Problem 1. Consider the system
\[
\begin{align*}
\dot{x}_1 &= x_2 - x_3 \\
\dot{x}_2 &= -x_1 x_3 - x_2 + u \\
\dot{x}_3 &= -x_1 + u
\end{align*}
\]
with output \( y = h(x) \), a scalar function chosen by you. Design a feedback linearizing control law \( u(x, v) \) based on your choice of \( h \), and discuss the effect of different choices of \( h \).

Problem 2. Consider the system
\[
\begin{align*}
\dot{x}_1 &= -x_1 + x_2 - x_3 \\
\dot{x}_2 &= -x_1 x_3 - x_2 + u \\
\dot{x}_3 &= -x_1 + u
\end{align*}
\]
with output \( y = x_3 \). Design a feedback linearizing control law. What can you say about the stability of the overall system when this control law is used?

Problem 3: Linearization of a Translational Oscillator.

The dynamics of the translational oscillator with rotating actuator (TORA) are given below.

To get a little background and see a picture of the TORA, please refer to the paper: R. T. Bupp, D. S. Bernstein, and V. T. Coppola, “A Benchmark Problem for Nonlinear Control Design”, Int. J. of Robust and Nonlinear Control, 8, 307-310, 1998, available at the link:


\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{-x_1 + \epsilon x_4 \sin x_3}{1 - \epsilon^2 \cos^2 x_3} + \frac{-\epsilon \cos x_3}{1 - \epsilon^2 \cos^2 x_3} u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{1 - \epsilon^2 \cos^2 x_3} (\epsilon \cos x_3 (x_1 - \epsilon x_4^2 \sin x_3) + u)
\end{align*}
\]

where \( x_1 \) and \( x_2 \) are the displacement and the velocity of the platform, \( x_3 \) and \( x_4 \) are the angle (\( \theta \)) and the angular velocity of the rotor carrying the mass \( m \), and \( u \) is the control torque applied to the rotor. The parameter \( \epsilon < 1 \) depends on properties of the rotor, spring constant, as well as the masses \( m \) and \( M \).

This system is a simplified model of a dual-spin spacecraft and has been used to study the resonance capture phenomenon. It has also been studied to investigate the utility of a rotational proof-mass actuator for stabilizing translational motion.

With \( y = x_3 \) as the output, determine the relative degree and the zero dynamics. Give a physical interpretation of the zero dynamics.