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Frequency domain telegraph

Periodic noise

What if interference pattern is not "clean"?

Sources of periodic interference patterns:
- Coupling and amplification of low level signals in electro-optical scanners' electronic circuits.
Approach:

1. First isolate principal contributions (spikes) of the interferon pattern.

2. Subtract a variable weighted portion of the pattern from the corrupt image.

Objective: i.e. e.g. 'minimize local variance of a processed image.'
\[ g(x,y) \leftrightarrow G(\omega_1, \omega_2) \quad \text{observed degraded signal} \]

\[ f(x,y) \leftrightarrow F(\omega_1, \omega_2) \quad \text{clean signal, original signal, undegraded} \]

\[ \hat{f}(x,y) \leftrightarrow \hat{F}(\omega_1, \omega_2) \quad \text{proceed to version of } g \text{ approximated } f \]
Step 1. Put a notch filter $H(w_1, w_2)$ at location of each spike.

Find filter

$$N(w_1, w_2) = H(w_1, w_2) G(w_1, w_2)$$

$$\gamma(x, y) = \sum \gamma_{x_0 y_0} H(w_0, w_2) G(w_0, w_1)$$

noise in space domain.
Step (2)

\[ \hat{f}(x,y) = g(x,y) - w(x,y) \theta(x,y) \]

Optimization: Choose local weights \( w(k,y) \) to minimize local variance of \( f \) at \((x,y)\)

\[
\text{neighborhood } (2a+1) \times (2b+1) \]

Local variance over this neighborhood:

\[
\sigma_{xy}^2 = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \left( \hat{f}(x+s, y+t) - \hat{f}(x,y) \right)^2
\]
\[ \hat{f} = \text{local mean} = \hat{f}(x, y) = \frac{a + b}{(2a+1)(2b+1)} \sum \sum \hat{f}(x+s, y+t) \]

Plug in \( \hat{f} = g - w \) into

Assume \( W(x, y) \) is constant over \([2a+1] \times [2b+1] \) region.

\[ W(x+s, y+t) = W(x, y) \]

\( s, t \in [-a+1] \times [-b+1] \)
Estimating the degradation

Observations

1. Go to parts of g that you have a priori knowledge.
   See how it looks.
   Compare to what it should have looked like.
   Make some intelligent guess about
   data and noise

2. Experiment. Put a known signal into your system to calibrate.
Modelling


c\(H_{\text{air}, \text{vis}} = e^{\frac{1}{k}(w_0^2 + w_i^2)}\)
Blur $\rightarrow$ motion blur $\rightarrow$ out of focus blur $\rightarrow$ atmospheric turbulence.

**Motion Blur.** Assume scene translates at constant velocity $V_{\text{relative}}$ under an angle $\phi$ radians w.r.t. horizontal axis.

- Exposure time $[0, t_{\text{exposure}}]$
- Length of motion $L = V_{\text{relative}} \cdot t_{\text{exposure}}$

PSF $d(x, y) = \begin{cases} \frac{1}{L^2} & \text{if } \sqrt{x^2+y^2} \leq \frac{L}{2} \quad , \quad \frac{x}{y} = -\tan \phi \\ 0 & \text{otherwise} \end{cases}$
Out of focus blur

\[ d(x,y) = \begin{cases} \frac{1}{HR^2} & \text{if } \sqrt{x^2+y^2} \leq R^2 \\ 0 & \text{otherwise} \end{cases} \]

\[ R \text{ is parameter for how much out of focus} \]

\[ d(u_i,v_i) = \begin{cases} \frac{1}{C} & \text{if } \sqrt{u_i^2+v_i^2} \leq R^2 \\ 0 & \text{otherwise} \end{cases} \]

Atmospheric blur

\[ d(x,y) = c \exp \left\{ -\frac{x^2+y^2}{2\sigma^2} \right\} \]
Motion Blur modeling

\[ g(x,y) = \int_0^T f(x-x_0(t),y-y_0(t)) \, dt \]

\[ g(x,y) = \mathcal{F}\{g(x,y)e^{-j2\pi (w_x x + w_y y)}\} \]

F.T. \[ \mathcal{E}\{g(x,y)\} = C(w_x, w_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) e^{-j2\pi (w_x x + w_y y)} \, dx \, dy. \]
\[
G(\omega_x, \omega_y) = \int_0^T \left( \int_0^\infty \int_0^\infty f(x-x_0(t), y-y_0(t)) e^{-j2\pi(\omega_x x + \omega_y y)} \right) dx \, dy \, dt
\]

\[
= \int_0^T F(\omega_x, \omega_y) e^{-j2\pi(\omega_x x_0(t) + \omega_y y_0(t))} dt
\]

\[
G(\omega_x, \omega_y) = F(\omega_x, \omega_y) \int_0^T e^{H(\omega_x, \omega_y)} dt
\]

- \(x_0(t) = 0\) \(\Rightarrow\) \(x(t) = 0\) \(\Rightarrow\) \(g(x, y) = T + (x, y)\)

- \(x_0(t) = \frac{at}{T} = \text{constant speed along} \ x \ \text{direction}\) \(\Rightarrow\) \(y_0(t) = 0\)

\[
\begin{cases}
G(\omega_x, \omega_y) = F(\omega_x, \omega_y) + H(\omega_x, \omega_y)
\end{cases}
\]
\[ H(\omega_x, \omega_y) = \int_0^T e^{-j \omega_y \omega t} \, dt \]
\[ = \left[ e^{-j \omega_y \omega t} \right]_0^T \]
\[ = e^{-j \omega_y \omega T} - 1 \]
\[ H(\omega_x, \omega_y) = \frac{T \sin (\pi \omega_x a)}{\pi \omega_x a} e^{-j \pi \omega_x a} \]
\( G \) \( \epsilon (w_x, w_y) = F(w_x, w_y) H(w_x, w_y) + \text{Noise} \)
Techniques for deconvolution

1. Blur fn is known
2. Blur fn is unknown
3. Blind deconvolution

3 classes of Restoration/Deconvolution Algs.

1. Inverse Filtering
   - Weiner
2. Least square filters (CLS)
3. Iterative filters

Inverse Filtering

\[ h_{inv} \ast d = s(u_1, u_2) \]  
\[ H_{inv}(w_1, w_2) = \frac{1}{D(w_1, w_2)} \]
Problem 2

H(m, w) = \begin{cases} 0 & \text{if } H(m, w) \text{ is noise} \\ \text{High-pass filter} & \text{accurate value} \end{cases}

D(m, w) = \begin{cases} D(\frac{1}{2}, w) & \text{if } D(\frac{1}{2}, w) \text{ is an even number} \\ \text{otherwise} & \text{if } D(\frac{1}{2}, w) \text{ is an odd number} \end{cases}

Figs. 3, 4, and 5, p. 239 of [Reference].