NOISE IN LINEAR TWO-PORTS

USING THERMODYNAMIC ARGUMENTS, IT CAN BE SHOWN THAT A RESISTOR AT TEMP T GENERATES WHITE NOISE (FLAT POWER SPECTRAL DENSITY) WITH AVAILABLE POWER

$$P_A = KTB$$

WHERE B IS THE BANDWIDTH OF OBSERVATION.

FURTHERMORE, DUE TO THE QUANTIZED NATURE OF CHARGE, A "DC" CURRENT CROSSING A JUNCTION IS ACTUALLY A NOISY CURRENT WITH WHITE NOISE ~~COMPONENT~~ RMS VALUE

$$I_n = 2qI_{DC}$$

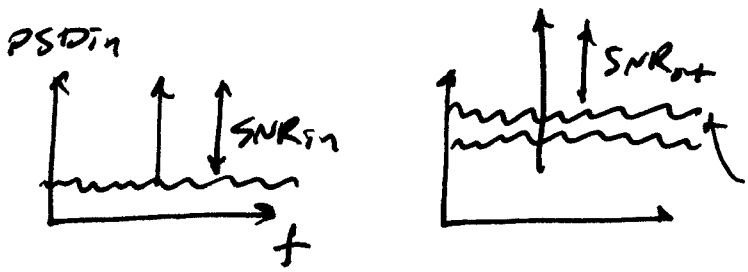
SO DIODES AND BIPOLAR TRANSISTORS HAVE THIS "SHOT NOISE". THIS NOISE IS DUE TO RANDOM "ARRIVAL" TIME OF CHARGE CARRIERS CROSSING A JUNCTION. NOTE THAT IF THE CHARGES ARRIVED UNIFORMLY, THE PSD OF A "DC" CURRENT WOULD HAVE SPECTRAL LINES AT THE HARMONIC OF 1/(ARRIVAL PERIOD).

THUS EVERY PHYSICAL CIRCUIT IS ACCOMPANIED BY NOISE.

<u>NOISE FIGURE</u>: A MEASURE OF THE DEGRADATION IN THE SNR DUE TO A TWO-PORT NETWORK.

$$F \triangleq \frac{SNR_{in}}{SNR_{out}} \geq 1$$

EQUALITY FOR A NOISELESS 2-PORT



NOISE "GAIN" LARGER THAN SIGNAL GAIN

$$SNR_{in} = P_{in}/N_{in}$$

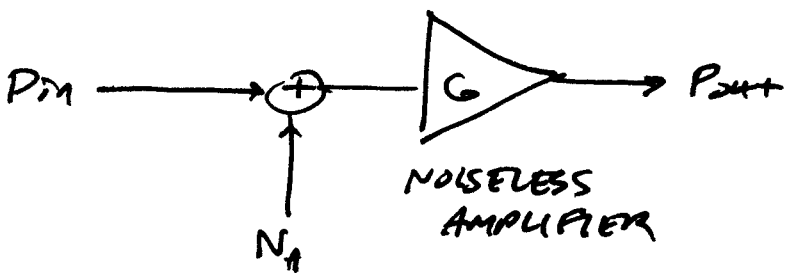
$$SNR_{out} = P_{out}/N_{out}$$

$$F = \frac{P_{in}}{P_{out}} \frac{N_{out}}{N_{in}} = \frac{1}{G\cancel{P_{in}}} \frac{G(N_{in}+N_A)}{N_{in}}$$

$$= 1 + \frac{N_A}{N_{in}}$$

INTERNAL NOISE INTO AMP

EXT NOISE INTO AMP

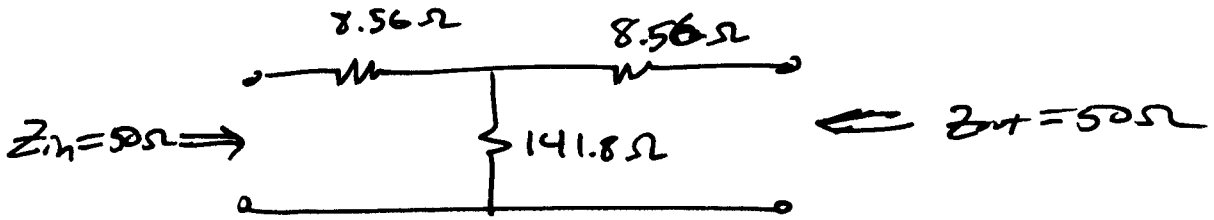


<u>CASCADE</u>

$$F = F_1 + \frac{F_2 - 1}{G_1} + \cdots + \frac{F_n - 1}{G_1 G_2 \cdots G_N}$$

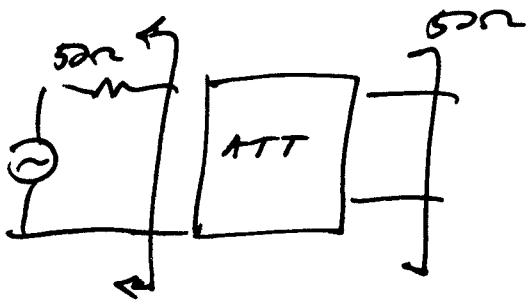
→ MATCHED INTERFACE

EG A "MATCHED" ATTENUATOR



$S_{11} = S_{22} = 0$ $\quad S_{12} = S_{21} = 0.707$

$\gamma = 3\,dB$ ATTENUATOR



THE AVAILABLE NOISE FROM GENERATOR SIGNAL RESISTOR IS 4KTB

SINCE THE ATTENUATOR ALSO HAS SAME SOURCE RESISTANCE

$P_A = 4KTB$

SO $P_A = 4KTB \Rightarrow N_{out} = 4KTB$

$S_{out} = S_{in} \cdot \gamma$

SIGNAL IS ATTENUATED

$$\frac{S_{out}}{N_{out}} = \frac{S_{in} \cdot \gamma}{N_{in}} = \frac{S_{out}}{N_{in}}$$

$$\frac{SNR_{in}}{SNR_{out}} = \frac{1}{\gamma} = 2$$

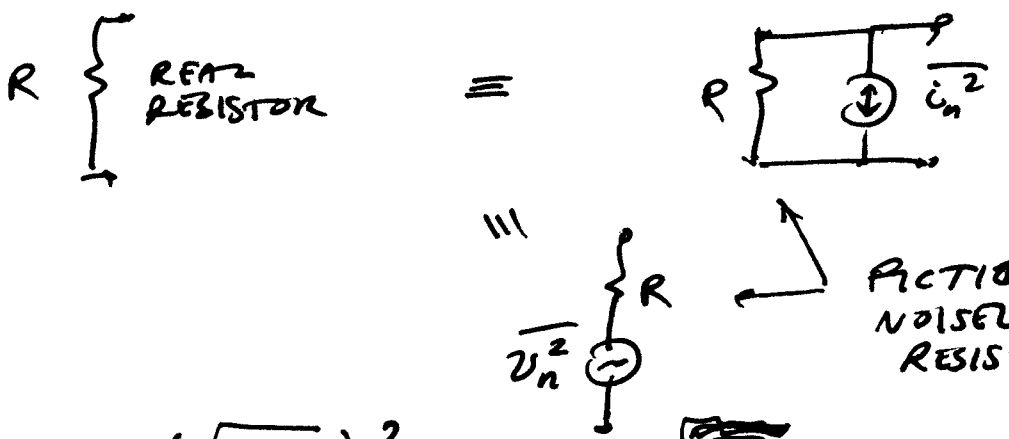
(HALF SIGNAL POWER FOR SAME NOISE)

~~DOUBLE THE NOISE FOR FIXED SIG/NO~~

~~FOR ANY 1-PORT NETWORK, THE AVAILABLE NOISE POWER IS 4KTB~~

~~FOR A GENERAL~~

THE NOISE OF A RESISTOR CAN ALTERNATIVELY BE REPRESENTED BY A SIGNAL SOURCE AND A NOISELESS RESISTOR:



$$\longrightarrow \quad P_A = \left(\frac{\sqrt{\overline{v_n^2}}}{2}\right)^2 \frac{1}{R} = \frac{\overline{v_n^2}}{4R} = KTB$$

$$\boxed{\overline{v_n^2} = 4KTBR}$$

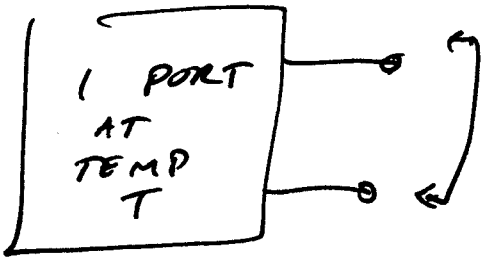
NOISE IS RMS (NOT 8)

EQ. NOISE VOLTAGE SQUARED

$$\longrightarrow \quad P_A = \left(\frac{\sqrt{\overline{i_n^2}}}{2}\right)^2 R = KTB$$

$$\frac{\overline{i_n^2}}{4} R = KTB \qquad \boxed{\overline{i_n^2} = \frac{4KTB}{R}}$$

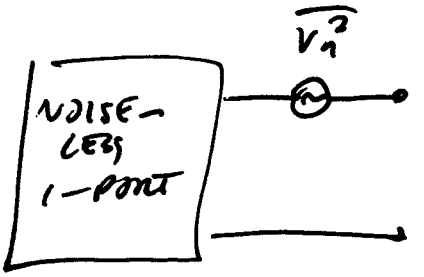
IN FACT, ~~FOR~~ ANY 1-PORT CIRCUIT CAN BE REPRESENTED BY A LOSSLESS 1-PORT AND AN EQ. NOISE GENERATOR:

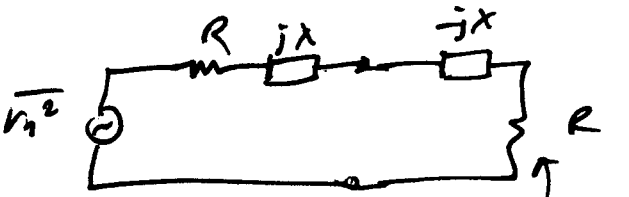


$Z_{in} = R + jX$

$P_A = KTB$

$P_L =$





NOISE POWER TO LOAD SHOULD BE KT·B

$P_L = \dfrac{R}{R - jX}$

$$P_L = \frac{\overline{v_n^2}}{4R} = KTB$$

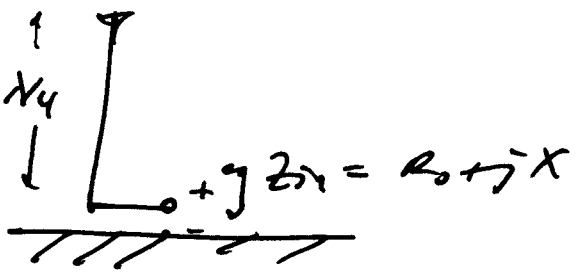
$$\overline{v_n^2} = 4KTB \cdot R$$

$$\overline{v_n^2} = 4KTB \; Re(Z)$$

<u>MYSTERY</u>: AN ANTENNA HAS $Z_{in} = R_0 + jX_0$

~~AS RESONANCE AND $Z_{in} = R_0$~~

WITH $Re(Z_{in}) = R_0$



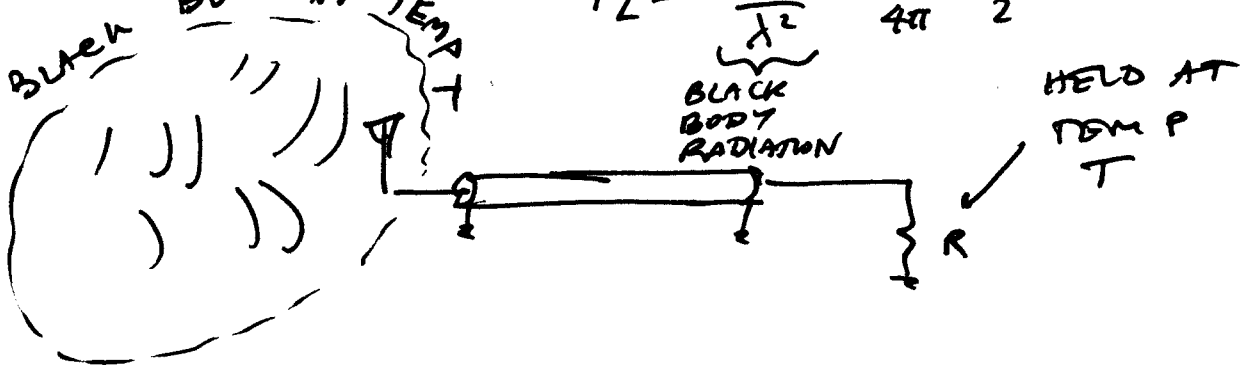
IF THE MATALLIZATION IS A PERFECT CONDUCTOR, THEN THE PHYSICAL ANTENNA RESISTANCE IS ZERO!

WHERE DOES THE NOISE COME FROM?

INCOMING RADIATION!

ALL BODIES AT TEMP T RADIATE BLACK BODY RADIATION. IN FACT, THIS IS A CLEVER PROOF OF THE EXISTENCE OF RESISTOR THERMAL NOISE

$$P_L = \underbrace{\frac{2KT}{\lambda^2}}_{\text{BLACK BODY RADIATION}} \overbrace{\frac{\lambda^2}{4\pi}}^{\text{ANTENNA CROSS SECTION}} \frac{1}{2} \underbrace{4\pi}_{\text{SOLID ANGLE}} = KT$$

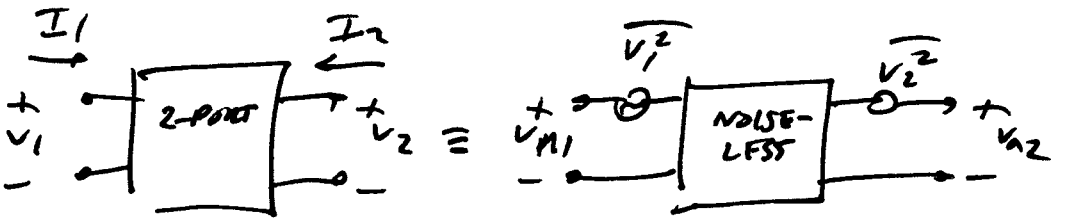


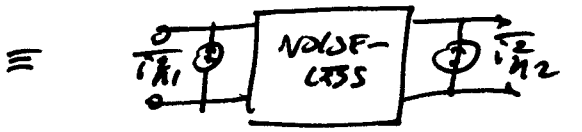
SINCE THE ANTENNA IS IRRADIATED BY BLACK BODY RADIATION, IT DELIVERS POWER P LOAD RESISTOR R.

BUT SINCE THE BLACK BODY IS IN THERMAL EQUILIBRIUM WITH THE RESISTOR, THE RESISTOR MUST LIKEWISE DELIVER POWER P_{BACK} BLACK-BODY.

A NOISY TWO PORT CAN ALSO BE CHARACTERIZED BY A NOISE-LESS 2-PORT AND EQUIVALENT NOISE GENERATORS:

(RESULT BY ROTHE AND DAHLKE)



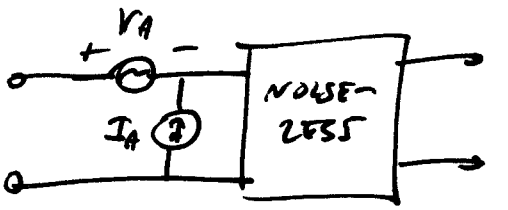


$$I_1 = y_{11} V_1 + y_{12} V_2 + i_{n1}$$
$$I_2 = y_{21} V_1 + y_{22} V_2 + i_{n2}$$

$$V_1 = z_{11} I_1 + z_{12} I_2 + V_{n1}$$
$$V_2 = z_{21} I_1 + z_{22} I_2 + V_{n2}$$

ABCD - REPRESENTATION

$$V_1 = A V_2 + B I_2 + V_A$$
$$I_1 = C V_2 + D I_2 + I_A$$



CAN SHOW:

$$I_A = -\frac{i_{n2}}{y_{21}} = -v_{n} - \frac{v_{n2} z_{11}}{z_{21}}$$

$$I_A = i_{n1} - \frac{i_{n2} y_{11}}{y_{21}} = -\frac{v_{n2}}{z_{21}}$$

IN GENERAL THE NOISE VOLTAGE AND CURRENT ARE CORRELATED

$$I_A = I_u + I_c$$

UNCORRELATED TO V_A — FULLY CORRELATED TO V_A

$$I_c = Y_{cor} V_A$$

COR. ADMITTANCE

$$I_A = I_u + Y_{cor} V_A$$

$$\langle \overline{V_A}, I_A \rangle = \langle \overline{V_A}, I_u \rangle + \langle \overline{V_A}, Y_{cor} V_A \rangle$$

$$\langle \overline{V_A}, I_A \rangle = 0 + Y_{cor} \overline{V_A^2}$$

$$Y_{cor} = \frac{\langle \overline{V_A}, I_A \rangle}{\overline{V_A^2}}$$

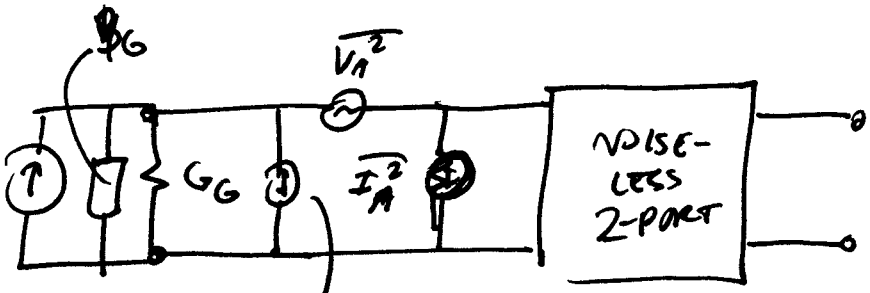
$$c \equiv \frac{\langle \overline{V_A}, I_A \rangle}{\sqrt{\langle \overline{V_A}, V_A \rangle \langle \overline{I_A}, I_A \rangle}} = \frac{\langle \overline{V_A}, I_A \rangle}{\sqrt{\overline{V_A^2}\ \overline{I_A^2}}}$$

$$= Y_{cor} \cdot \sqrt{\frac{\overline{V_A^2}}{\overline{I_A^2}}}$$

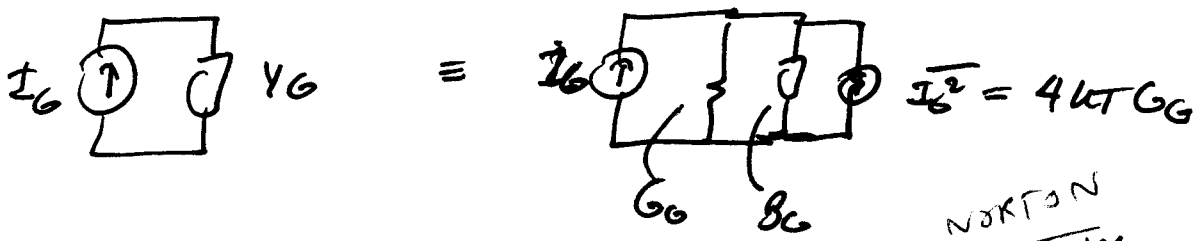
CONVENIENT TO EXPRESS $\overline{V_A^2}$ AND $\overline{I_A^2}$ IN TERMS OF R_n AND G_n

$$R_n \equiv \frac{\overline{V_A^2}}{4KTB}$$

$$G_n \equiv \frac{\overline{I_A^2}}{4KTB} \qquad G_u \equiv \frac{\overline{I_u^2}}{4KTB}$$



$$\overline{I_G^2} = 4kTBG_G$$



$$F = 1 + \frac{N_{added}}{N_{input}}$$

NORTON ~~THEVININ~~ EQ SRC

$$= 1 + \left| \frac{I_A + Y_G V_A}{I_G} \right|^2$$

RESULT FOLLOWS AFTER SOME ALGEBRA

$$\cancel{\overline{I_n^2} = 4kT(Y_{cor} R_n + G_n) B}$$

$$I_A = Y_{cor} V_A + I_n$$

$$I_A + Y_G V_A = (Y_G + Y_{cor}) V_A + I_n$$

$$|I_A + Y_G V_A|^2 = |Y_G + Y_{cor}|^2 \overline{V_A^2} + \overline{I_n^2}$$

$$F = 1 + (Y_G + Y_{cor})^2 \frac{\cancel{4kT} R_n \cancel{B}}{\cancel{4kT} G_G \cancel{B}} + \frac{\cancel{4kT} G_n}{\cancel{4kT} G_G}$$

$$= 1 + \frac{G_U}{G_G} + \frac{R_n}{G_G}\left[(G_G + G_{cor})^2 + (B_G + B_{cor})^2\right]$$

NOTE THAT F IS A FUNCTION OF G_G AND B_G FOR A FIXED 2-PORT

$$\frac{dF}{dG_G} = -\frac{G_u}{G_G^2} - \frac{R_n}{G_G^2}\left(Y_G + Y_{con}\right)^2$$

$$+ \frac{R_n}{G_G} \cdot 2\left(G_G + G_{con}\right) \cancel{+ B_G + B_{con}} = 0$$

$$\cancel{\frac{dF}{dB_G} = -\frac{G_u}{G_G^2} - \frac{R_n}{G_G^2}\left(Y_G + Y_{con}\right)^2}$$

$$\frac{dF}{dB_G} = + \frac{R_n}{G_G} \cdot 2\left(B_G + B_{con}\right) = 0$$

$$\Rightarrow \quad B_{G_{OPT}} = -B_{con}$$

$$0 = -\frac{G_u}{G_G^2} - \frac{R_n}{G_G^2}\left(G_G + G_{con}\right)^2 + \frac{R_n}{G_G} \cdot 2\left(G_G + G_{con}\right)$$

$$0 = -G_u - R_n\left(G_G + G_{con}\right)^2 + R_n G_G \cdot 2\left(\cancel{G_G + G_{con}}\right)$$

$$\cancel{G_u + R_n G_{con} = -R_n G_G + 2 R_n G_G}$$

$$\cancel{G_G = \frac{G_u}{R_n} + R_n}$$

$$0 = -G_u - R_n\left(G_G^2 + G_{con}^2 + \cancel{2 G_G G_{con}}\right)$$

$$+ 2 R_n\left(G_G^2 + \cancel{G_G G_{con}}\right)$$

$$G_u + R_n G_{con}^2 = \cancel{G_G^2 R_n}$$

$$G_{G,OPT} = \sqrt{\frac{G_u}{R_n} + G_{cor}^2}$$

NOTE : $Y_{G,OPT} \neq \overline{Y_{in}}$ ← POWER MATCH

$$\cancel{F_{min} = \left(1 + \frac{G_u}{G_G} + \frac{R_n G_{cor}^2}{G_G} + R_n G_G + 2 G_{cor} R_n\right)\Big|_{G_G = G_{G,opt}}}$$

$$F_{min} = 1 + \frac{G_u}{G_G} + \frac{R_n}{G_G}\underbrace{(G_G + G_{cor})^2}_{G_G^2 + 2G_G G_{cor} + G_{cor}^2}\Big|_{G_G = G_{G,OPT}}$$

$$= 1 + \frac{G_u}{G_G} + \frac{R_n G_{cor}^2}{G_G} + R_n G_G + 2 G_{cor} R_n \Big|_{G_G =}$$

$$= 1 + \frac{G_u + R_n G_{cor}^2}{G_{G,OPT}} + 2 G_{cor} R_n + R_n G_{G,OPT}$$

$$G_{G,OPT}^2 = \frac{G_u}{R_n} + G_{cor}^2$$

$$= 1 + R_n G_{G,OPT} + R_n G_{G,OPT} + 2 G_{cor} R_n$$

$$F_{min} = 1 + 2\sqrt{R_n G_u + (R_n G_{cor})^2} + 2 G_{cor} R_n$$

CAN SHOW:

$$\boxed{F = F_{min} + \frac{R_n}{G_G}\left|Y_G - Y_{OPT}\right|^2}$$

$Y_G = G_G + jB_G$ SOURCE ADMITTANCE

PROP OF DEVICE

- Y_{OPT} OPTIMUM SOURCE ADMITTANCE RESULTING IN MINIMUM NF
- F_{min} MINIMUM NOISE FIGURE OF TRANSISTOR
- R_n EQUIVALENT NOISE RESISTANCE OF TRANSISTOR

G_G REAL PART OF SOURCE ADMITTANCE

$$Y_G = \frac{1}{Z_0}\,\frac{1-\Gamma_G}{1+\Gamma_G}$$

$$Y_{opt} = \frac{1}{Z_0}\,\frac{1-\Gamma_{opt}}{1+\Gamma_{opt}}$$

$$|Y_G - Y_{opt}|^2 = \frac{4}{Z_0}\,\frac{|\Gamma_G - \Gamma_{opt}|^2}{|1+\Gamma_G|^2\,|1+\Gamma_{opt}|^2}$$

$$G_G = Re\{Y_s\} = \frac{1}{2Z_0}\left(\frac{1-\Gamma_G}{1+\Gamma_G} + \frac{1-\overline{\Gamma_G}}{1+\overline{\Gamma_G}}\right)$$

$$= \frac{1}{Z_0}\,\frac{1-|\Gamma_G|^2}{|1+\Gamma_G|^2}$$

$$F = F_{min} + \frac{4R_n}{Z_0}\,\frac{|\Gamma_G - \Gamma_{opt}|^2}{(1-|\Gamma_G|^2)\,|1+\Gamma_{opt}|^2}$$

FOR FIXED F, THIS DEFINES A CIRCLE ON THE Γ_G PLANE

$$N = \text{NOISE FIGURE PARAMETER} = \frac{|\Gamma_G - \Gamma_{opt}|^2}{1 - |\Gamma_G|^2}$$

$$= \frac{F - F_{min}}{4 R_n / Z_0} |1 + \Gamma_{opt}|^2$$

$$(\Gamma_G - \Gamma_{opt})(\overline{\Gamma_G} - \overline{\Gamma_{opt}}) = N \left(1 - |\Gamma_G|^2\right)$$

$$\Gamma_G \overline{\Gamma_G} - (\Gamma_G \overline{\Gamma_{opt}} + \overline{\Gamma_G} \Gamma_{opt}) + \Gamma_{opt} \overline{\Gamma_{opt}}$$

$$= N - N|\Gamma_G|^2$$

$$\Gamma_G \overline{\Gamma_G} - \frac{(\Gamma_G \overline{\Gamma_{opt}} + \overline{\Gamma_G} \Gamma_{opt})}{N+1} = \frac{N - |\Gamma_{opt}|^2}{N+1}$$

ADD $|\Gamma_{opt}|^2 / (N+1)^2$ TO BOTH SIDES

$$\left| \Gamma_G - \frac{\Gamma_{opt}}{N+1} \right| = \frac{\sqrt{N(N+1 - |\Gamma_{opt}|^2)}}{N+1}$$

$\rightarrow$ CIRCLE $\quad C_F = \dfrac{\Gamma_{opt}}{N+1}$

$$R_F = \frac{\sqrt{N(N+1 - |\Gamma_{opt}|^2)}}{N+1}$$

<u>NOISE CIRCLE</u> !

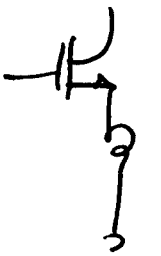
THE NOISE CIRCLE LET'S US ~~FIND~~ TRADE OFF BETWEEN VARIOUS VALUES OF GAIN FOR FIXED NOISE FIGURE

EFFECT OF FB

PARASITICS COMMONLY PROVIDE FB AT HF

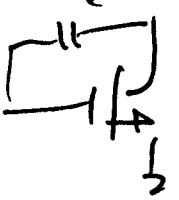
SUTER MMWAVE J. FEB '83

HARTMAN/STRUTT EDT '73



SERIES FB

DEFINE TRANSFORMATION MATRIX



SHUNT FB

$$n = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}$$

$$R_n' = R_n \left| n_{11} + n_{12} Y_{cor} \right|^2 + G_n \left| n_{12} \right|^2$$

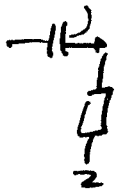
$$G_n' = \frac{G_n R_n}{R_n'} \left| n_{11} n_{22} - n_{12} n_{21} \right|^2$$

$$Y_{cor}' = \frac{R_n}{R_n'} \left(n_{21} + n_{22} Y_{cor} \right)\left(\overline{n_{11}} + \overline{n_{12}}\, \overline{Y_{cor}} \right) + \frac{G_n}{R_n'} n_{22} \overline{n_{12}}$$

$$F_{min} = 1 + 2 R_n' \left(G_{cor}' + G_{on}' \right)$$

$$\text{opt} \begin{cases} R_n = R_n' \\ G_{on} = \sqrt{\dfrac{G_n'}{R_n'} + G_{cor}'^2} \\ B_{on} = -B_{cor}' \end{cases} \quad \text{AS B/F}$$

SERIES FB $\quad Z_S$



$Z_S = R_S + jX_S$

$$n = \begin{pmatrix} 1 & Z_0 \dfrac{S_{21}M - S'_{21}N}{S_{21}C_1' + S'_{21}C_1} \\ 0 & \dfrac{S_{21}C_1'}{S_{21}C_1' + S'_{21}C_1} \end{pmatrix}$$

$$S'_{11} = S'_{22} = \frac{-1}{1+2Z_S}$$

$$S'_{12} = S'_{21} = \frac{2Z_S}{1+2Z_S}$$

$$M = (1+S'_{11})(1-S'_{22}) + S'_{12}S'_{21}$$

$$N = (1+S_{11})(1-S_{22}) + S_{12}S_{21}$$

$$C_1 = (1-S_{11})(1-S_{22}) - S_{12}S_{21}$$

$$C_1' = (1+S'_{11})(1-S'_{22}) - S'_{12}S'_{21}$$

FORMULAS ALSO FOR SHUNT FB & COMMON GATE

SEE ALSO VENDELIN ET AL

MICROWAVE CIRCUIT DESIGN