Detour into Numerical Analysis

Problem

- Well Conditioned
- Ill Conditioned

Algorithm applied to problem

- Well Conditioned
- Ill Conditioned

Condition $\# = \text{measure of how "well conditioned" an algorithm on a problem is}$
Problem → output
Problem → signal
Problem → intensity pattern on wafer
Desired mask
Desired intensity pattern on wafer
Exposure
Sample of FTM→
Matrix $A$ vector $b$ → Solving linear system of equations $Ax = b$ → $x$ unknown

Initial conditions → Weather

Rain, temp, satellite image → Tomorrow, temp, fair...

$a, b, c, d$ → Find roots of poly

$a x^3 + b x^2 + c x + d$ → root
Algorithm: Method to solve a particular problem.

\[ \mathbf{A} \mathbf{x} = \mathbf{b} \]

List of possible Algorithms:

1. Gaussian Elimination. Ill conditioned.
2. Compute Inverse. Terrible.
3. Gaussian Elimination with Pivoting.
4. SVD excellent.
5. QR Decomposition. Excellent.
Condition # = \[
\frac{\text{Perturbation in output}}{\text{Perturbation in input}} \]

\[A, b \rightarrow A\Delta x = \Delta b \]

Input \[\rightarrow\] Problem \[\rightarrow\] Output

\[x \rightarrow \text{relative change output} \]

\[\text{Condition #} = \frac{10^6, 10^8, 10^{12}}{\text{relative change input}} \]

If condition # is large \[\rightarrow\] problem is ill-posed.

If condition # is small \[\rightarrow\] problem is well-posed.
\[ A \mathbf{y} = b \]

\[ A \rightarrow \text{Condition } \# = \frac{\text{largest singular value}}{\text{smallest singular value}} \]

\[ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \]

Condition \# of problem depends on the "Intrinsic" aspect of the problem itself.

- Well conditioned \( \Rightarrow \) good accuracy
- Ill conditioned \( \Rightarrow \) messed up answer
- Well Alg. \( \Rightarrow \) out of luck
Condition # of Alg

Input $\rightarrow$ Alg $\rightarrow$ Output

Condition # = \frac{\text{relative change in output}}{\text{relative change in input}}

$A \times x = b$

$A = \text{very bad condition}$
Ren from FTM

- Even though theoretically Hayf/McCollom showed uniqueness in practice \( \Rightarrow \) ill conditioned

- 2D polynomials can almost closely be approximated by factors

- Close form \( \rightarrow \) Izraelevitz + Lin

- Iterative

Oppenheim + Mersereau \( \rightarrow \) 1972 Proceedings of IEEE P.S.T.
Recon from F.T. Phase

\[ X(n_1, n_2) \xrightarrow{D.T.F.T} X(\omega_1, \omega_2) = \sum_{n_1} \sum_{n_2} x(n_1, n_2) e^{-j\omega_1 n_1 - j\omega_2 n_2} \]

\[ \phi(\omega_1, \omega_2) \]

Patrick Van Hove \( \approx 1982 \)

Need to have samples of \( \phi \) at more than \( N^2 \) continuous valid variables.
Two Alg. \rightarrow \text{iterative}

\text{direct}

\text{1982} \rightarrow \text{Even quantizing phase to one bit & yet nearly signal successful
Iteration

N x N signal.
Know 1D/FOS: N x N.
Samples at 2N x 2N
up phase of F. 7.

\( x = \text{original sign} \)

Initial Guess
\( y(n_{1..N}) \)
N x N

Set signal to
zero outside
N x N region

\( F^{-1} \) to get
another 2N x 2N space domain

Loop mag. y.
Replace phase
with given phase
2N x 2N.

\( \Phi_x = \Phi_y \)
\( \Phi_y \)
Yes

STSP

No