Local Enhancement
Using local histogram Analysis

Define square or rectangle neighborhood over each pixel, move center of "window" across the image.

At each location: compute histogram in the window. Do hist. eq.
Use local stats for snapshot:

Stat → mean, variance:

global mean: \( M_G = \frac{1}{l-1} \sum_{i=0}^{l-1} r_i \cdot p(r_i) \)

global variance: \( \text{Var}_G = \frac{1}{l-1} \sum_{i=0}^{l-1} (r_i - M_G)^2 \cdot p(r_i) \)

Local mean:

Let \( S_{xy} \) be the neighborhood, centering at \((x, y)\).

local mean: \( m = \frac{1}{S_{xy}} \sum_{(s,t) \in S_{xy}} r_{st} \cdot p(r_{st}) \)

local variance: \( \sigma^2 = \frac{1}{S_{xy}} \sum_{(s,t) \in S_{xy}} (r_{st} - m_{S_{xy}})^2 \cdot p(r_{st}) \)
Goal: Enhance dark areas while leaving the bright areas unchanged as possible.

- Detect regions that have both:
  - dark \( \rightarrow \) local mean has to be small compared to global mean.
  - low contrast

\[
\begin{align*}
M_{S_{xy}} & \leq K_0 M_G \\
\text{local mean} & \leq K_0 \leq 1 \\
\text{global mean} & \text{constant} \\
S_{S_{xy}} & \leq K_2 \text{ Var}_G \\
\text{local var} & \text{constant} \\
K_2 & \leq 0.4
\end{align*}
\]
(c) hot too low of contour
leave that region unchained

\[ g(x_1, y) = \begin{cases} \varepsilon_g & \text{if } m_5 \leq k_0 M_6 \\
0 & \text{else} \end{cases} \]

\[ \kappa \varepsilon_g \leq \varepsilon_g \]

\[ k \varepsilon_g \leq \varepsilon_g \]
Emulation is an arithmetic/logic operation:

logic: \( \text{AND}, \text{OR}, \text{NOT} \rightarrow \text{functionally complete} \)

\( \text{NAND} \rightarrow \text{functionally complete} \)

\( \text{NOT} \rightarrow \text{negative information} \)

\( \text{AND}, \text{OR} \rightarrow \text{masking} \)

image subtraction

\[ g(x, y) = f(x, y) - h(x, y) \]

mask.
Emblunt Averaging

\[ g(x,y) = f(x,y) + \eta(x,y) \]

Series of noisy images \( \{g_i(x,y)\} \).

Estimated \( \bar{g}(x,y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x,y) \)

\[ E[\bar{g}(x,y)] = f(x,y) \]

\[ \text{Var} [\bar{g}(x,y)] = \frac{\sigma^2}{\bar{g}(x,y)} = \frac{1}{K} \sigma^2(x,y) \]

Variance of \( \bar{g} \) shrinks as \( K \to \infty \).
Spatial Filtering

\[ g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x+s, y+t) \]

If \( w(s, t) \to LPF \to \text{blurred} \to \text{noise} \)

\[ w(s, t) \to HPF \to \text{sharpening} \]
Order Statistics:

- Median
- Max
- Min

Salt + Pepper noise
Impulsive noise
Shaping Spatial Filters

Objective: highlight on certain detail

LSI → FIR → H.P.

Derivative operator

1st order: \( \frac{df}{dt} \)

2nd order: \( \frac{d^2f}{dx^2} \)
2nd order derivative

Simplest isotropic. \( \nabla^2 f = \text{hessian} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \)

\[
\begin{align*}
\frac{\partial^2 f}{\partial x^2} &= f(x+1, y) - f(x-1, y) - f(x, y) \\
\frac{\partial^2 f}{\partial y^2} &= f(x, y+1) + f(x, y-1) - 2 f(x, y)
\end{align*}
\]
\[ \Delta f = f(x_0 + 1, y) - f(x_0, y) + f(x + 1, y_0) - f(x, y_0) \]

Show, include drawings.