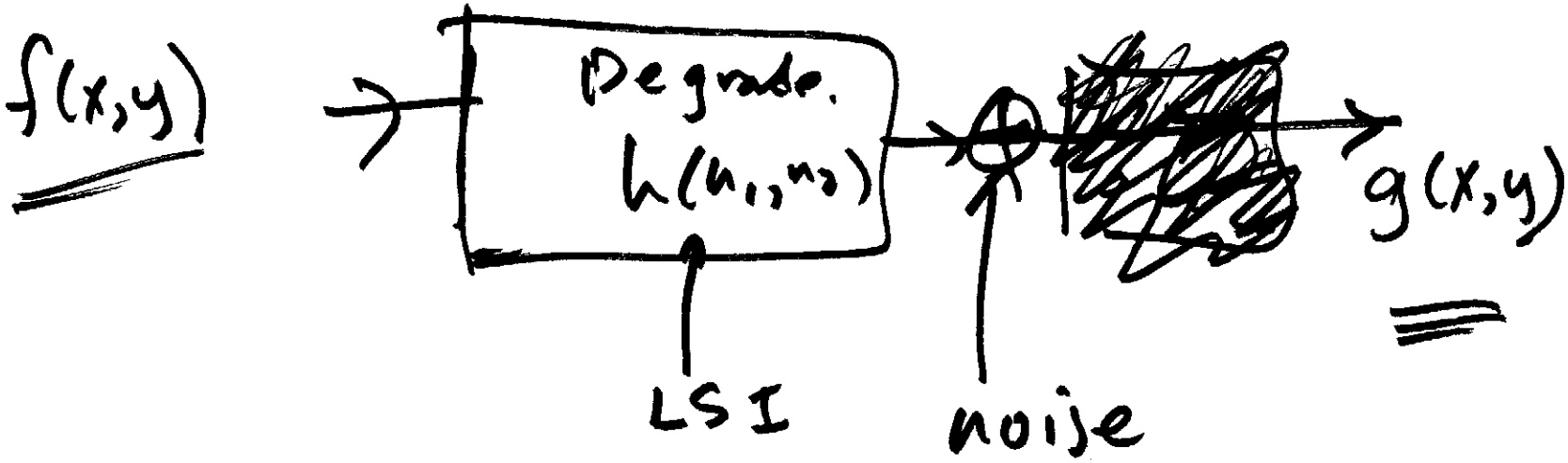
Image Restoration

1. Enhancement → image "look" better
   ↓
   subjective improvement.

2. Restoration.
   Image has been degraded by something.
   - noise
   - blur
   - Atmospheric turbulence.

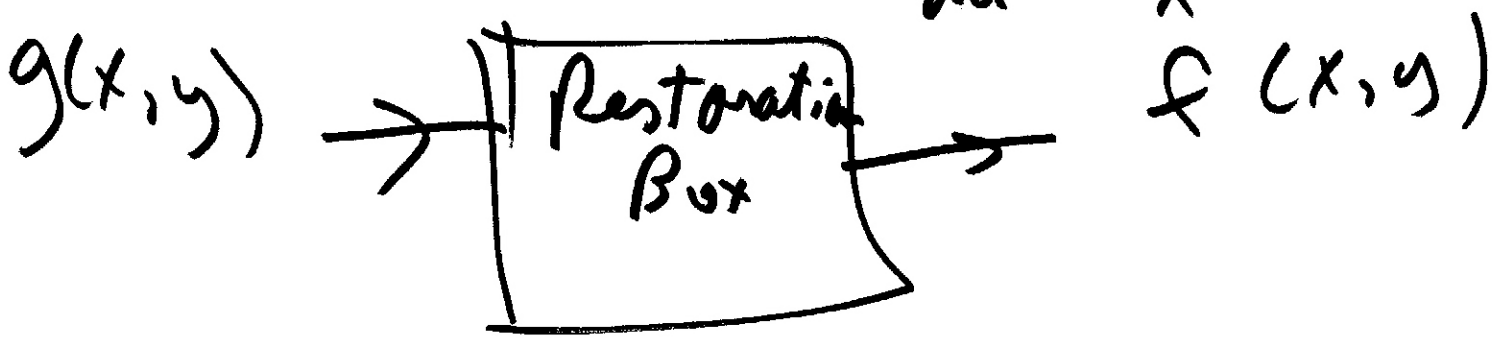
objective models → Error → MSE



$$g(x,y) = h * f + \eta$$

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

noise



minimize $E\left[\left(f(x,y) - \hat{f}(x,y)\right)^2\right]$

Today.

H is identity.

only corruption $\longrightarrow$ noise.

$$g(x,y) = f(x,y) + \eta(x,y)$$

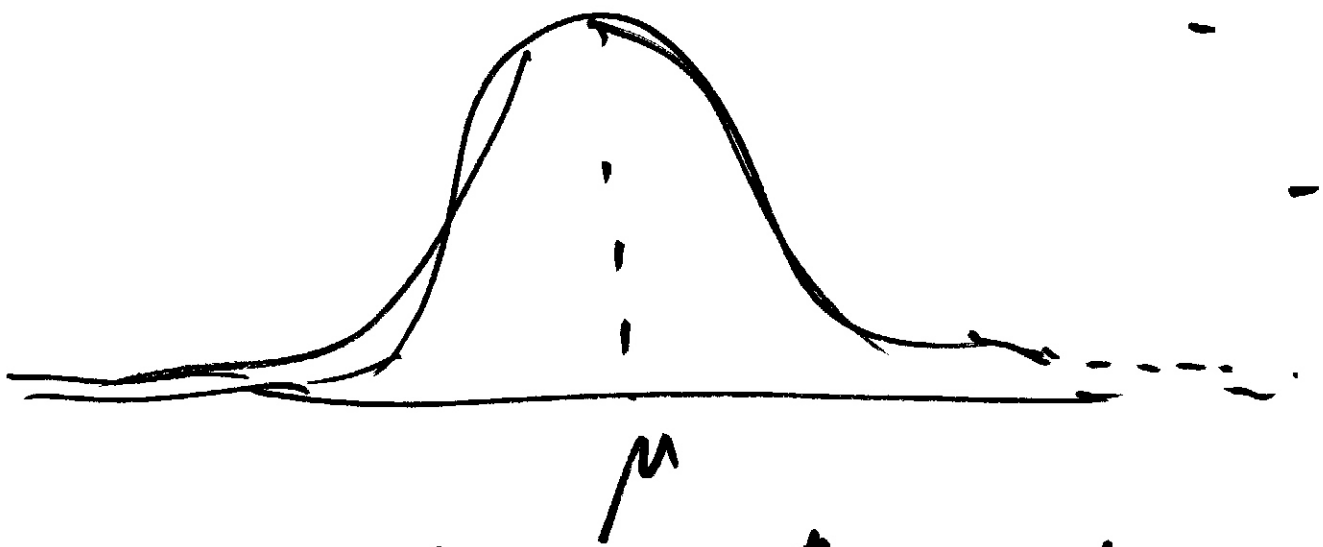
observed. original image noise

Assume noise is independent of spatial coordinates
uncorrelated w.r.t. image itself.

① Gaussian

$$p(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(z-\mu)^2}{2\pi\sigma^2}}$$

var = σ^2 mean = μ



- electronic noise
- sensor noise due to poor illumination or high Temperature

70% of values are within one sigma $[\mu-\sigma, \mu+\sigma]$

95% within 2 sigma.

② Rayleigh.

$$p(z) = \begin{cases} \frac{z}{b}(z-a)\, e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$

$$\mu = a + \sqrt{\frac{b\pi}{4}} \qquad \sigma^2 = \frac{b(4-\pi)}{4}$$

noise in range imaging apps.

③ Erlang (Gamma) noise.

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1} e^{-az}}{(b-1)!} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$\mu = b/a$ $\quad \sigma^2 = \dfrac{b}{a^2}$

<u>laser imaging.</u>

④ Exponential special case of Erlang.

$$p(z) = \begin{cases} a e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

Not working

(5) Uniform.

$$p(z) = \begin{cases} \frac{1}{b-a} & a \le z \le b \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu = \frac{a+b}{2} \qquad \text{var} = \frac{(b-a)^2}{12}$$

→ random # generation.

(6) Impulse, salt/pepper noise.
quick transient, faulty switching.

$$p(z) = \begin{cases} P_a & z = a \\ P_b & z = b \\ 0 & \text{otherwise.} \end{cases}$$

if $b > a$, b show up as light dots
a " " " dark dot.

① Mean Filters

1. Arithmetic Mean.

$S_{xy} \rightarrow$ window.

$m \times n$.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

2. Geometric Mean.

similar to arithmetic but loose less detail

$$\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn}$$

3. Harmonic mean filter.

works for salt noise. fails for pepper

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

4. Contra Harmonic.

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

$Q>0 \rightarrow$ remove pepper noise

$Q<0 \rightarrow$ remove salt noise

order statistics filter

1. Median

look at S_{xy}. find median replace. ~~at~~ (x,y) with the median value.

2. Max

also removes some dark pixels.

$$\hat{f}(x,y) = \max_{S_{xy}} g(s,t)$$

3. Min

also removes some white pixels

$$\hat{f}(x,y) = \min_{S_{xy}} g(s,t)$$

4. Midpoint

$$\frac{1}{2}\{\max + \min\}.$$

Alpha Trimed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_{\ominus}(x,y)$$

S_{xy} $m \times n$.

$g_r(s,t) \longrightarrow g(s,t)$ excluding.

$d/2$ brightest grayleve

and $d/2$ darkest " "

5	4	3	2
1	1	.	.
.		.	.
		.	.

grayleve

darkest ——— brightest

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Adaptive local noise Reduction

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_N^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

m_L = local mean $= \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$

σ_L^2 = local variance.

σ_N^2 = noise variance.

if $\sigma_N^2 << \sigma_L^2 \longrightarrow \hat{f} \simeq g$ good

if $\sigma_N^2 \gg \sigma_L^2$ then approximate $\frac{\sigma_N^2}{\sigma_L^2} \simeq 1$

$\longrightarrow \hat{f} \simeq m_L$

Image Restoration

Spatial domain

Frequency domain

"additive Gaussian noise"

salt/pepper

median

Adaptive Median filters

Adaptive Median Filter

- looking at neighborhood $S_{x,y}$ around pixel (x,y)
- z_{xy} = pixel value at center of $S_{x,y}$
- z_{max} = max value of intensity over S_{xy}

- z_{min} = min value in S_{xy}
- z_{med} = median value in S_{xy}

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outline

- Keep increasing window size until z_{med} is <u>not an</u> <u>impulse</u>.

  $z_{min} < z_{med} < z_{max}$

- When this happen, check z_{xy}.
  - If z_{xy} is not an impulse → output z_{xy}
  - If z_{xy} is an impulse → output z_{med}

    Since z_{med} is not an impulse.

Pseudo code

Part A if $z_{min} < z_{med} < z_{max}$

Then go to part B. → z_{med} is not an impulse.

else. if windowsize $< S_{max}$

window ← window + 1, go to part A

else. output z_{xy}.

part B. If $z_{min} < z_{xy} < z_{max}$ → z_{xy} is not an impulse

output z_{xy}

else. output z_{med}