Euge Rontoul

Frequency domain techniques

Periodic noise

What if interference pattern is not "clean"?

Sources of periodic interference patterns:

- Coupling and amplification of low level signals in electrical/optical scanners' electronic circuits.
Approach

1) First isolate principal contributions (spikes) of the interface pattern.

2) Subtract a variable weighted portion of the pattern from the corrupt image.

Objective: i.e. e.g. minimize local variance of a processed image.
\[ g(x,y) \leftrightarrow G(\omega_1, \omega_2) \quad \text{observed degraded signal.} \]

\[ f(x,y) \leftrightarrow F(\omega_1, \omega_2) \quad \text{clean signal.} \]

\[ \hat{f}(x,y) \leftrightarrow \hat{F}(\omega_1, \omega_2) \quad \text{proceed to reconstruction of \( f \).} \]
Step 1: Put a notch filter \( H(w_1, w_2) \) at location of each spike.

Find filter

\[
N(w_1, w_2) = H(w_1, w_2) G(w_1, w_2)
\]

noise

\[
y(x, y) = \mathcal{F}^{-1} \left\{ \mathcal{F} \left\{ H(w, w_2) G(w, w_1) \right\} \right\}
\]

noise in space domain.
Step 2

\[ f(x, y) = g(x, y) - w(x, y) \cdot \nabla f(x, y) \]

**Optimization:** Choose local weight \( w(x, y) \) to minimize local variance of \( f \) at \( (x, y) \)

**Weight function**

Choose local weight \( w(x, y) \) to minimize local variance of \( f \) at \( (x, y) \)

**Neighborhood** \((2a + 1) \times (2b + 1)\)

Local variance over this neighborhood:

\[
\sigma^2_{xy} = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left( f(x+s, y+t) - \hat{f}(x, y) \right)^2
\]
\[ \bar{f} = \text{local mean} = \hat{f}(x, y) = \frac{1}{(2a + 1)(2b + 1)} \sum \sum_{s=-a \atop \text{even}}^{a} \hat{f}(x+s, y+t) \]

Plug in \( \hat{f} = g - w(x, y) \) into

Assume \( w(x, y) \) is constant over \([2a + 1] \times [2b + 1]\) region.

\[ w(x+s, y+t) = w(x, y) \]

\( s, t \in [-a + b] \times [-b, b] \)
Estimating the degradation for

Observation

Go to parts of g

that you have
apriori knowledge.

See how it looks.

Compare to what it should have looked like.

Make some intelligent guess about

\( \text{signal and noise} \)

Experiment: Put a known signal

into your system to calibrate.
Modeling

Steele + Huthzegel.

Atmospheric turbulence:

\[
H(\omega, \omega) = e^{-k \left( \omega^2 + \omega_i^2 \right)^{5/6}}
\]

Motion blur.
Restoration

Degradation $\rightarrow g$ $\rightarrow$ Restoration $\rightarrow \hat{f}$

as close as possible.

Degradation can be modelled as:

$\hat{f} + \text{noise} \xrightarrow{LSTM} \hat{f} \rightarrow g$

Use domain specific knowledge to model degradation.
Motion Blur modeling

\[ g(x, y) = \int_T f(x - x_0(t), y - y_0(t)) \, dt \]

\[ g(x, y) = \int_T f(x - x_0(t), y - y_0(t)) \, dt \]

\[ g(x, y) = \int_T f(x - x_0(t), y - y_0(t)) \, dt \]

\[ G(w_x, w_y) = \int \int g(x, y) \, e^{-j2\pi (w_x x + w_y y)} \, dx \, dy \]

F.T. \[ G(x, y) = \int \int g(x, y) \, e^{-j2\pi (w_x x + w_y y)} \, dx \, dy \]
\[
G(w_x, w_y) = \int_0^T \left( \int_0^\infty \left( \int_{-\infty}^{\infty} f(x-x_0(t), y-y_0(t)) e^{-j2\pi(w_x x + w_y y)} \, dx \, dy \right) \, dt \right) \, dt
\]

\[
= \int_0^T F(w_x, w_y) e^{-j2\pi(w_x x_0(t) + w_y y_0(t))} \, dt.
\]

\[
G(w_x, w_y) = F(w_x, w_y) \int_0^T e^{-j2\pi(w_x x_0(t) + w_y y_0(t))} \, dt
\]

\[
H(w_x, w_y)
\]

- \( x_0(t) = 0 \quad y_0(t) = 0 \implies G(x, y) = T + (x, y) \)

- \( x_0(t) = \frac{at}{T} = \text{constant speed along x direction} \quad y_0(t) = 0 \)

\[
G(w_x, w_y) = F(w_x, w_y) + H(w_x, w_y)
\]
\[ H(\omega_x,\omega_y) = \int_0^T e^{-j\omega_y t} e^{j\omega_x t} dt \]

\[ = \int_0^T e^{j(\omega_x - \omega_y) t} dt \]

\[ H(\omega_x,\omega_y) = \frac{T}{\sin(\pi \omega_x \alpha)} e^{-j\pi \omega_x \alpha} \]

| \[ H(\omega_x,\omega_y) \] |
\[ G(w_x, w_y) = \underbrace{F(w_x, w_y) \cdot H(w_x, w_y)}_{\frac{1}{H(w)}} + \text{Noise} \]
Stationarity

\[
P_{x(t_1), x(t_2), \ldots, x(t_n)}(X_1, X_2, X_3, \ldots, X_n) =
\]

\[
P_{x(0), x(t_2-t_1), \ldots, x(t_n-t)}(X_1, X_2, \ldots, X_n)
\]