Asymptote about T(r)

\[ \text{T(r)} = S \]

- Lead.
  - 
  - 
  - 
  - 
  - 

- Environ of light. 
  - 
  - 0 = dark = block

- Remembered it (0,1)

- -

- -

- Environ containing values of intensity.

- Consider continuing values of intensity.

- Historical aspects/evaluation of
(a) $T(r)$ single valued and monotonically increasing in the interval $0 \leq r \leq 1$

for inverse transform $T$ to exist

preserve increasing order from black to white in outputting.
\[ P_s(s) = \mathbf{P}_n(r) \]

Result after proof:

\[ T_1(s) \text{ satisfies condition (a)} \]

\[ T_2(s) = r \]

\[ T_3(s) = s \]

\[ T_4(s) = \mathbf{1} \]

An initial insight: In the same way,
Consider CDF: as a Transformation.

\[ s = T(r) = \int_0^r P_r(w) \, dw \]

\[ \frac{ds}{dr} = \frac{d}{dr} \left( \int_0^r P_r(w) \, dw \right) = P_r(r) \]

\[ P_s(s) = P_r(r) \left| \frac{1}{P_r(r)} \right| = 1 \quad 0 \leq s \leq 1 \]

Words: If \( T(r) \) is just a CDF on just the integral of input \( \text{pdf} \) \( (P_r(r)) \) then applying \( T(r) \) results in a image whose \( \text{pdf} \) \( (P_s(s)) \) is Uniform.
Discrete Case

$P_k = \text{discrete intensity values}$

$k = 0, \ldots, L-1$

$n_k = \# \text{ of pixels that have intensity } r_k$

$S_k = T(r_k) = \sum_{j=0}^{\infty} \sum_{j=0}^{\infty} \frac{1}{W_j W_{j'}}$

$k = 0, \ldots, L-1$
$T(r) = S$

Rather than $P_3(s)$ uniform, we want $P_3(s)$ to match a "desired" pdf given.

$r = \text{pixel value before matching}$

$z = \text{pixel } \ldots \text{ after matching}$

Can compute $P_z(r)$ from given image.

Given, given $P_z(z)$

Goal: What is the trajectory $r ightarrow z$?
Approach:

\[ S = T(r) = \int_0^r P_T(r') \, dr' \]

\[ v = G(z) = \int_0^z P_z(t) \, dt \]

Histogram matching

\[ G(z) = T(r) \]

\[ z = G^{-1}(r) \]

\[ T(r) \]

\[ T(0), T(z_0), T(z_1) \]

\[ v = \text{CDF of } z \]

\[ \text{how to find } z_0 \]
Lookuptable

<table>
<thead>
<tr>
<th>( r_0 )</th>
<th>( z_0 )</th>
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<tbody>
<tr>
<td>( r_1 )</td>
<td>( z_1 )</td>
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\[ \text{Matched history of } P_{\text{\text{arr}}} (z) \text{ desired} \]
Local Enhancement

Using local histogram analysis

Define square or rectangle neighborhood over each pixel, move center of "window" across the image.

At each location: compute histogram in the window.

Do hist. eq.
Use local stats for each.

\[
\begin{align*}
\text{Stat} & \rightarrow \text{mean}, \text{variance} \\
\text{Global mean:} & \quad \mu_G = \frac{1}{N} \sum_{i=0}^{N} \mu_i \\
\text{Global variance:} & \quad \sigma_G^2 = \frac{1}{N} \sum_{i=0}^{N} (\mu_i - \mu_G)^2 \\
\text{Local mean:} & \quad \mu_{x,y} = \frac{1}{S_{x,y}} \sum_{(s,t) \in S_{x,y}} x_{s,t} \\
\text{Local variance:} & \quad \sigma_{x,y}^2 = \frac{1}{S_{x,y}} \sum_{(s,t) \in S_{x,y}} (x_{s,t} - \mu_{x,y})^2 \\
\end{align*}
\]
Goal: Ensure dark areas white against
The bright areas unchanged as possible.

- Detect regions that have both.
  (a) dark \rightarrow local mean has to be
      small compared to global mean.

\[
\begin{align*}
    M_{xy} & \leq K_0 M_G \\
    0 \leq k_0 \leq 1 & \quad \text{Constant} \quad k_0 \approx 0.4 \\
    \sigma_{xy} & \leq K_2 \sigma_{G} \\
    k_0 \text{ constant} & \quad k_2 \approx 0.4
\end{align*}
\]
(c) not too low of contrast, leave that region unclipped

\[ \sigma_{xy} \geq k_1 \sigma \text{ Var } \]

\[ k_1 = 0.01 \]

\[ q(x, y) = \begin{cases} 
E[f(x, u)] & \text{if } M_{sxy} \leq k_0 M_c \\
\text{otherwise.} & \sigma_{xy} \leq \sigma_{sxy} \leq k_2 \sigma_c 
\end{cases} \]
Emulation is another Arithmetical/Logic operator:

- **Logic**: AND, OR, NOT
- **Functionally complete**: NAND
- **Negative Inflection**: NOT
- **Masking**: AND, OR

Image subtraction:

\[ g(x, y) = f(x, y) - h(x, y) \]

No mask.