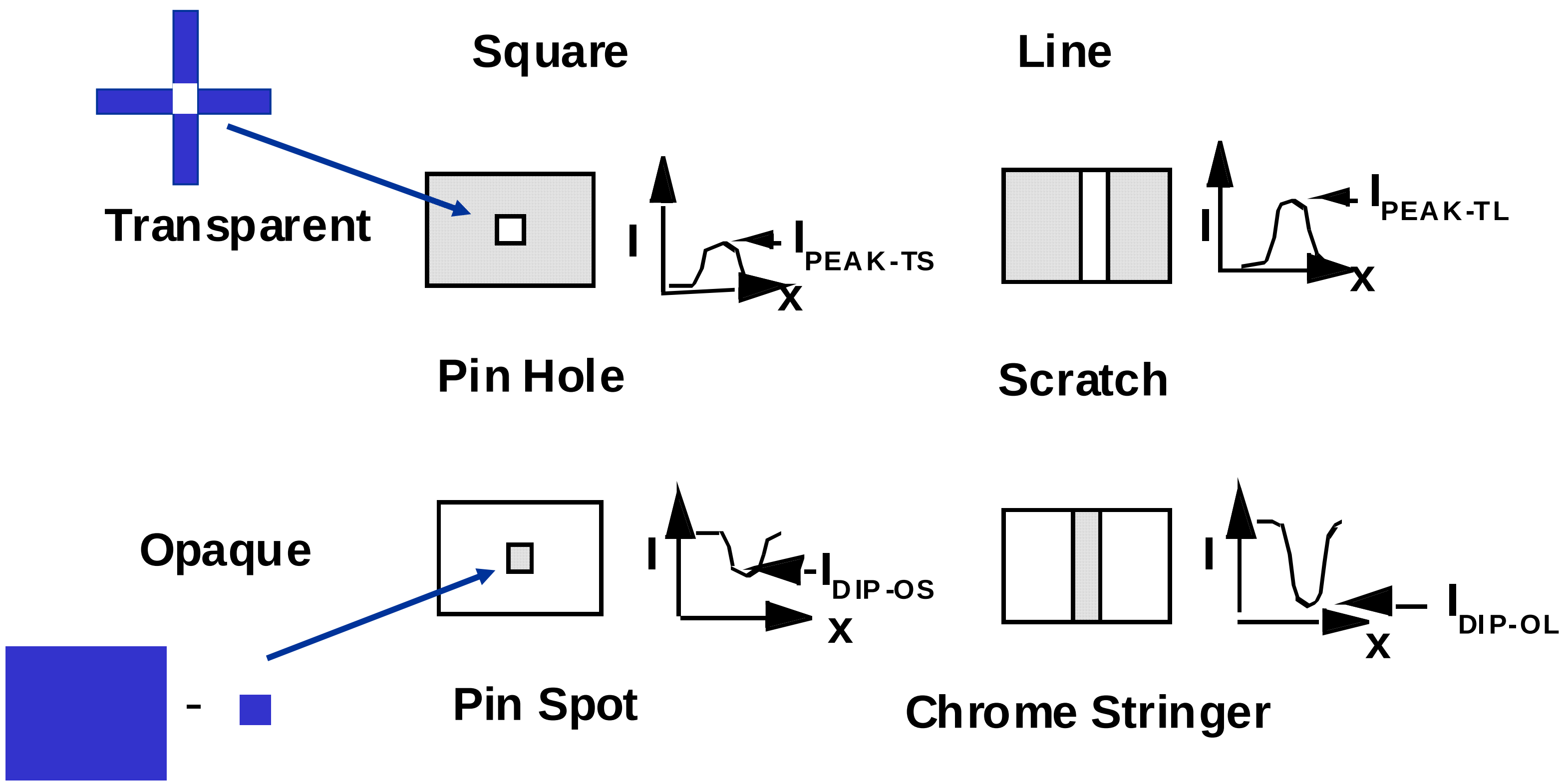
$$I = \overline{E}_C \cdot \overline{E}_C^{\,*} = \left|E_L\right|^2 + E_L E_S \, 2\,\mathrm{Re}\left[e^{j(\theta_S - \theta_L)}\right] + \left|E_S\right|^2$$

$$I = I_L + \sqrt{I_L}\,\sqrt{I_S}\,2\,\mathrm{Re}\left[e^{j(\theta_S - \theta_L)}\right] + I_S$$

Small

Very Small

Basic Types of Small Features



Intensity Models for Small Features

S = Square, L = Line T = Transparent, O = Opaque

$$I_{PEAK\text{-}TS} = 8.5\,[d/(\lambda/NA)]^4$$

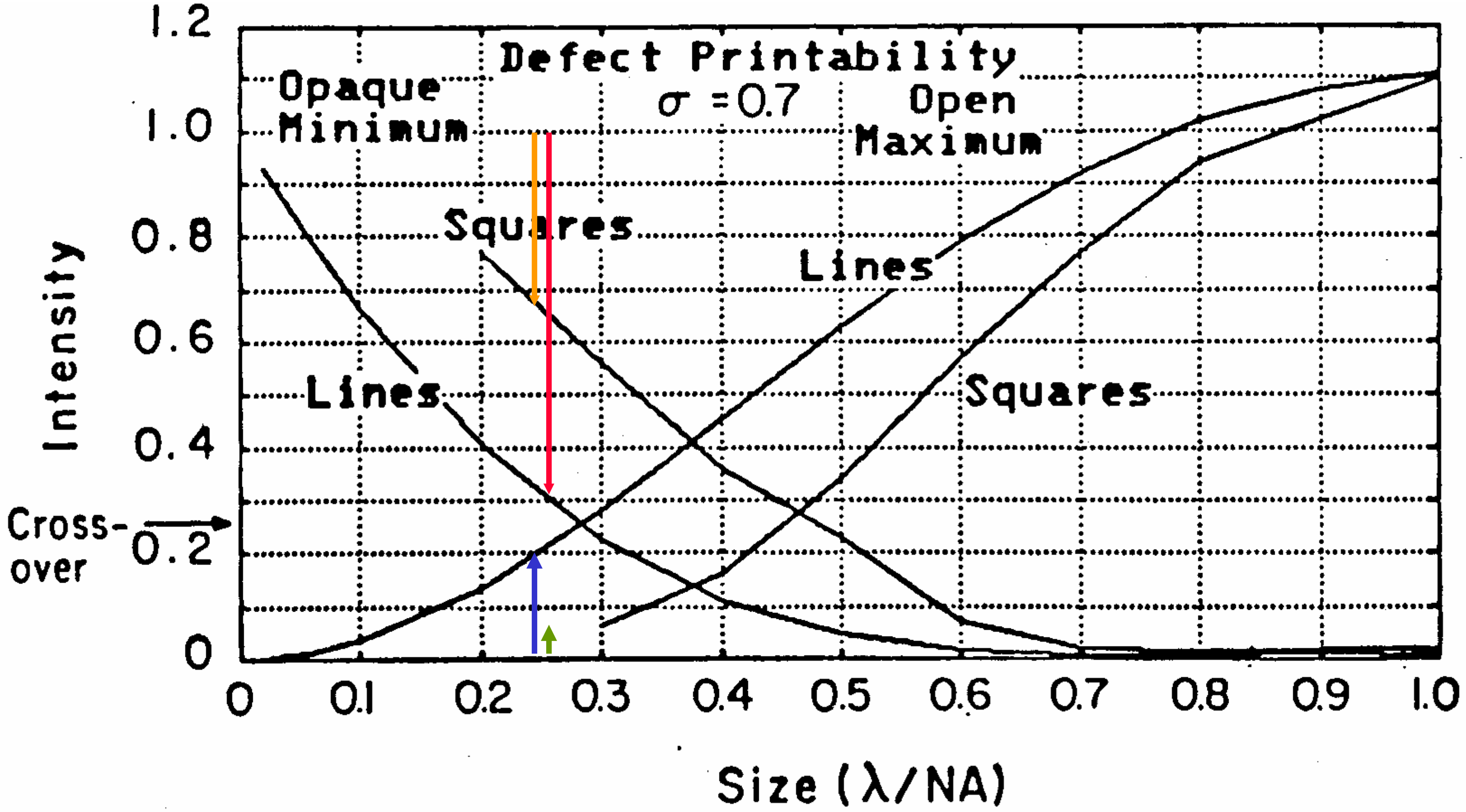
$$I_{PEAK\text{-}TL} = \mathrm{sqrt}(I_{PEAK\text{-}TS})$$

$$I_{DIP\text{-}OS} = 1 - 2\,\mathrm{sqrt}(I_{PEAK\text{-}TS})$$

$$I_{DIP\text{-}OL} = 1 - 2\,\mathrm{sqrt}[\mathrm{sqrt}(I_{PEAK\text{-}TS})]$$

Small mask openings produce a point spread shaped distribution with an intensity proportional to the square of the area.

Peak and Minimum Intensity for Small Features



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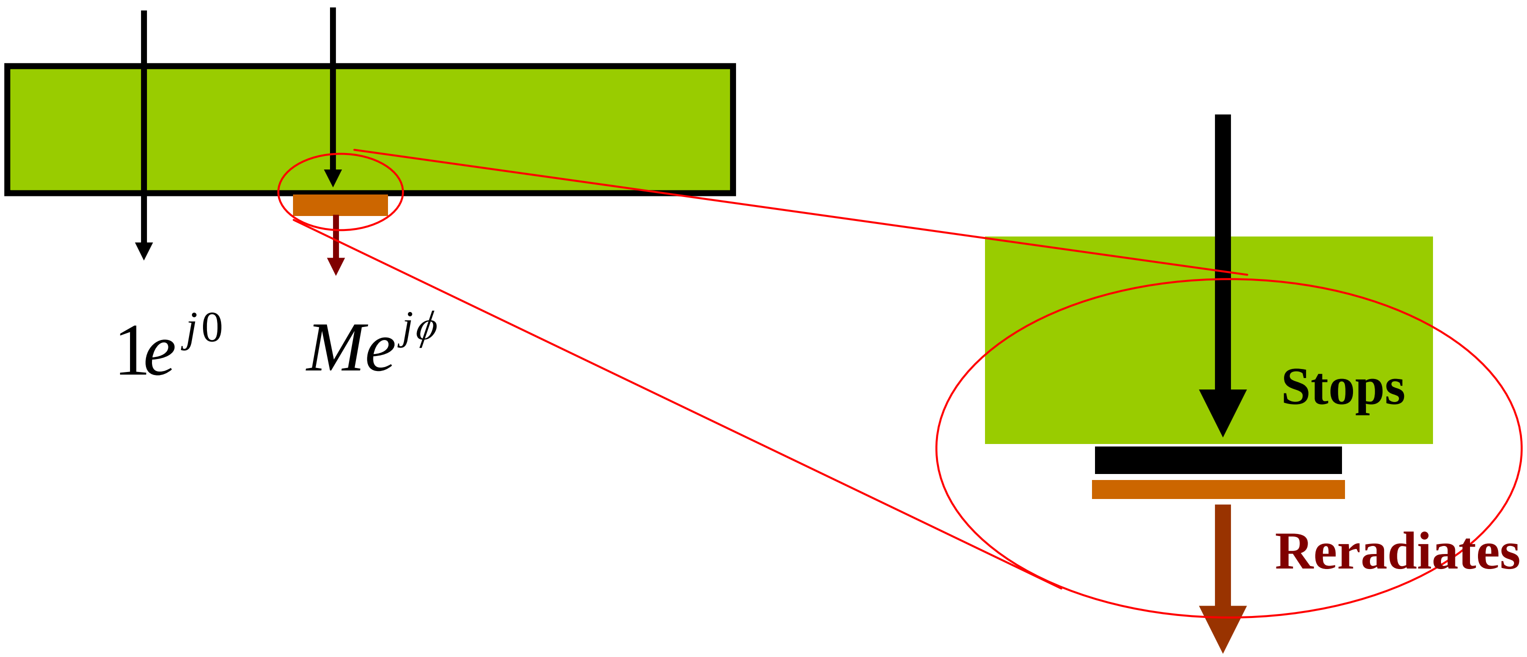
Phase-Defects

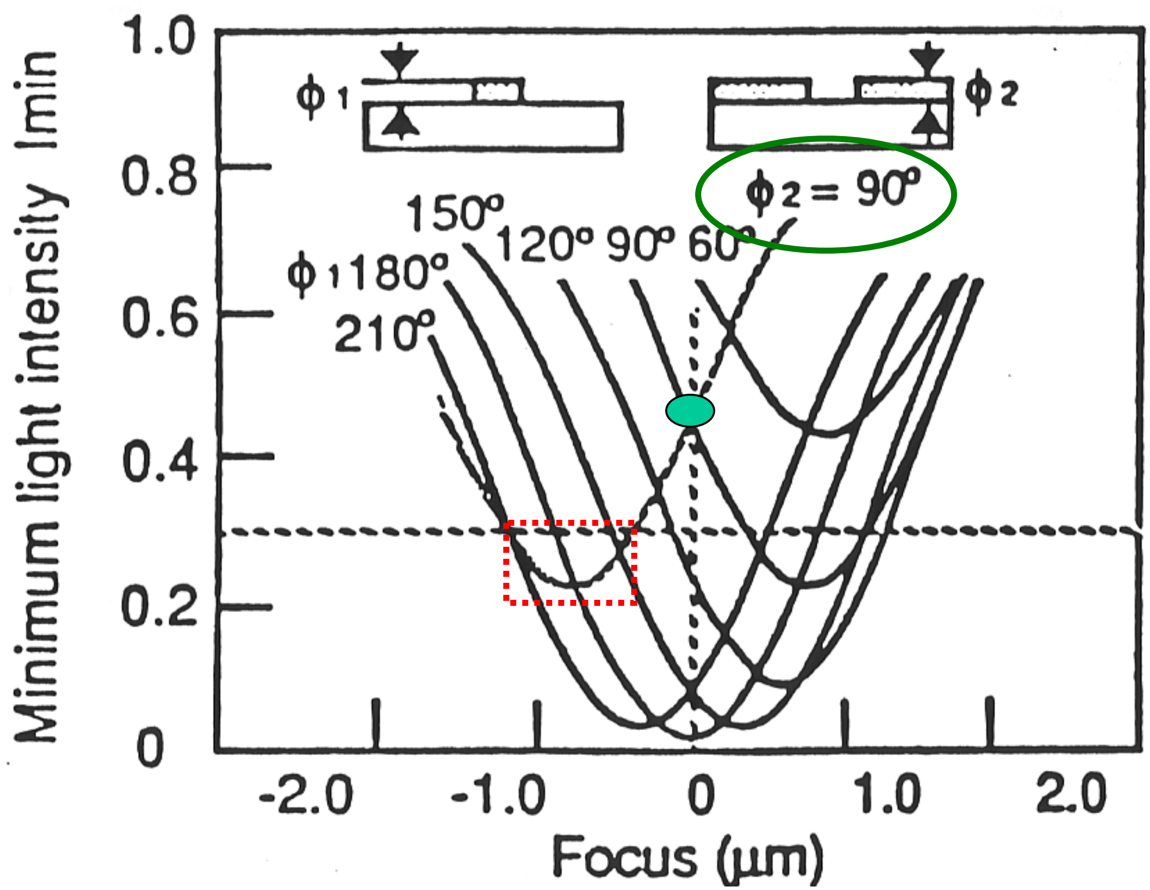


Severity Factor for interaction is

[1-Mcos(ϕ)]

Worst case is a **factor of 2** when **ϕ = 180º**

Phase Defects May Print Worse Out of Focus



M = E field transmission amplitude

Severity Factor [1-Mcos(ϕ)] becomes [2] when phase of defocus is included.

Wantanabe, et al.

Optical Path Difference

The optical path difference (OPD) is the phase error over the pupil between rays for the actual lens and those for a perfect diffraction limited lens.

Born and Wolf 7th ed, Ch 9

The OPD is usually normalized to wavelengths.

The OPD contributes an additional phase factor of $e^{j(2\pi/\lambda)OPD}$ in the integration of the rays in computing the electric field at the image.

For primary aberrations like tilt, defocus, spherical, coma, and astigmatism a power series is used

$$\Phi = A'_{n,m}\rho^n \cos^m \theta$$

Zernike introduced the so called "circle polynomials" that are orthonormal on the unit circle to describe aberrations.

$$\Phi = A_{00} + \frac{1}{\sqrt{2}}\sum_{n=2}^{\infty} A_{n0}R_n^{\infty}(\rho) + \sum_{n=1}^{\infty}\sum_{m=1}^{n} A_{nm}R_n^m(\rho)\cos m\theta$$

$$R_n^{\pm m}(1) = 1$$

Primary Coma => orthogonal components => tilt + balanced coma

Expansion of Zernike

Table 2 Fringe Zernike Polynomial Coefficients and Corresponding Aberrations*

Term	*Fringe Zernike Polynomial*	*Aberration*
1	1	Piston
2	$r\cos(\alpha)$	X Tilt
3	$r\sin(\alpha)$	Y Tilt
4	$2r^2 - 1$	Defocus
5	$r^2\cos(2\alpha)$	3rd Order astigmatism
6	$r^2\sin(2\alpha)$	3rd Order 45° astigmatism
7	$(3r^3 - 2r)\cos(\alpha)$	3rd Order X coma
8	$(3r^3 - 2r)\sin(\alpha)$	3rd Order Y coma
9	$(6r^4 - 6r^2) + 1$	3rd Order spherical
10	$r^3\cos(3\alpha)$	
11	$r^3\sin(3\alpha)$	
12	$(4r^4 - 3R^2)\cos(2\alpha)$	5th Order astigmatism
13	$(4R^4 - 3R^2)\sin(2\alpha)$	5th Order 45° astigmatism
14	$(10r^5 - 12r^3 + 3r)\cos(\alpha)$	5th Order X coma
15	$(10r^5 - 12r^3 + 3r)\sin(\alpha)$	5th Order Y coma
16	$20r^6 - 30r^4 + 12r^2 - 1$	5th Order spherical
17	$r^4\cos(4\alpha)$	
18	$r^4\sin(4\alpha)$	
19	$(5r^5 - 4r^3)\cos(3\alpha)$	
20	$(5r^5 - 4r^3)\sin(3\alpha)$	
21	$(15r^6 - 20r^4 + 6r^2)\cos(2\alpha)$	7th Order astigmatism
22	$(15r^6 - 20r^4 + 6r^2)\sin(2\alpha)$	7th Order 45° astigmatism
23	$(35r^7 - 60r^5 + 30r^3 - 4r)\cos(\alpha)$	7th Order X coma
24	$(35r^7 - 60r^5 + 30r^3 - 4r)\sin(\alpha)$	7th Order Y coma
25	$70r^8 - 140r^6 + 90r^4 - 20r^2 + 1$	7th Order spherical
26	$r^5\cos(5\alpha)$	
27	$r^5\sin(5\alpha)$	
28	$(6r^6 - 5r^4)\cos(4\alpha)$	
29	$(6r^6 - 5r^4)\sin(4\alpha)$	

The Zernike polynomials are individual functional variations in radius r and angle α in an orthonormal expansion over the lens pupil normalized to radius 1.

Z2 Tilt in x is r cos(α)

Z7 is Coma in x with Tilt removed or so called balanced coma.

Sheats and Smith, pp. 224

Strehl Ratio

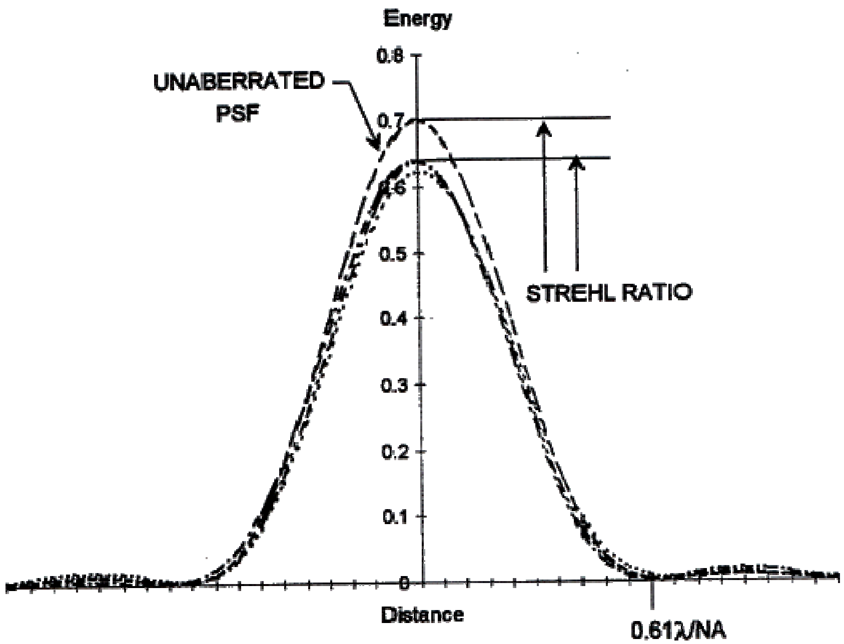


Figure 46 Strehl ratio for an aberrated point image.

The peak value of the point spread function decreases proportionally to the square of the RMS value of the aberrations present.

The Strehl Ratio is defined as the ratio of the peak value with aberrations to the peak value without aberrations.

The Strehl Ratio is approximately
1- $4\pi^2$(RMS OPD)^2

RMS OPD = (P-V OPD)/3.5

Sheats and Smith pp 255

Finding the Strehl Ratio

$$E(\bar{x}) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 E(r,\alpha)e^{-j\bar{k}\cdot\bar{x}}e^{-jkOPD(r,\alpha)}r\delta r\delta\alpha$$

To find the Strehl ratio (relative value the peak intensity in the presence of an aberration) we only need look at x = 0 and assume that E(r,a) is produced by a small pin hole and thus constant. This greatly simplifies the integral to

$$E(0) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 e^{-jkOPD(r,\alpha)}r\delta r\delta\alpha$$

When the OPD is small

$$E(0) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 [1 - jkOPD(r,\alpha) - \frac{k^2}{2}OPD^2(r,\alpha)]r\delta r\delta\alpha = 1 - jk\overline{\Phi} - \frac{k^2}{2}\overline{\overline{\Phi}}$$

Where $\overline{\overline{\Phi}}$ is the average of OPD^2 over the pupil

The intensity is given by **$4\pi^2$** **Orthogonal for Zernike's**

$$\left|E(0)\right|^2 = \left|1 - jk\overline{\Phi} - \frac{k^2}{2}\overline{\overline{\Phi}}\right|^2 = \left|1 - \frac{k^2}{2}\overline{\overline{\Phi}} - jk\overline{\Phi}\right|^2 \approx 1 - k^2\overline{\overline{\Phi}} + k^2\left(\overline{\Phi}\right)^2 = 1 - 4\pi^2\overline{\overline{\Phi}} = 1 - 4\pi^2\sum_{n,m}\overline{\overline{\Phi}}_{n,m} = 1 - 4\pi^2\sum_{n,m}A_{n,m}^2$$

Zero for Zernike's

Zernike Phase Contrast Microscopy

1935 Nobel Prize 1953

$$F(x) = e^{i\phi(x)} \approx 1 + i\phi(x)$$

$$F(x) = \sum_n c_m e^{i\frac{2\pi}{P}x}$$

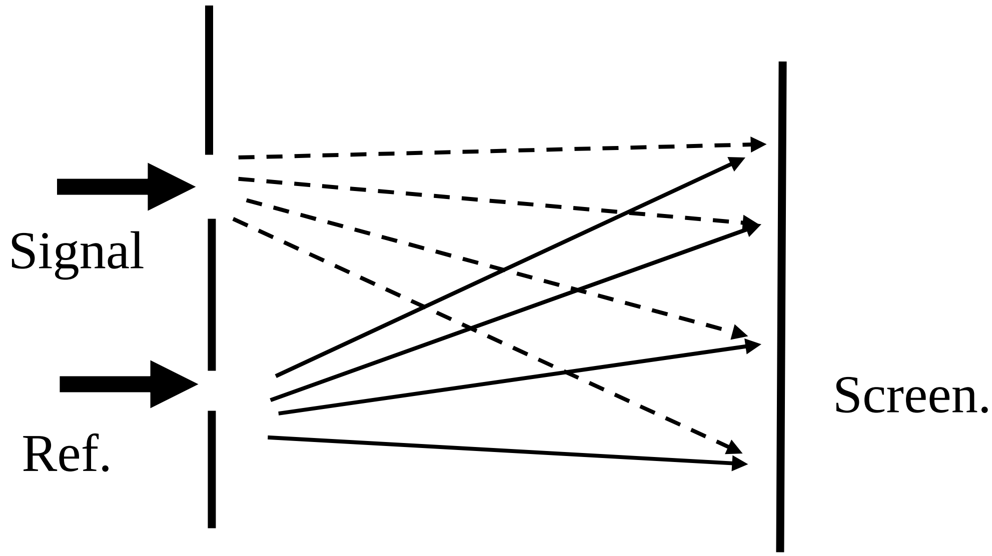
$$c_0 = 1, c_{-m} = -c_m^*$$

$$G(x) = \pm i + i\phi(x)$$

$$I(x) = |G(x)|^2 = 1 \pm 2\phi(x)$$

- **A phase object must be imaged in many applications such as in Biology**
- **For a periodic phase object ϕ(x) with the Taylor series approximation the spectrum is 1 for the D.C. term and imaginary for the higher order terms.**
- **A phase plate is added to shift the D.C. order by + or – 90 degrees**
- **A real (but fictitious) image proportional to the phase is produced.**

Point Spread Interferometry



- A reference signal is introduced by using light from a pin hole or scatterer
- The signal under test then interacts with the reference signal spreading from the point
- Computer analysis of the resulting interference fringes on the screen allow the phase of the signal to be determined.

Mathematics of Aberrations and OPD

The electric field produce by the convergence of the cone of plane waves from the pupil at any point x on the wafer is given by integrating the waves over the pupil.

$$E(\bar{x}) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 E(r,\alpha)e^{-jkOPD(r,\alpha)}e^{-j\bar{k}\cdot\bar{x}}r\delta r\delta\alpha$$

Pupil

Spectrum

Optical Path Difference

Kernel Green's Function

Area element

When OPD is small $e^{jkOPD} \cong 1 - jkOPD$

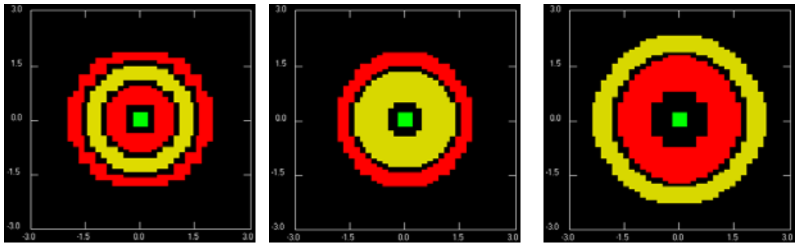
$$E(\bar{x}) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 E(r,\alpha)[1 - jkOPD(r,\alpha)]e^{-j\bar{k}\cdot\bar{x}}r\delta r\delta\alpha$$

Unaberrated E-Field

Aberration Ghost E-Field with lateral spillover

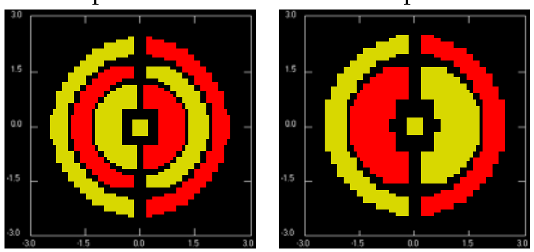
Pattern-and-Interferometric-Probe Aberration Monitors

Garth Robins



Defocus Spherical HO Spherical

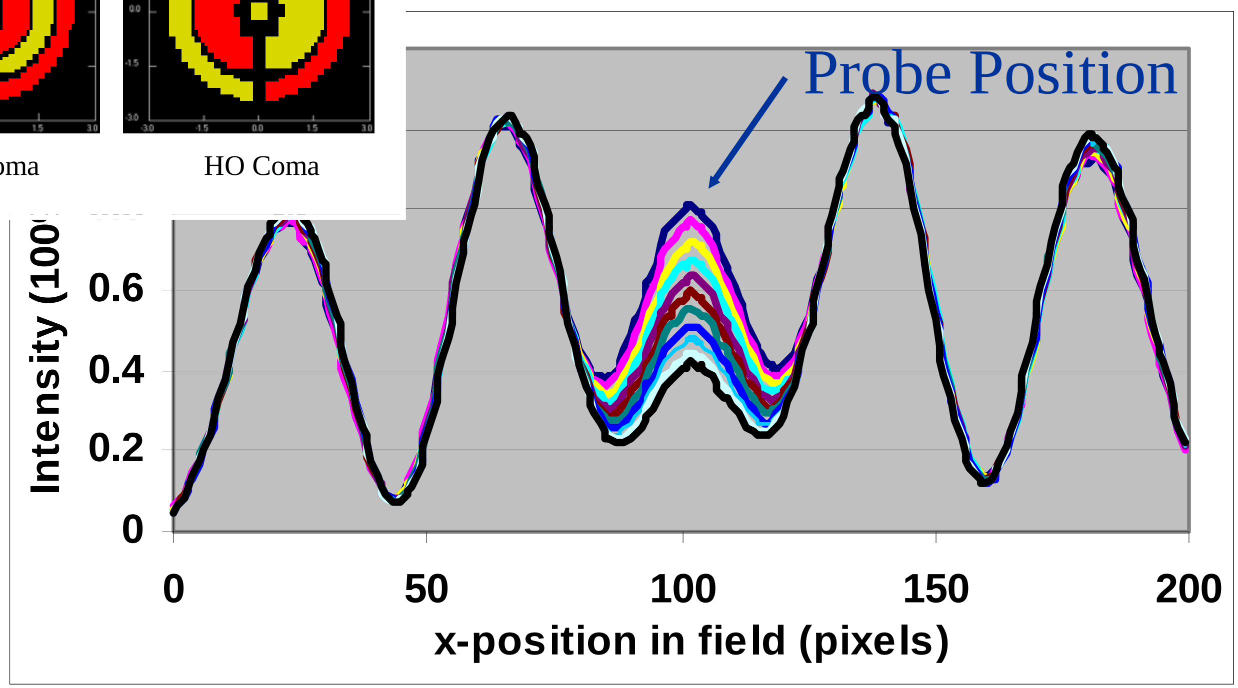
Mask phases
- **yellow = 0°**
- **green = 90°**
- **red = 180°**



Coma HO Coma

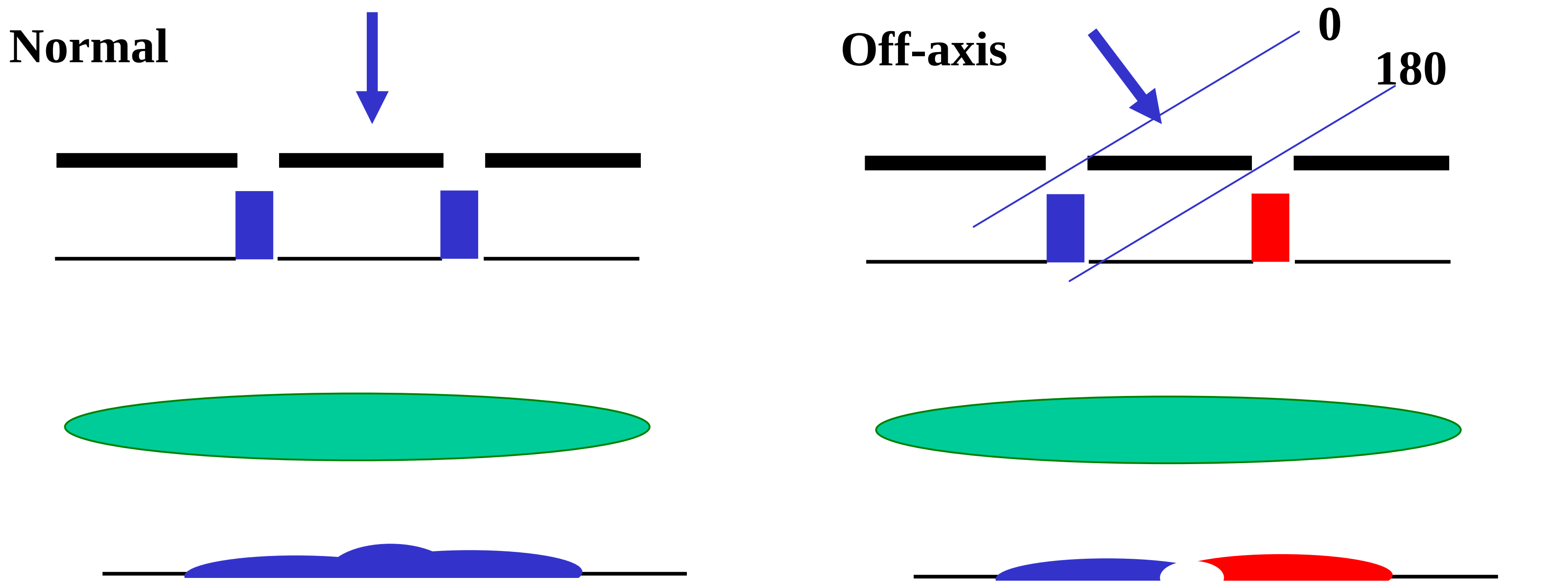
Discovered through simulation
Lead to a new theory
Becoming a practical technology

Defocus target Experiment on AIMS at low NA looks just like simulation!



Illumination Controls Proximity Effects

Two Pinholes in a mask



$$I_{TOTAL} = E_1^2 + 2E_1E_2 + E_2^2$$

$$I_{TOTAL} = E_1^2 - 2E_1E_2 + E_2^2$$

- Wafer
  - Tails of electric field overlap (spillover)
  - Relative phases depend on phase of illumination

Mutual Coherence of Illumination

$$\mu_{21} = \mu(\bar{x}_2, \bar{x}_1) = \sum_m \sum_n \frac{E_m(\bar{x}_1) E_n(\bar{x}_2)^*}{|E_m(\bar{x}_1)||E_n(\bar{x}_2)|}$$

$$m \neq n \Rightarrow 0$$

$$E_n(\bar{x}_2) = |E_n(\bar{x}_1)| e^{-j\bar{k}_n(\bar{x}_2 - \bar{x}_1)}$$

$$\mu_{21} = \mu(\bar{x}_2 - \bar{x}_1) = \sum_n e^{+j\bar{k}_n(\bar{x}_2 - \bar{x}_1)}$$

$$\mu_{21} = F.T.(Source\_Shape)|_{(\bar{x}_2 - \bar{x}_1)}$$

- Defined as time-average normalized cross-product
- Assuming that illumination waves from different angles are incoherent the cross terms all drop out
- For a given source angle (k-vector) field at x_2 found from field at x_1
- Sum for a uniform source is just the Fourier transform or the source shape

Mutual Coherence Function

$$I_{TOTAL}(x) = E_1^2(x) + 2\,\mathrm{Re}\{\mu(x_2 - x_1)E_1(x)E_2(x)\} + E_2^2(x)$$

- Measures the complex degree of coherence between any two points on the mask.
- Is a function only of the distance and angle between the two points and not their absolute positions.
- The value is given by the inverse Fourier transform of the wave angles and magnitudes that make up illumination. (The temporal phases are incoherent)
- For a disk of illumination of radius σNAk_0 it is the Airy function with argument $|x|/\sigma$.
- Thus the signal is relatively coherent over most of a spot $1/\sigma$ times the point spread function (i.e. $(1.22/\sigma)\lambda/NA$).

Mutual Coherence Function: Top Hat

$$\mu(x_1 - x_2) = \frac{2J_1\left(2\pi\left(\frac{\lambda}{\sigma NA}\right)(x_1 - x_2)\right)}{\left(2\pi\left(\frac{\lambda}{\sigma NA}\right)(x_1 - x_2)\right)}$$

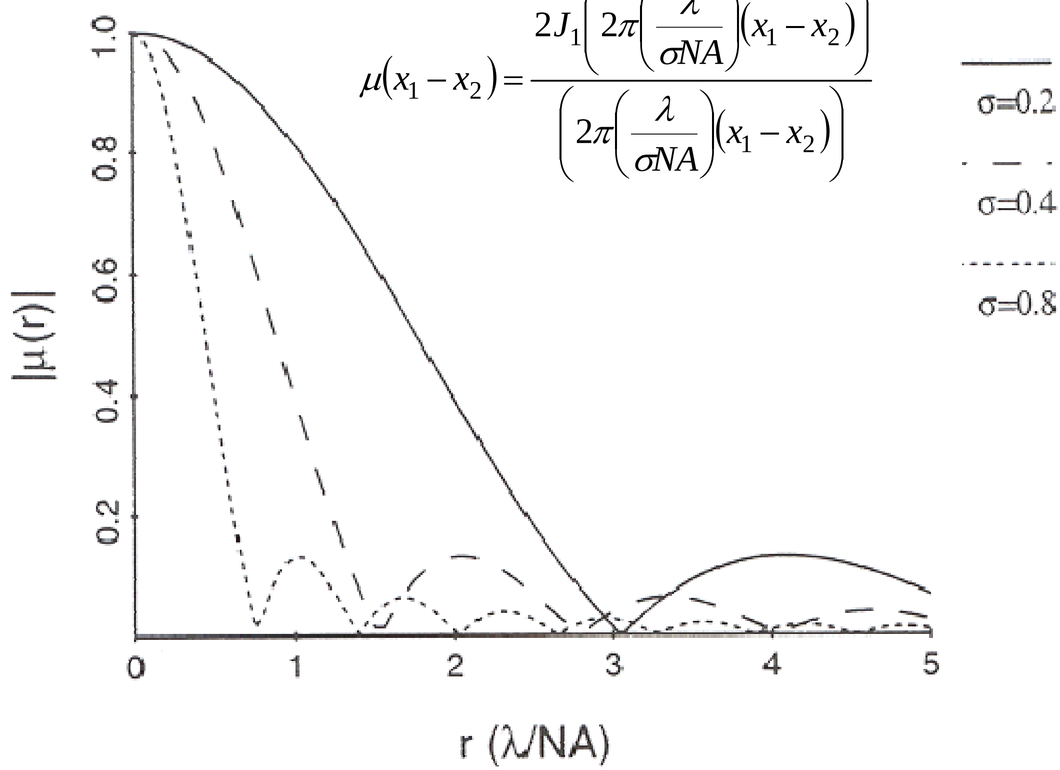


Figure 2.22: For any partial coherence factor, the degree of coherence $|\mu(\hat{r})|$ has the same shape except for a scaling of the x-axis by $(1/\sigma)$.

Aerial Image Intensity for Knife Edge