Periodic Orbits

Problem 1

Consider the system
\[
\dot{x}_1 = -x_1 + x_1^3 + x_1x_2^2 \\
\dot{x}_2 = -x_2 + x_2^3 + x_1^2x_2
\]

Find its equilibria and sketch the system in the plane. Hint: Use polar coordinates.

Problem 2

Consider the system
\[
\dot{x}_1 = x_1x_2 \\
\dot{x}_2 = x_2
\]

Show that this system has no limit cycles.

Problem 3

Show that the system
\[
\dot{x} = -y + x(1 - x^2 - y^2)^2 \\
\dot{y} = x + y(1 - x^2 - y^2)^2
\]

has a semi-stable limit cycle. Sketch the phase portrait for this system.

Problem 4

Show that the linear system \( \dot{x} = Ax \) with
\[
A = \begin{bmatrix} 0 & -4 \\ 1 & 0 \end{bmatrix}
\]
has a continuous band of cycles \( \Gamma_\alpha : \gamma_\alpha(t) = (\alpha \cos(2t), \alpha/2 \sin(2t)) \) for \( \alpha \in (0, \infty) \). What is the period of each of those cycles?