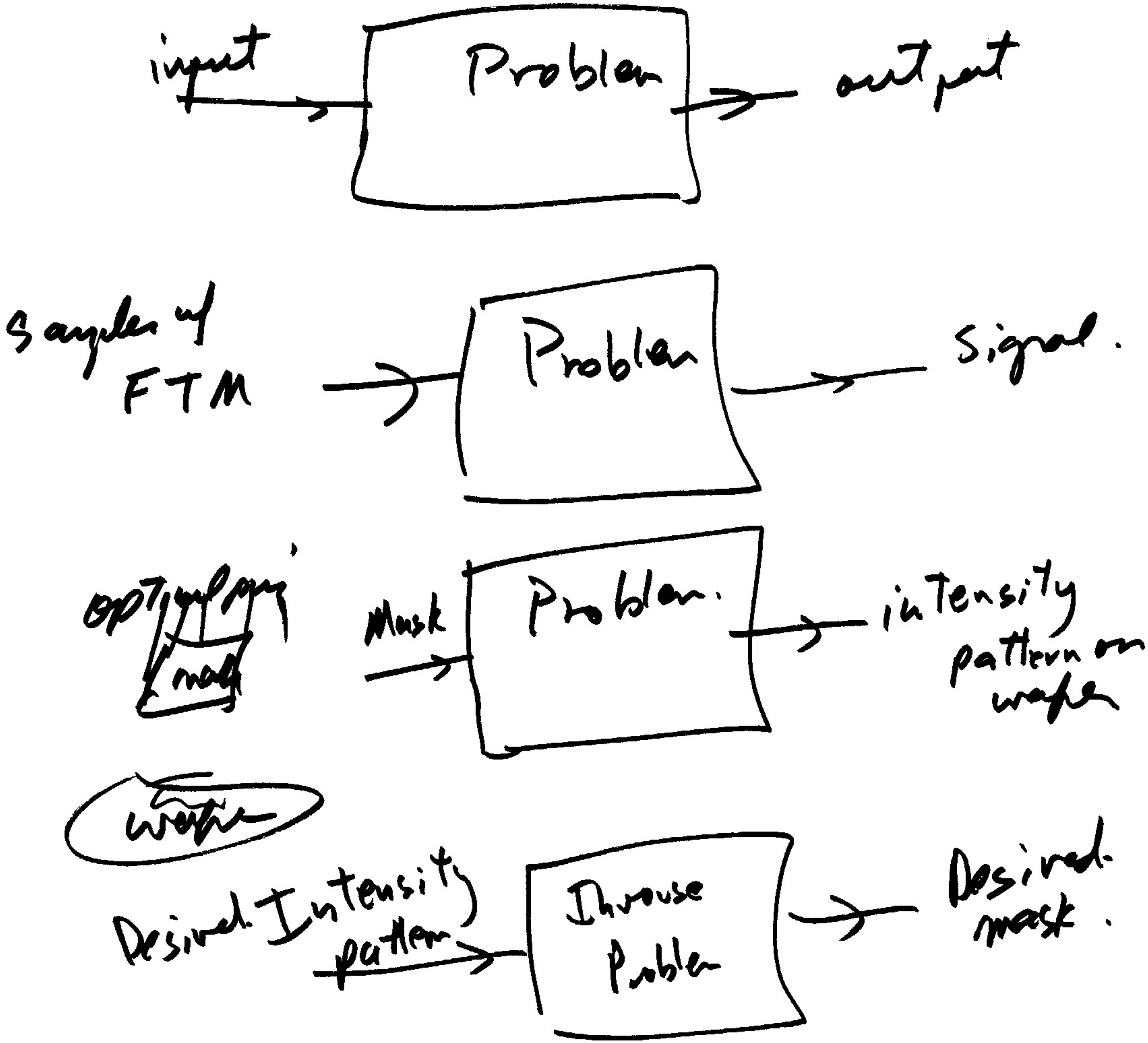
Detour into Numerical Analysis

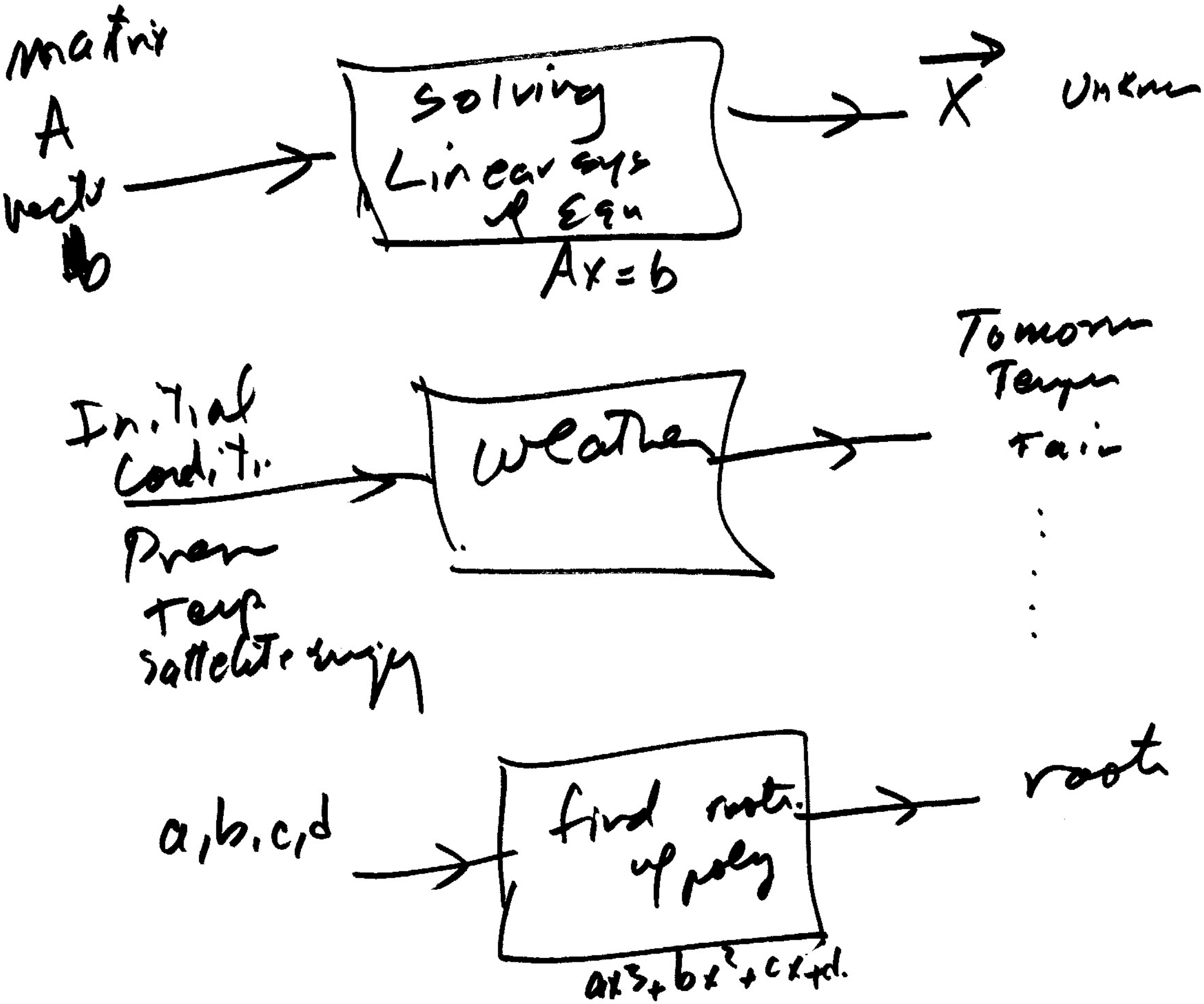
Feb 1, 06
EE225B

Problems → well Conditioned
Problems → ill Conditioned.

Algorithms applied to problem → Well Condi
Algorithms applied to problem → Ill Conditioned.

Condition # = measure of how "well conditioned" either alg or a problem is.


input
Problem
output
samples of FTM
Problem
Signal.
OPTical proj. mask
Mask
Problem.
intensity pattern on wafer
wafer
Desired Intensity pattern
Inverse Problem
Desired mask.


matrix A
vector b
Solving Linear sys of Equ
Ax = b
$\vec{X}$
Unknown
Initial Condition
Pressure
Temp
sattelite imaging
weather
Tomorrow Temp
rain
a, b, c, d
find roots of poly
ax^3+bx^2+cx+d
roots

Algorithm: Method to solve a particular problem.

Ex $A\vec{x} = \vec{b}$

List of possible Algorithm

1. Gaussian Elimination. Ill cond Alg.
2. Compute Inverse Terrible.
3. Gauss Elimination with pivoting good
4. SVD excellent
5. QR Decomposition. excellent
6. Iterative alg. large problem

Condition of Problem

Condition # = $\dfrac{\text{Perturbation in output}}{\text{perturbation in input}}$.

$A, \vec{b}$ $\quad A\vec{x} = \vec{b}$ $\quad \vec{x}$

input $\longrightarrow$ $\boxed{\text{Problem}}$ $\longrightarrow$ output.

Condition # = $\dfrac{\text{relative chg output}}{\text{relative chg input}}$

If condition # is large ($10^6, 10^{10}, 10^{12}$) $\longrightarrow$ problem is ill conditioned

" " " " small (~ 1) $\longrightarrow$ problem is well conditioned

$Ax = b$

$$A \longrightarrow \text{Condition \#} = \frac{\text{largest singular value}}{\text{smallest singular value}}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Condition # of problem depends on the "Intrisic" aspects of the problem itself.

$$\text{Problem} \longrightarrow \text{well condition} \xrightarrow{\text{well Alg}} \text{good answer}$$

$$\text{well condition} \xrightarrow[\text{Alg.}]{\text{ill}} \text{messed up answer.}$$

$$\text{Problem} \longrightarrow \text{Ill condition} \longrightarrow \text{out of luck.}$$

Condition # of Alg

$$\text{input} \longrightarrow \boxed{\text{Alg}} \longrightarrow \text{output}$$

$$\text{Condition \#} = \frac{\text{relative change in output}}{\text{relative change in input.}}$$

$A = $ very large condition

$$A x = b$$

Recon from FTM

- Even Though Theoretically Hayes/McClellan showed uniqueness
  in practice $\Longrightarrow$ ill conditioned
- 2D polynomials can almost closely be approximated by factors

- Close form $\rightarrow$ Izraelivitz + Lim
- Iterative

Oppenheim + Mersereau $\longrightarrow$ 1972 Proceedings of IEEE
P.S.T.

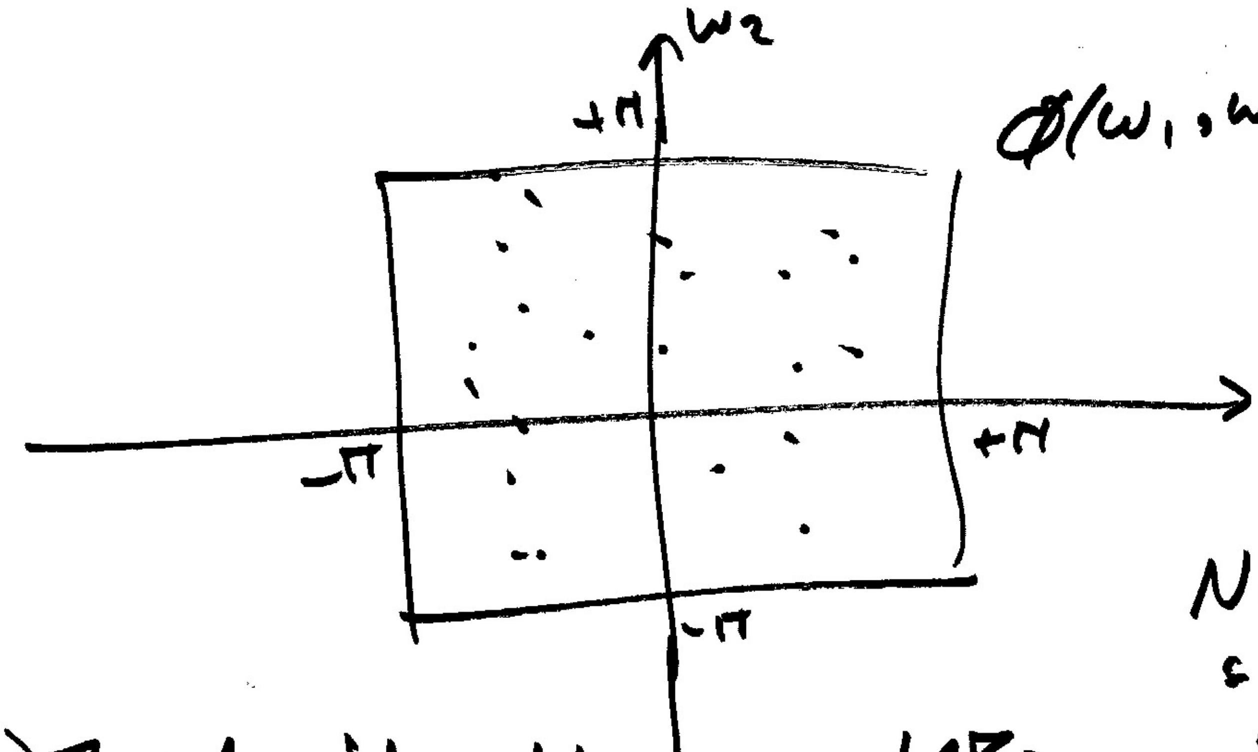
Recon from F.T. Phase

$$x(n_1,n_2) \xrightarrow{\text{D.T.F.T}} X(\omega_1,\omega_2) = |\quad| \, e^{j \measuredangle \phi}$$

$N \times N$

$$X(\omega_1,\omega_2) = \sum_{n_1} \sum_{n_2} x(n_1,n_2)\, e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

$\phi(\omega_1,\omega_2)$



$\omega_1 \rightarrow$ continuous valued variables

Need to have samples of ϕ at more than N^2

Patrick Van Hove $\approx$ 1982

Two Alg. $\rightarrow$ iterative.

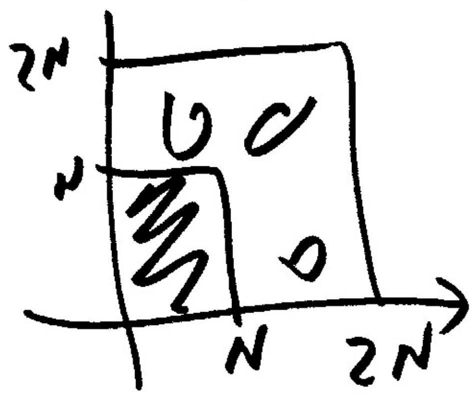
$\searrow$ direct.

~~the~~ 1982 $\longrightarrow$ Even quantizing phase of F.T. to **one** **bit** & yet reconst. signal successful.

Iterative

x = original sigh

$N \times N$ signal.
Know ROS : $N \times N$.
→ Samples at $2N \times 2N$ of phase of F.T.
ϕ_x



~~Samples of~~