EE21A Section 1
August 30, 2019

1 Preface

2 Methods of proof

1. Direct proof
   Logically derive the conclusion by combining the definitions, assumptions, lemmas, theorems etc.

2. Proof by contrapositive
   Establish the conclusion “if A then B” by showing the equivalent contrapositive statement “if not B then not A”.

3. Proof by contradiction
   First assume that the conclusion is false, and then derive a logical contradiction. Therefore, the conclusion must be true.

4. Proof by construction
   Construct an example that shows the fallacy or validity of a statement. Usually useful for disproving an assertion such as “all X are Y” or confirming a statement such as “there exists a W such that Z.”

5. Proof by induction (if the conclusion involves the natural number \( n \in \{1, 2, 3, \cdots \} \))
   First prove that the conclusion is true when \( n = 1 \). Then, assuming the conclusion is true whenever \( n \leq k \), prove that the conclusion is true when \( n = k + 1 \).

For a description of other methods, see http://en.wikipedia.org/wiki/Mathematical_proof.

Definition 1. A number \( n \in \mathbb{Z} \) is even if \( n = 2k \) for some \( k \in \mathbb{Z} \).

Definition 2. A number \( q \) is rational if there exist \( a, b \in \mathbb{Z} \) with \( b \neq 0 \) such that \( q = \frac{a}{b} \).
Problem 1 (Direct method). Prove that the sum of two odd numbers is even.

Problem 2 (Direct method). Prove that the product of an even number and any other number is even.

Problem 3 (Proof by contrapositive). Show that, if $x^2$ is even, then $x$ is even.

Problem 4 (Proof by contrapositive). If $ab$ is even, then either $a$ or $b$ is even.

Problem 5 (Proof by contradiction). Prove that $\sqrt{2}$ is an irrational number.

Problem 6 (Proof by construction). Prove that not all odd numbers are prime.

Problem 7 (Proof by induction). Prove that $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$. 