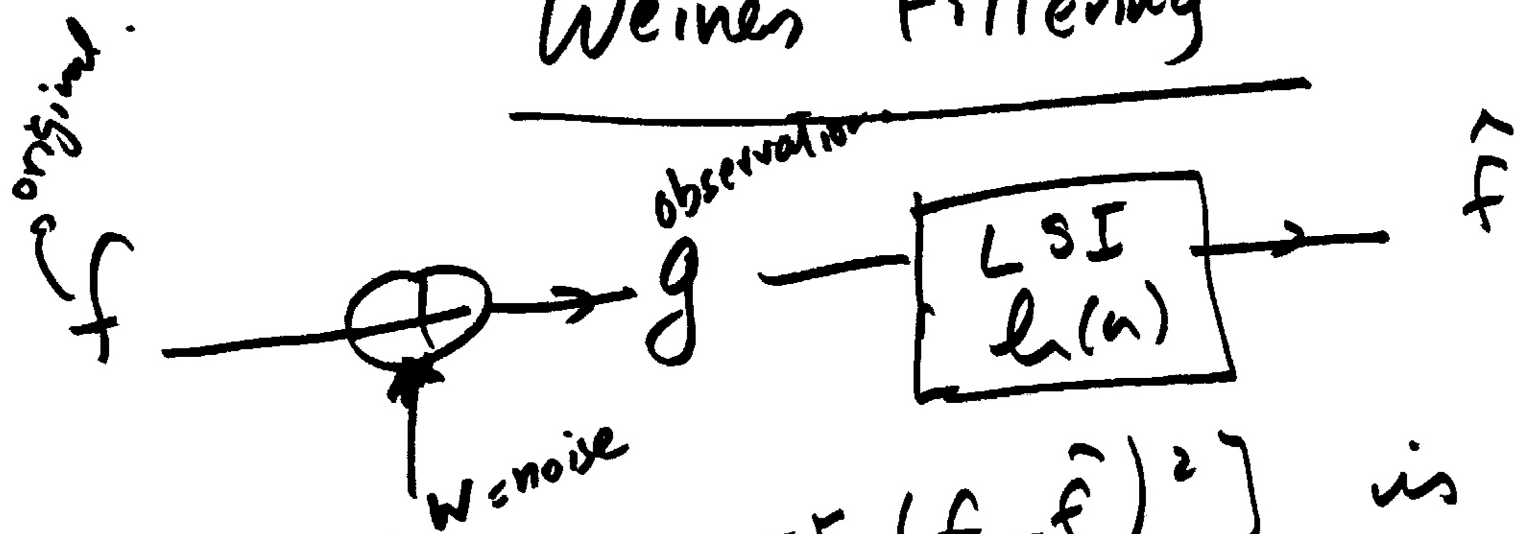
Weiner Filtering

04/05/06

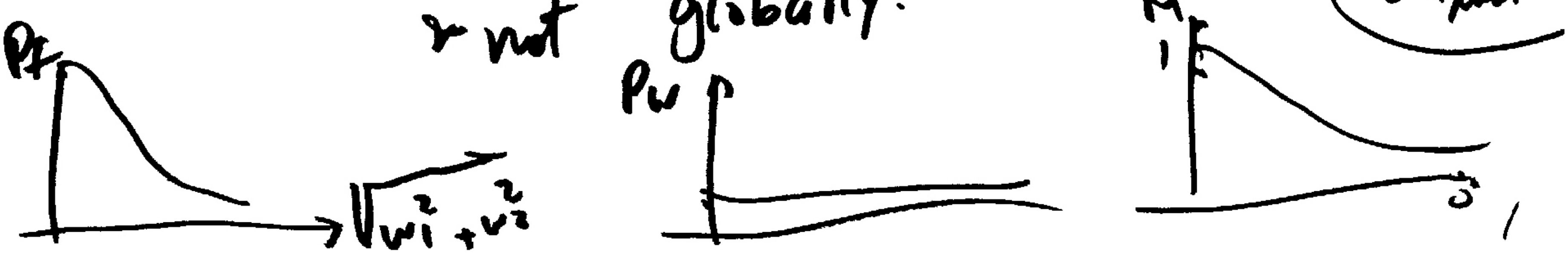


Choose $h(n)$ s.t. $E[(f-\hat{f})^2]$ is minimized

$$H(\omega_1, \omega_2) = \frac{P_f(\omega_1, \omega_2)}{P_f(\omega_1, \omega_2) + P_w(\omega_1, \omega_2)}$$

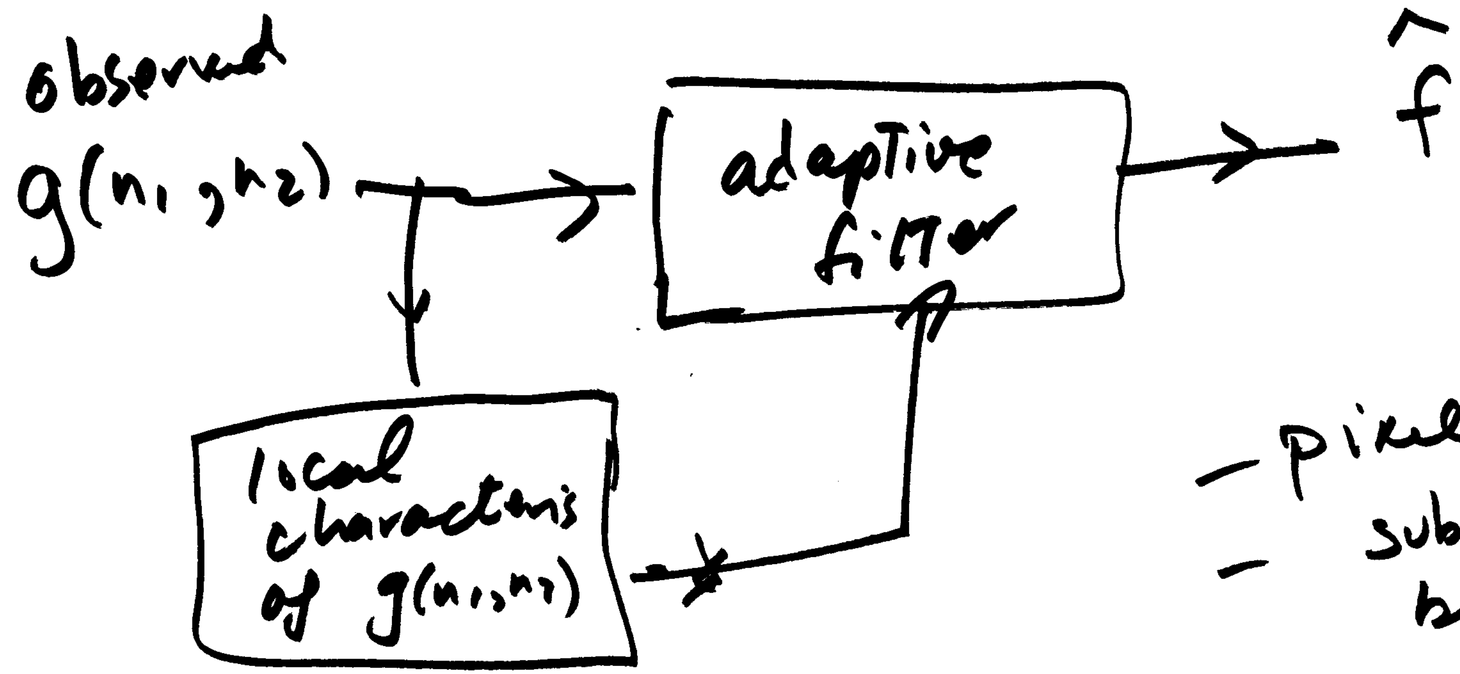
Images are only locally stationary $\Rightarrow$ Blurry outputs

& not globally.



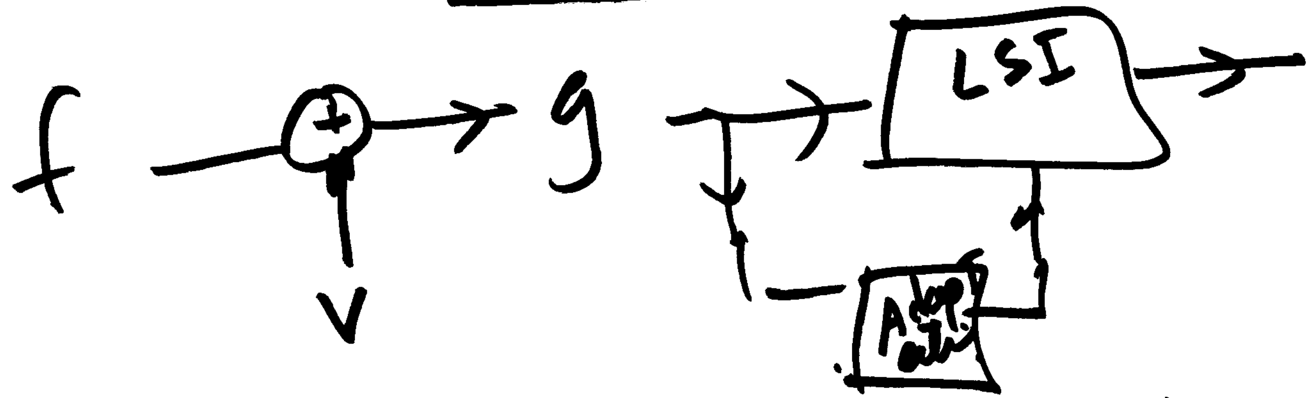
Adaptive Weiner Filter :

Basic idea : no image stationary.



- pixel by pixel
- sub image by subimage
- block by block.

One possible Adaptive filter



- Assume, noise v is white, zero mean, variance σ_v^2

$$P_v(\omega_1, \omega_2) = \sigma_v^2$$

- Assume signal satisfies the following model:

$$f(n_1, n_2) = m_f + \sigma_f\, w(n_1, n_2)$$

- w is a white noise, zero mean, unit variance process.

$\Rightarrow$ Weiner filter assuming zero mean input.

$$H(\omega_1, \omega_2) = \frac{P_f(\omega_1, \omega_2)}{P_f(\omega_1, \omega_2) + P_v(\omega_1, \omega_2)}$$

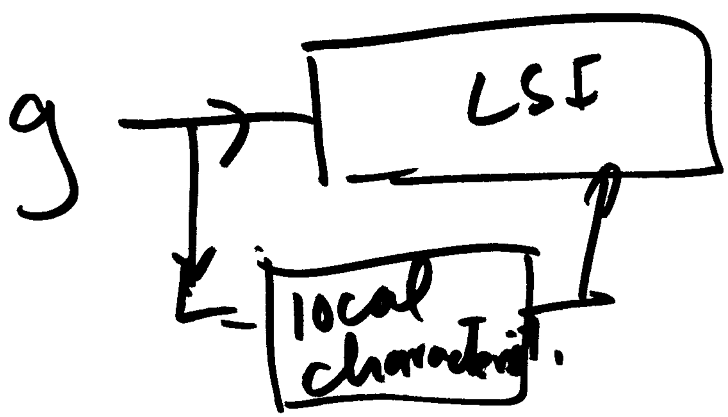
$$H(\omega_1, \omega_2) = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2}$$

$$\Rightarrow h(n_1, n_2) = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2} \delta(n_1, n_2)$$

"Taking care of the mean"

$$\hat{f}(n_1, n_2) = m_f + \left(g(n_1, n_2) - m_f\right) * h(n_1, n_2)$$

$$\hat{f}(n_1, n_2) = m_f + \left(g(n_1, n_2) - m_f\right) * \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2} \delta(n_1, n_2)$$



m_f, σ_f^2 } are both fns of (n_1, n_2)

$$\hat{f}(n_1,n_2) = m_f(n_1,n_2) + \left(g(n_1,n_2) - m_f(n_1,n_2)\right)\frac{\sigma_f^2(n_1,n_2)}{\sigma_f^2(n_1,n_2)+\sigma_v^2}$$

$$\boxed{\hat{f}(n_1,n_2) = m_f(n_1,n_2) + \left[g(n_1,n_2) - m_f(n_1,n_2)\right)\frac{\sigma_f^2(n_1,n_2)}{\sigma_v^2+\sigma_f^2(n_1,n_2)}} \quad (*)$$

2 cases: (1) $\sigma_f^2 \ll \sigma_v^2 \Rightarrow \hat{f} \simeq \underline{m_f(n_1,n_2)}$

flat parts of image.

for (n_1,n_2) or regions where image is "flat"

$\Rightarrow$ output is just mean.

(2) $\sigma_f^2 \gg \sigma_v^2 \Rightarrow \hat{f} \simeq g(n_1,n_2)$

Textured part of image

How to estimate m_f?

$$f \longrightarrow \oplus \longrightarrow g$$
$$\uparrow$$
$$v$$

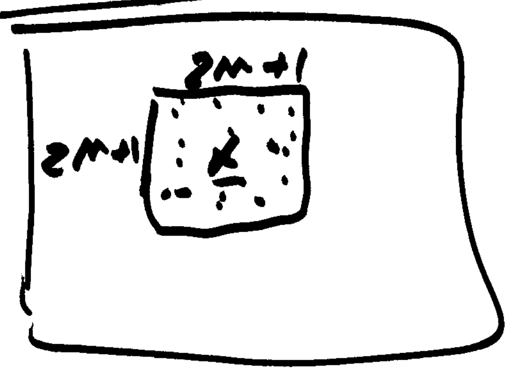
$\cdot\, m_v = 0$

$$m_g = m_v + m_f$$

$$\Longrightarrow \hat{m}_f = \bar{m}_g =$$

$$\boxed{\hat{m}_f^{(n_1,n_2)} = m_g = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} g(k_1,k_2)}$$

plug this into ⓚ

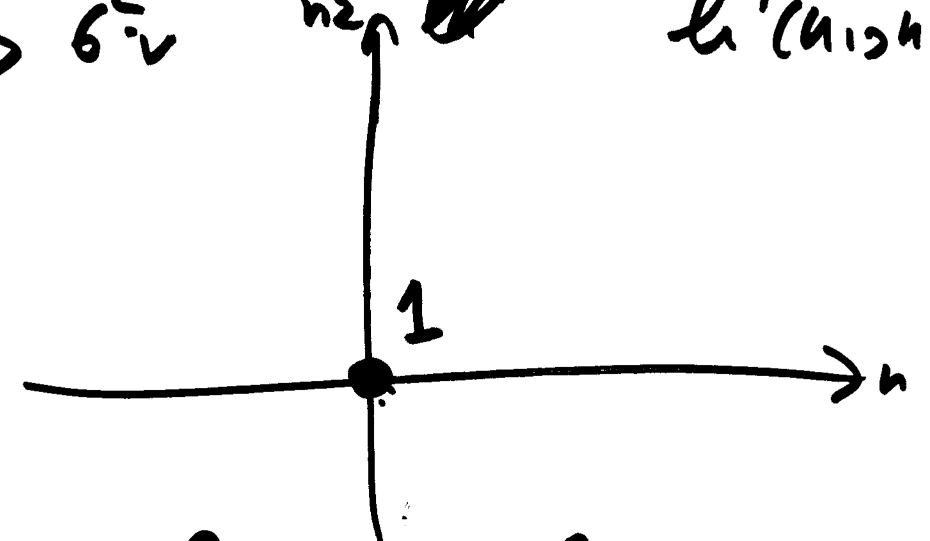


g

$$\hat{f} = g * h'$$

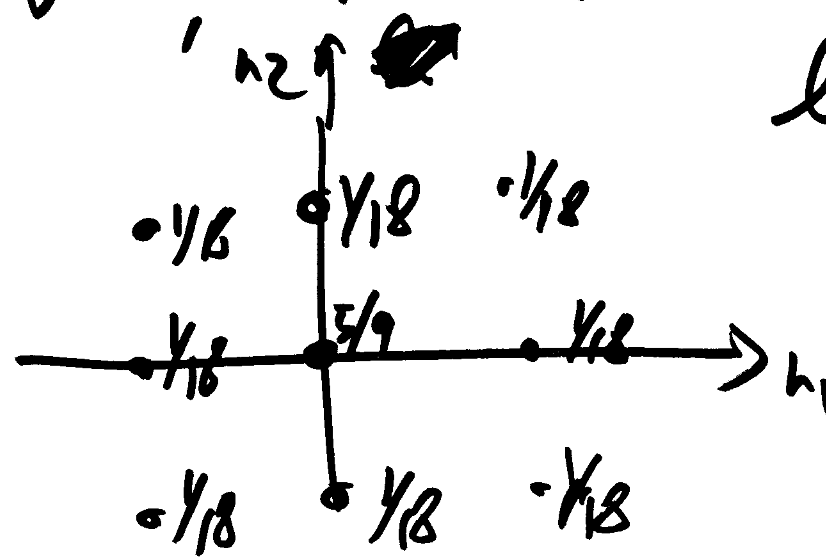
$$h'(n_1, n_2) = \begin{cases} \dfrac{\sigma_f^2 + \dfrac{\sigma_v^2}{(2M+1)^2}}{\sigma_f^2 + \sigma_v^2} & n_1 = n_2 = 0 \\[2ex] \dfrac{\dfrac{\sigma_v^2}{(2M+1)^2}}{\sigma_f^2 + \sigma_v^2} & |n_1| \le M,\ |n_2| \le M \text{ except } n_1 = n_2 = 0 \\[2ex] 0 & \text{otherwise} \end{cases}$$

3 cases (1) $\sigma_f^2 \gg \sigma_v^2$ $\Longrightarrow$ $h'(n_1,n_2)$



If energy/variance of f is much larger than noise, let the signal through.

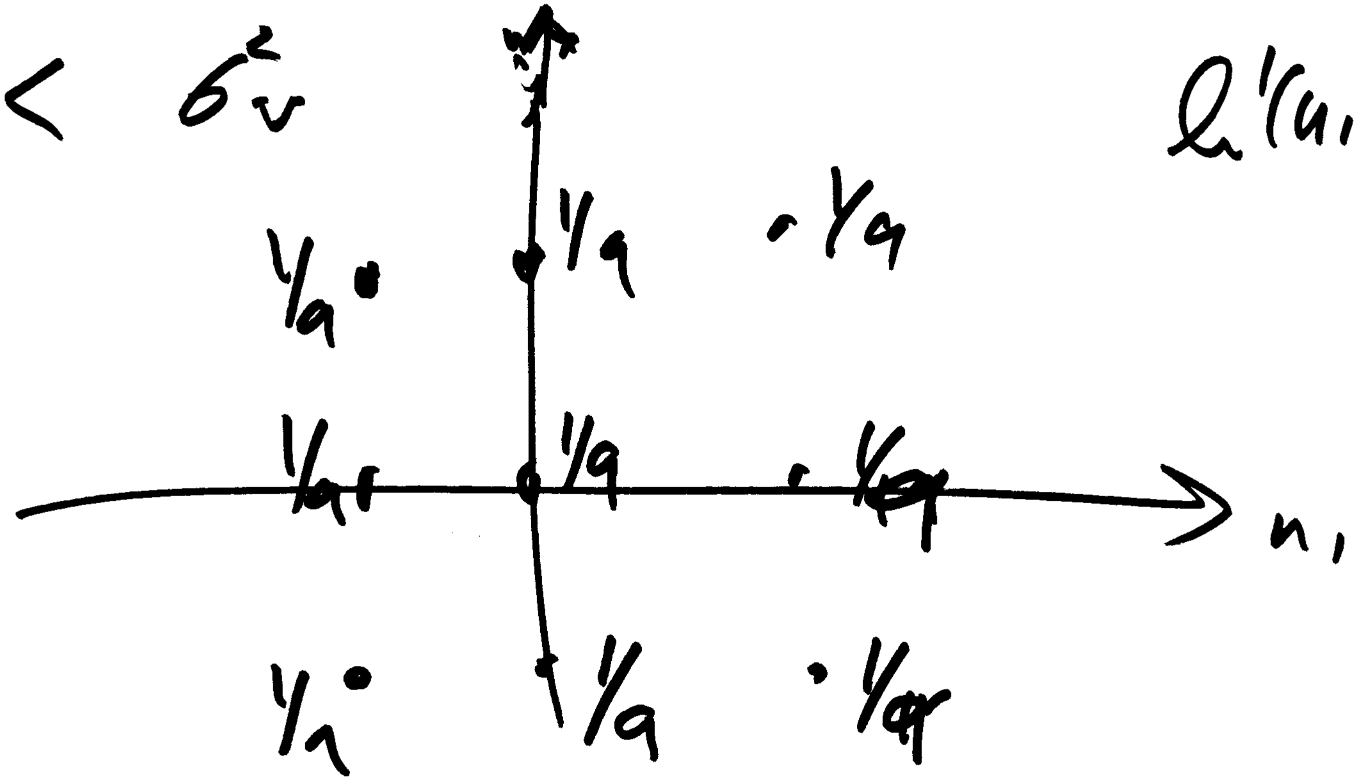
(2) $\sigma_f^2 \approx \sigma_v^2$, $M=1 \Rightarrow$ h'

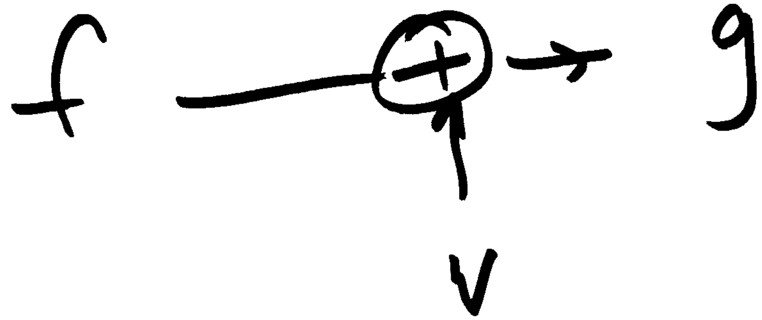


③ $\sigma_f^2 << \sigma_v^2$

$M = 1$

$h'(n_1, n_2)$





f, v are independent

$g = v + f \Longrightarrow \sigma_g^2 = \sigma_v^2 + \sigma_f^2$

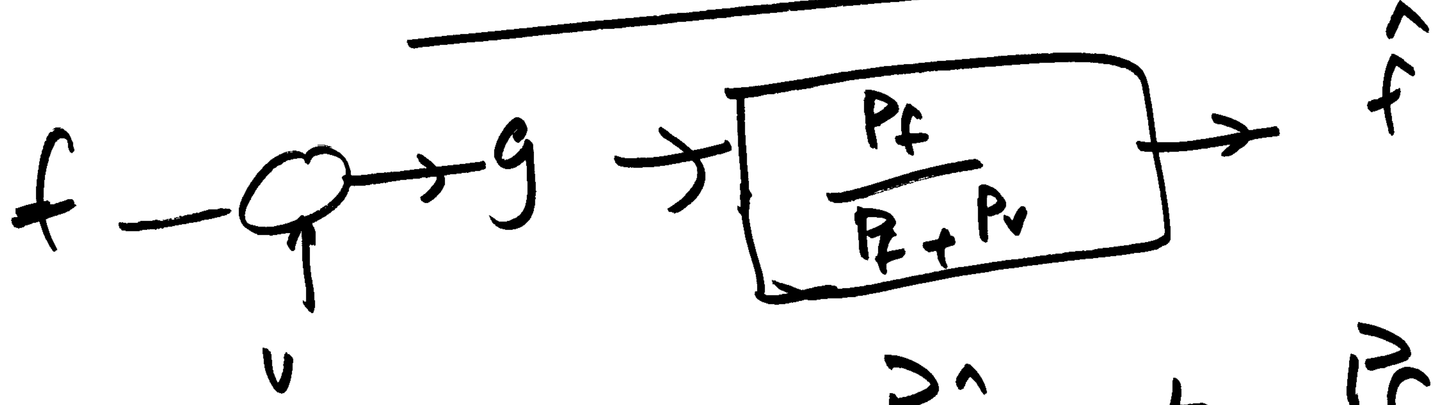
$\Longrightarrow \cancel{\hat{\sigma}_f^2 = \sigma_g^2} - \sigma_v^2$??

$$\hat{\sigma}_f^2 = \begin{cases} \sigma_g^2 - \sigma_v^2 & \text{only if } \sigma_g^2 > \sigma_v^2 \\ 0 & \text{otherwise.} \end{cases}$$

$\hat{\sigma}_g^2$

$$\hat{\sigma}_g^2(n_1, n_2) = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} \left[g(k_1, k_2) - \hat{m}_f(n_1, n_2) \right]^2$$

Power Spectrum Filtering



$$P_{\hat{f}} \neq P_f$$

- observation

$$\left\{ \begin{aligned} P_{\hat{f}} &= P_g(\omega_1, \omega_2) \frac{P_f^2(\omega_1, \omega_2)}{\left[P_f(\omega_1, \omega_2) + P_v(\omega_1, \omega_2)\right]^2} \\ P_g(\omega_1, \omega_2) &= P_f + P_v \end{aligned} \right.$$

$$P_{\hat{f}} = \frac{\cancel{\left[P_f(\omega_1, \omega_2) + P_v(\omega_1, \omega_2)\right]} P_f^2(\omega_1, \omega_2)}{\left[P_f(\omega_1, \omega_2) + P_v(\omega_1, \omega_2)\right]^{\cancel{2}}}$$

$$\neq P_f$$

$$f \longrightarrow \bigcirc \longrightarrow g \longrightarrow \boxed{H(\omega_1, \omega_2)} \longrightarrow \hat{f}$$

$$\uparrow v$$

Choose filter H s.th. $P_f = P_{\hat{f}}$

$\Longrightarrow$ resulting filtering "Power spectrum filtering".

$$P_{\hat{f}} = P_f = P_g \, |H(\omega_1, \omega_2)|^2 \Longrightarrow$$

$$|H(\omega_1, \omega_2)| = \left(\frac{P_f}{P_g} \right)^{1/2}$$

$$|H(\omega_1, \omega_2)| = \left(\frac{P_f(\omega_1, \omega_2)}{P_f(\omega_1, \omega_2) + P_v(\omega_1, \omega_2)} \right)^{1/2}$$

$\Longrightarrow$ zero phase.

Parametric Weiner filter

"Traditional Weiner filter" = $\dfrac{P_f}{P_f + P_v}$

Parametric. $\left(\dfrac{P_f}{P_f + \alpha P_v}\right)^{\beta}$

$\alpha = \beta = 1 \longrightarrow$ Traditional

$\alpha = 1, \beta = 1/2 \longrightarrow$ power spectrum.

$\alpha > 1$ ~~[illegible]~~ $\beta = 1$