EE224A Fall 2009: Homework #6

Assigned: 10/29/2009 --- Due: 11/12/2009 in class *before* lecture

Problem 1

$$y[n] = h_0 x[n] + h_1 x[n-1] + w[n], \quad n \geq 1$$

1. Consider the 2-tap channel with i.i.d. BPSK input symbols starting at time 0. For symbol energy $E = 1$ and noise variance $\sigma^2 = 1$, generate on MATLAB a random realization of the output of the channel for 10 time instants, i.e. $y[0]$ up to $y[9]$. Run the Viterbi algorithm by hand or by computer, showing the trellis and the shortest paths at each stage. Also, give the final maximum-likelihood sequence. In the sequence, are there any input symbols for which the algorithm will not change its guess even on observation of more outputs after time 9?

2. Now suppose instead of BPSK, 4-PAM symbols are used. Draw the corresponding finite state machine and the trellis. Describe how the Viterbi algorithm would work but there is no need to run any simulations.

3. Redo the last part using BPSK symbols but with a channel with 3 taps.

Problem 2

Consider the 2-tap ISI channel:

$$y[n] = h_0 x[n] + h_1 x[n-1] + w[n], \quad n \geq 1.$$

We wish to communicate using the OFDM approach.

1. What is the minimum length of the cyclic prefix L needed?

2. For an OFDM symbol of length N_c, denote the OFDM channel coefficients, as in the lecture notes, by $\tilde{h}_0, \ldots, \tilde{h}_{N_c-1}$. Choose $h_0 = 1$ and $h_1 = 0.5$ and suppose that that the sampling period $1/W$ is equal to 1μs.

   (a) With $N_c = 2$, evaluate the OFDM channel coefficients $\tilde{h}_0$ and $\tilde{h}_1$.

   (b) Plot the magnitude squared of the channel coefficients versus the tone number for different values of N_c; in particular, plot for $N_c = 32$, 64, 128 and 256. Specifically, plot $|\tilde{h}_k|^2$ at point $\frac{kW}{N_c}$ for $k = 0, \ldots, N_c - 1$.

   (c) Consider the *discrete-time Fourier transform* of the ISI channel defined as follows:

$$H(f) \stackrel{\text{def}}{=} \sum_{l=0}^{1} h_l \exp\left(-\frac{j2\pi l f}{W}\right), \quad f \in [0, W].$$

   Plot $|H(f)|^2$ as a function of f. How does this plot relate to the previous plots you have drawn for different values of N_c, earlier in this question?

Problem 3

A (d, k) run-length limited code is one each coded sequence is a binary string satisfying the property that the 1's are separated by at least d 0's and at most k 0's. They are often in magnetic-recording applications.

1. Explain how information is conveyed using a run-length limited code.
2. Show that a (d, k) run-length limited code can be represented as a finite state machine. How many states are needed?
3. Suppose the coded sequence passes through a bit-flipping channel such that each symbol is flipped independently with probability p. Derive the Viterbi algorithm for a $(1, 3)$ code. Draw the trellis and explain the processing at each stage.