NOISE IN LINEAR TWO-PORTS

USING THERMODYNAMIC ARGUMENTS, IT CAN BE SHOWN THAT A RESISTOR AT TEMP T GENERATES A NOISE (FLAT POWER SPECTRAL WHITE DENSITY) WITH AVAILABLE POWER

\[ P_a = kTb \]

WHERE \( b \) IS THE BANDWIDTH OF OBSERVATION.

FURTHERMORE, DUE TO THE QUANTIZED NATURE OF CHARGE, A "DC" CURRENT IS ACTUALLY A NOISY CURRENT WITH A NOISE COMPONENT RMS VALUE

\[ I_n = 2qI_{dc} \]

SO DIODES AND BIPOLAR TRANSISTORS HAVE THIS "SHOT NOISE". THIS NOISE IS DUE TO RANDOM "ARRIVAL" TIME OF CHARGE CARRIERS CROSSING A JUNCTION. NOTE THAT IF THE CHARGES ARRIVED UNIFORMLY, THE PSD OF A "DC" CURRENT WOULD HAVE SPECTRAL LINES AT THE HARMONIC OF 1/(ARRIVAL PERIOD).

THIS EVERY PHYSICAL CIRCUIT IS ACCOMPANIED BY NOISE.
**Noise Figure**: A measure of the degradation in the SNR due to a two-port network.

\[ F = \frac{SNR_{in}}{SNR_{out}} \geq 1 \]

Equality for a noiseless 2-port

**Noise 'Gain' Larger than Signal Gain**

\[ SNR_{in} = \frac{P_{in}}{N_{in}} \]
\[ SNR_{out} = \frac{P_{out}}{N_{out}} \]

\[ F = \frac{P_{in}}{P_{out}} \frac{N_{out}}{N_{in}} = 1 + \frac{N_{A}}{N_{in}} \]

Internal noise into amp

\[ = 1 + \frac{N_{A}}{N_{in}} \]

External noise into amp

\[ P_{in} \rightarrow A \rightarrow P_{out} \]

Noiseless Amplifier

\[ F = F_1 + \frac{F_2-1}{G_2} + \ldots + \frac{F_n-1}{G_1 G_2 \ldots G_n} \]

\[ \rightarrow \text{Matched Interface} \]
A "MATCHED" ATTENUATOR

\[ Z_{in} = 50 \Omega \rightarrow 8.56 \Omega \rightarrow 8.56 \Omega \rightarrow 141.8 \Omega \rightarrow Z_{out} = 50 \Omega \]

\[ S_{11} = S_{22} = 0 \quad S_{12} = S_{21} = 0.707 \quad \delta = 3 \text{dB ATTENUATOR} \]

THE AVAILABLE NOISE FROM GENERATOR SIGNAL RESISTOR IS 4 KTB

SINCE THE ATTENUATOR ALSO HAS SAME SOURCE RESISTANCE

\[ P_a = 4 \text{KTB} \]

So \[ P_a = 4 \text{KTB} \Rightarrow N_{out} = 4 \text{KTB/} \text{Hz} \]

\[ \text{Sout} = \sin \cdot \delta \]

\[ \frac{\text{Sout}}{N_{out}} = \frac{\sin \cdot \delta}{N_{in}} \]

\[ \text{SNR}_{in} = \frac{1}{\delta} = 2 \]

\[ \text{SNR}_{out} \]

\[ \text{HFA \ SIGNAL \ POWER FOR SAME \ NOISE} \]

\[ \text{DOUBLE \ THE \ FIXED \ S16/Hz} \]
For any 1-port network, the available noise power is $4kTB$

For a general

The noise of a resistor can alternatively be represented by a signal source and a noiseless resistor:

\[
R \begin{cases}
\text{Real resistor} & = \quad \text{R} \\
\end{cases}
\]

\[
\rightarrow P_A = (\frac{\sqrt{V_n}}{2})^2 \frac{1}{R} = \frac{\sqrt{V_n^2}}{4R} = 4kTB
\]

\[
\sqrt{V_n^2} = 4kTB R
\]

\[
\rightarrow P_A = \left(\sqrt{\frac{\overline{i_n^2}}{2}}\right)^2 R = \quad kTB
\]

\[
\frac{\overline{i_n^2}}{4} R = kTB
\]

\[
\overline{i_n^2} = \frac{4kTB}{R}
\]
In fact, for any 1-port circuit can be represented by a lossless 1-port and an eq. noise generator:

\[ Z_m = R + jX \]

\[ P_A = kT B \]

\[ P_L = \] 

\[ V_n^2 = 4kTB \cdot R \]

\[ V_n^2 = 4kTB \cdot \text{Re}(Z) \]

\[ P_L = \frac{V_n^2}{4R} = kTB \]

\[ V_m^2 = 4kTB \cdot \text{Re}(Z) \]

**Mystery**: An antenna has \( Z_m = R_0 + jX_0 \)

\[ Z_0 = R_0 + jX_0 \]

with \( \text{Re}(Z_m) = R_0 \)

If the materialization is a perfect conductor, then the physical antenna resistance is zero!
WHERE DOES THE NOISE COME FROM?
INCOMING RADIATION!

ALL BODIES AT TEMP T RADIATE BLACK BODY RADIATION. IN FACT, THIS IS A CLEVER PROOF OF THE EXISTENCE OF RESISTOR THERMAL NOISE.

\[ P_L = \frac{2KT}{\lambda^2} \]

\[ \frac{1}{4\pi} \frac{1}{2} \]

\[ Y_{01} = KT \]

SINCE THE ANTENNA IS IRRADIATED BY BLACK BODY RADIATION, IT DELIVERS POWER TO LOAD RESISTOR R.

BUT SINCE THE BLACK BODY IS IN THERMAL EQUILIBRIUM WITH THE RESISTOR, THE RESISTOR MUST LIKEWISE DELIVER POWER TO BLACK-BODY.

A NOISY TWO PORT CAN ALSO BE CHARACTERIZED BY A NOISE-LESS 2-PORT AND EQUIVALENT NOISE GENERATORS:

(Result by Rothe and Dahlke)
\[ I_1 = y_{11} V_1 + y_{12} V_2 + i_{n1} \]
\[ I_2 = y_{21} V_1 + y_{22} V_2 + i_{n2} \]
\[ V_1 = Z_{11} I_1 + Z_{12} I_2 + V_{n1} \]
\[ V_2 = Z_{21} I_1 + Z_{22} I_2 + V_{n2} \]

**ABCD - REPRESENTATION**

\[ V_1 = AV_2 + 8I_2 + V_A \]
\[ I_1 = CV_2 + DI_2 + I_A \]

Can show:
\[ I_A = -\frac{i_{n2}}{y_{21}} = -V_{n2} - \frac{V_{n2} Z_{11}}{Z_{21}} \]
\[ I_A = i_{n1} - \frac{i_{n2} y_{11}}{y_{21}} = -\frac{V_{n2}}{Z_{21}} \]
In general the noise voltage and current are correlated

\[ I_A = I_u + I_C \]

\[ I_C = Y_{cor} V_A \]

\[ I_A = I_u + Y_{cor} V_A \]

\[ \langle V_A, I_A \rangle = \langle V_A, I_u \rangle + \langle V_A, Y_{cor} V_A \rangle \]

\[ \langle V_A, I_A \rangle = 0 + Y_{cor} V_A^2 \]

\[ Y_{cor} = \frac{\langle V_A, I_A \rangle}{V_A^2} \]

\[ C = \frac{\langle V_A, I_A \rangle}{\sqrt{\langle V_A, V_A \rangle \langle I_A, I_A \rangle}} = \frac{\langle V_A, I_A \rangle}{\sqrt{V_A^2 I_A^2}} \]

\[ C = Y_{cor} \cdot \sqrt{\frac{V_A^2}{I_A^2}} \]

Convenient to express \( V_A^2 \) and \( I_A^2 \) in terms of \( R_n \) and \( G_n \)

\[ R_n = \frac{V_A^2}{4kT} \]

\[ G_n = \frac{I_A^2}{4kT} \]

\[ G_u = \frac{I_u^2}{4kT} \]
\[ I_c^2 = 4kTBG_0 \]

\[ I_c = \frac{I_A + Y_G V_A}{I_G} \]

\[ F = 1 + \left( \frac{I_A + Y_G V_A}{I_G} \right)^2 \]

\[ \Delta I_c^2 = 2kT \left( Y_G + Y_{in} + 6u \right) \]

\[ I_A = Y_{cor} V_A + I_u \]

\[ V_{BS} \]

\[ I_A + Y_G V_A = (Y_G + Y_{cor}) V_A + I_u \]

\[ \left| I_A + Y_G V_A \right|^2 = \left( Y_G + Y_{cor} \right)^2 V_A^2 + I_u^2 \]

\[ F = 1 + \left( Y_G + Y_{cor} \right)^2 \frac{AKT R_i B}{AKT G_0 B} + \frac{AKT G_u}{AKT G_0 B} \]

\[ = 1 + \frac{G_U}{G_0} + \frac{R_n}{G_0} \left[ (G_0 + G_{cor})^2 + (G_0 + 2u)^2 \right] \]
NOTE THAT \( F \) IS A FUNCTION OF \( G_0 \) AND \( B_0 \) FOR A FIXED 2-PORT.

\[
\frac{dF}{dG_0} = -\frac{G_u}{G_0^2} - \frac{R_n}{G_0} \left( Y_0 + Y_{\text{con}} \right)^2
\]

\[+ \frac{R_n}{G_0} \cdot 2 \left( G_0 + Y_{\text{con}} \right) Y_{\text{con}} + B_0 = 0\]

\[
\frac{dF}{dB_0} = \frac{R_n}{G_0} \cdot 2 \left( G_0 + Y_{\text{con}} \right) = 0
\]

\[\Rightarrow B_{0_{\text{opt}}} = -Y_{\text{con}}\]

\[0 = -\frac{G_u}{G_0^2} - \frac{R_n}{G_0^2} \left( G_0 + Y_{\text{con}} \right)^2 + \frac{R_n}{G_0} \cdot 2 \left( G_0 + Y_{\text{con}} \right)\]

\[0 = -\frac{G_u}{G_0} - \frac{R_n}{G_0} \left( G_0 + Y_{\text{con}} \right)^2 + \frac{R_n}{G_0} \cdot 2 \left( G_0 + Y_{\text{con}} \right)\]

\[
G_0 = \frac{G_u}{R_n} + R_n
\]

\[
B_{0_{\text{opt}}} = \frac{R_n G_{\text{con}}}{G_0} + G_0
\]

\[
d = -G_u - R_n \left( G_0^2 + G_{\text{con}}^2 + 2 G_0 G_{\text{con}} \right)
\]

\[+ 2 R_n \left( G_0^2 + G_{\text{con}} \right) G_0\]

\[G_u + R_n G_{\text{con}}^2 = G_0^2 \cdot R_n\]
\[
G_{opt} = \sqrt{\frac{G_u}{R_n} + G_{con}^2}
\]

**Note:** \(Y_{G, opt} \neq Y_m\)

\[
F_{min}^2 = (1 + \frac{G_u}{G_0} + \frac{R_n G_{con}}{G_0} + \frac{R_n G_0}{G_0} + 2 G_{con} R_n)
\]

\[
F_{min} = 1 + \frac{G_u}{G_0} + \frac{R_n G_{con}}{G_0} + \frac{R_n G_0}{G_0} + 2 G_{con} R_n \quad | \quad G_0 = G_{opt}
\]

\[
G_{opt}^2 = \frac{G_u}{R_n} + G_{con}^2
\]

\[
F_{min} = 1 + \frac{R_n G_{opt}^2 + R_n G_0}{G_{opt}} + 2 G_{con} R_n + R_n G_{opt}
\]

\[
G_{opt} = \frac{G_u}{R_n} + G_{con}
\]

\[
F_{min} = 1 + 2 \sqrt{R_n G_u + (R_n G_{con})^2} + 2 G_{con} R_n
\]

Can show:
\[
F = F_{min} + \frac{R_n}{G_0} |Y_G - Y_{opt}|^2
\]
\[ Y_0 = G_0 + jB_0 \]

**Source Admittance**

OPTIMUM SOURCE ADMITTANCE RESULTING IN MINIMUM NF

MINIMUM NOISE FIGURE OF TRANSISTOR

EQUIVALENT NOISE RESISTANCE OF TRANSISTOR

REAL PART OF SOURCE ADMITTANCE

\[
Y_0 = \frac{1}{20} \frac{1 - \Gamma_0}{1 + \Gamma_0}
\]

\[
Y_{opt} = \frac{1}{20} \frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}}
\]

\[
|Y_0 - Y_{opt}|^2 = \frac{4}{20} \frac{|\Gamma_0 - \Gamma_{opt}|^2}{|1 + \Gamma_0|^2 (1 + \Gamma_{opt})^2}
\]

\[
G_0 = \text{Re} \left\{ Y_5 \right\} = \frac{1}{20} \left( \frac{1 - \Gamma_0}{1 + \Gamma_0} + \frac{1 - \Gamma_6}{1 + \Gamma_6} \right)
\]

\[
= \frac{1}{20} \frac{1 - |\Gamma_6|^2}{(1 + |\Gamma_6|^2)}
\]

\[
F = F_{min} + \frac{4R_n}{20} \frac{|\Gamma_6 - \Gamma_{opt}|^2}{(1 - |\Gamma_6|^2) (1 + \Gamma_{opt})^2}
\]
For fixed $F$, this defines a circle on the $\Gamma_0$ plane.

\[
N = \text{noise figure} = \frac{|\Gamma_0 - \Gamma_{\text{opt}}|^2}{1 - |\Gamma_0|^2} = \frac{F - F_{\text{min}}}{4 R_{\text{in}/20}} \left(1 + |\Gamma_{\text{opt}}|^2\right)
\]

\[
(\Gamma_0 - \Gamma_{\text{opt}})(\bar{\Gamma}_0 - \bar{\Gamma}_{\text{opt}}) = N \left(1 - |\Gamma_0|^2\right)
\]

\[
\Gamma_0 \bar{\Gamma}_0 - (\Gamma_0 \bar{\Gamma}_{\text{opt}} + \bar{\Gamma}_0 \Gamma_{\text{opt}}) + \Gamma_{\text{opt}} \bar{\Gamma}_{\text{opt}} = N - N |\Gamma_0|^2
\]

\[
\Gamma_0 \bar{\Gamma}_0 - (\Gamma_0 \bar{\Gamma}_{\text{opt}} + \bar{\Gamma}_0 \Gamma_{\text{opt}}) = \frac{N - |\Gamma_{\text{opt}}|^2}{N + 1}
\]

Add \(\frac{|\Gamma_{\text{opt}}|^2}{(N+1)^2}\) to both sides.

\[
\left|\Gamma_0 - \frac{\Gamma_{\text{opt}}}{N+1}\right| = \frac{\sqrt{N(N+1 - |\Gamma_{\text{opt}}|^2)}}{N+1}
\]

→ circle \(C_F = \frac{\Gamma_{\text{opt}}}{N+1}\)

\(R_F = \frac{\sqrt{N(N+1 - |\Gamma_{\text{opt}}|^2)}}{N+1} \)
SERIES F8  $Z_3$

\[ Z_3 = R_0 + jX_0 \]

\[
\mathbf{n} = \begin{pmatrix}
1 & 2 \frac{S_{11} M - S_{21} N}{S_{21} C_1^* + S_{21} C_1} \\
0 & \frac{S_{21} C_1}{S_{21} C_1^* + S_{21} C_1}
\end{pmatrix}
\]

\[ S_{11}' = S_{22} = \frac{-1}{\lambda + 2 \lambda S_3} \]

\[ S_{12}' = S_{21}' = \frac{2 \lambda S_3}{\lambda + 2 \lambda S_3} \]

\[ M = (1 + S_{11}')(1 - S_{22}) + S_{21}' S_{21}' \]

\[ N = (1 + S_{11}')(1 - S_{22}) + S_{12} S_{21} \]

\[ C_1 = (1 - S_{11})(1 - S_{22}) - S_{12} S_{21} \]

\[ C_1' = (1 + S_{11})(1 - S_{12}') - S_{12}' S_{21}' \]

FORMULAS ALSO FOR SHUNT F8 & COMMON GATE

SEE ALSO VENGELIN ET AL

MICROWAVE CIRCUIT DESIGN