May 3, 2006

Multi-resolution

- Multi-resolution introduced: Burt + Adelson 1982

Review of 1982 paper -> very negative

Initial motivation here to reduce complexity of motion estimation

$N_1$ $\rightarrow$ M.V. $ightarrow$ $N_2$

$N_2$ $\rightarrow$ M.V. $ightarrow$ $N_1$, $f_2$

$N$ $\rightarrow$ $N$, $f_0$
How To Build a pyramid

• Basic idea → successive low pass filtering + subsampling.

\[ f_i(n_1, n_2) \xrightarrow{\text{LPF}} \text{sumsample} \xrightarrow{} f_{i+1}(n_1, n_2) \]

Process of generation \((i+1)\)th level from \(i\)th level.

• How to filter?

\[ f_i^L(n_1, n_2) = f_i(n_1, n_2) \ast h(n_1, n_2) \]

• Subsample:

\[ f_{i+1}(n_1, n_2) = \begin{cases} 
\sum_{n=-\infty}^{\infty} f_i^L(n_1, n_2) & 0 \leq n_1, n_2 \leq 2^{-i} \\
0 & \text{otherwise}
\end{cases} \]
Gaussian pyramid:

\[ h(n_1, n_2) = h(n_1) h(n_2). \]

\[
h(n) = \begin{cases} 
  a & n = 0 \\
  \frac{1}{4} & n = \pm 1 \\
  \frac{1}{4} - \frac{a}{2} & n = \pm 2
\end{cases}
\]

usually \[ 0.3 < a < 0.6 \]

show fig 10.34, 10.36

J. Lim
Pyramid Coding

- Basic idea: code successive images down the pyramid from the one above it.

- Interpolate \( f_{i+1}(n_{1}, n_{2}) \) to get prediction \( \hat{f}_{i}(n_{1}, n_{2}) \)

\[
\hat{f}_{i}(n_{1}, n_{2}) = I[f_{i+1}(n_{1}, n_{2})]
\]

for \( f_{i} \)

- Code prediction error

\[
e_{i}(n_{1}, n_{2}) = f_{i}(n_{1}, n_{2}) - \hat{f}_{i}(n_{1}, n_{2})
\]

- Repeat until the bottom level image is reconstructed — i.e., original.

- Seq. \( f_{i} \to \) Gaussian Pyramid \( e_{i}(n_{1}, n_{2}) \to \) Laplacian pyramid.
Fig 10.38 + Fig 10.87 = J. S. Lim
Fig 7.1 = 7.3 = G/LW.

Subband Coding

Speech coding → Crosson, Esteban, Gaalad, 1976

Subhad: No expansion on # of samples
Cancel Aliasing, we must have:
\[ H_0(z) G_o(z) + H_1(-z) G_1(z) = 0 \]

To get perfect reconstruction, we must have:
\[ H_0(z) G_o(z) + H_1(z) G_1(z) = 2 \]

Define \( H_m(z) = \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} \)

To cancel aliasing + achieve PR, we must have
\( \begin{bmatrix} G_o(z) & G_1(z) \end{bmatrix} H_m(z) = \begin{bmatrix} 2 & 0 \end{bmatrix} \)
\( H_0, H_1 \rightarrow \text{Analytic filters} \\
G_0, G_1 \rightarrow \text{Synthetic filters.} \)

- Can show if \( \mathbb{O} \) is satisfied
  - Then analytic + synthetic filters are "bi-orthogonal"

\[
\left< h_i (2n-k), g_j (k) \right> = \delta(i-j) \delta(n)
\]

\[
\sum_k h_i (2n-k) g_j (k)
\]

\( i = 0, 1 \)
Example 2-channel filter bank with perfect reconstruction.

**Analysis**
Low pass: \((-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{2}, -\frac{1}{4})\)
High pass: \((-\frac{1}{4}, -\frac{1}{2}, \frac{1}{4})\)

**Synthesis**
Low pass: \((\frac{1}{4}, \frac{1}{2}, \frac{1}{4})\)
High pass: \((\frac{1}{4}, -\frac{1}{2}, -\frac{3}{4}, \frac{1}{2}, \frac{1}{4})\)

Bi-orthogonal 5/3 filters
Le Gall filters
Show Fryer layout in B.6 slides.
3 main classes of Perfect Reconstruction filter families

1) QMF = Quadrature Mirror Filters:
   A Aliasing Cancellation:
   \[ H_1(z) = H_0(-z) = G_0(z) = G_0(-z) \]
   Esteban 1976.

   - High pass band is the mirror of the
     loss pass band in freq. domain.

   - Only design 1 filter for Aliasing Cancellation.

   - To achieve PR, design "1" filter:
     \[ H_0^2(z) - H_0^2(-z) = 2 \]
(2) CQF = Conjugate Quadrature Filters

Achieve aliasing

\[ H_0(z) = G_0(z^{-1}) = f(z) \]
\[ H_1(z) = G_1(z^{-1}) = z f(-z^{-1}) \]

Smith, Barnwell 1986

time domain: \[ h_0(k) = g_0(-k) = f(k)^{k+1} \]
\[ h_1(k) = g_1(-k) = (-1)^k f(-k) \]

PR: \[ H_0(z) H_0(z^{-1}) + H_0(z) H_0(-z^{-1}) = 2 \]

Freq. Domain: \[ |H_0(\omega)|^2 + |H_0(\omega + \pi)|^2 = 2 \]
(3) Orthogonal:
\[
\begin{align*}
H_0(z) &= G_0(z^1) \\
H_1(z) &= G_1(z^1) \\
G_1(z) &= -z^{2K+1} G_0(-z^1) \\
G_0(z) G_0(z^1) + G_0(-z) G_0(-z^1) &= 2
\end{align*}
\]

Almost PR.

\[2k = \#\text{of filter taps in each filter.}\]

Time domain:
\[
\langle g_i(n), g_j(n+2m) \rangle = \delta(i-j) \delta(m)
\]
\[
g_1(n) = (-1)^n g_0(2K-1-n)
\]
\[
h_i(n) = g_i(2K-1-n) \quad i = 0, 1
\]