Image Restoration

1. Enhancement → Image "look" better
   Subjective improvement

2. Restoration
   Image has been degraded by smoothing
   - noise
   - blur
   - Atmospheric turbulence

   Objective model → Error → MSE
\[ g(x,y) = h \ast f + n \]

\[ g^\prime(x,y) = h(x,y) \ast f(x,y) + \text{noise} \]

\[ f(x,y) \]
minimize $E\left[ (f(x, y) - \hat{f}(x, y))^2 \right]$ 

Today.

$H$ is identity.
only corruption $\rightarrow$ noise.

$g(x, y) = f(x, y) + \eta(x, y)$

Assume noise is independent of spatial coordinates, uncorrelated w.r.t. image itself.
Gaussian

\[ p(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x - \mu)^2}{2\sigma^2}} \]

\[ \text{var} = \sigma^2 \]

\[ \mu \text{ is the mean} \]

- Sensor noise due to poor illumination or high temperature
- Electronic noise

70% of values are within one sigma 
\([-\sigma, \sigma]\) 

95% within two sigma.
(2) Rayleigh:

\[ p(z) = \begin{cases} \frac{2}{b} (2 - a) & z \geq a \\ 0 & z < a \end{cases} \]

\[ \mu = a + \sqrt{\frac{6\pi}{b}} \]

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\[ 6^2 = \frac{6(4 - \pi)}{4} \]

noise in range imaging apps.
3) Exponential (Gamma) noise:
\[ p(z) = \begin{cases} \frac{ze^{-\frac{z^2}{2\sigma^2}}}{(2\sigma)^{b-1}B(b-1)} & z > 0 \\ 0 & z < 0 \end{cases} \]

4) Exponential special case of Exponential:
\[ p(z) = \begin{cases} ae^{-az} & z > 0 \\ 0 & z < 0 \end{cases} \]

\( \mu = \frac{b}{a} \)
\( \sigma^2 = \frac{b}{a^2} \)

Laser imaging
3. Uniform:  
\[ p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \]

\[ \mu = \frac{a+b}{2} \quad \text{Random # generator} \]

6. Impulse, Salt/Pepper noise, quick transit, faulty switching  
\[ p(z) = \sum P_a \delta(z-a) + P_b \delta(z-b) \]

if \( b > a \), 0 show up as light dots  
\( \text{ otherwise } \) dark dot.
1. Arithmetic Mean.

\[
\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)
\]

2. Geometric Mean.

Similar to arithmetic but lose less detail.

\[
\hat{f}(x,y) = \left[ \prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn}
\]
3. Harmonic mean filter.

\[ \hat{F}(x, y) = \frac{m \cdot n}{\sum_{(s, t) \in S_{xy}} \frac{1}{g(s, t)}} \]

4. Contour Harmonic.

\[ \hat{f}(x, y) = \frac{1}{\sum_{(s, t) \in S_{xy}} g(s, t)} \sum_{Q \geq 0} g(s, t) \]

Q \geq 0 \rightarrow \text{removes pepper noise}

Q < 0 \rightarrow \text{removes salt noise}
1. Median:
   - Locate $S_{xy}$. Find median replaces $S_{xy}$ with the median value.

2. Max:
   - Also removes some dark pixels.

3. Min:
   - Also removes some white pixels.

4. Midpoint
   - $f(xy) = \frac{1}{2} \left\{ \max + \min \right\}.$
\[ \hat{f}(x, y) = \frac{1}{mn - p} \sum_{(s, t) \in S_{xy}} g_\Theta(x, y) \]

\[ S_{xy} \text{ max.} \]

\[ g_{r}(s, t) \rightarrow g(s, t) \text{ excluding, } d/2 \text{ brightest grey levels} \]

\[ \text{and } d/2 \text{ darkest } \]

\[ \text{Grayscale layout} \]
Adaptive local noise Reduction

\[ \hat{f}(x, y) = g(x, y) - \frac{\hat{\sigma}^2}{\sigma_L^2} (g(x, y) - m_L) \]

\[ m_L = \text{local mean} = \frac{1}{mn} \sum_{(x,t) \in S_{x,y}} g(x, t) \]

\[ \sigma_L^2 = \text{local variance}. \]

\[ \sigma_N^2 = \text{noise variance}. \]

if \( \sigma_N^2 \ll \sigma_L^2 \) \( \Rightarrow \hat{f} \approx g \) good

if \( \sigma_N^2 \gg \sigma_L^2 \) then. approx. \( \frac{\sigma_N^2}{\sigma_L^2} \approx 1 \)

\[ \Rightarrow f \ll m_L \]
Adaptive Median Filter
looking at neighborhood $S_{xy}$ around

$P_{xy} = \max_{i,j} I_{i,j}$

Spatial domain

Additive noise

Frequeny domain

Additive Gaussian noise
- $Z_{\text{min}} = \text{min value in } S_{xy}$
- $Z_{\text{med}} = \text{median value in } S_{xy}$

**Outline**

- Keep increasing window size until $Z_{\text{med}}$ is not an impulse. $Z_{\text{min}} < Z_{\text{med}} < Z_{\text{max}}$

- When this happens, check $Z_{xy}$.
  - If $Z_{xy}$ is not an impulse $\rightarrow$ output $Z_{xy}$
  - If $Z_{xy}$ is an impulse $\rightarrow$ output $Z_{\text{med}}$

Since $Z_{\text{med}}$ is not an impulse.
Psuedo code

part A
if $Z_{\text{min}} < Z_{\text{med}} < Z_{\text{max}}$
Then go to part B. \( \Rightarrow Z_{\text{med}} \) is not an impulse.
else if window size \( \leq S_{\text{max}} \)
window \( \leftarrow \) window + 1, go to part A
else output $Z_{xy}$

part B
if $Z_{\text{min}} < Z_{xy} < Z_{\text{max}}$
output $Z_{xy}$\( \Rightarrow Z_{xy} \) is not an impulse.
else output $Z_{\text{med}}$