1. Enhance $\Rightarrow$ image "look" better
   subjective improve.

2. Restoration

   Image has been degraded by smooth,
   - noise
   - blur
   - Atmospheric turbulence.

   objective model $\Rightarrow$ Error $\Rightarrow$ MSE
$f(x,y)$ \rightarrow \text{Degraded, } h(m,n) \rightarrow g(x,y)$

\[ g(x,y) = h \ast f + \eta \]
\[ g(x,y) = h(x,y) \ast f(x,y) + \eta(x,y) \]

$g(x,y)$ \rightarrow \text{Restoration Box} \rightarrow \hat{f}(x,y)$
minimize $E\left[ (f(x,y) - \hat{f}(x,y))^2 \right]$

Today.

$H$ is identity.

only corruption $\rightarrow$ noise.

$g(x,y) = f(x,y) + \eta(x,y)$

Assume noise is independent of spatial coordinate
uncorrelated w.r.t. image itself.
Gaussian distribution:

\[ p(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

- Mean: \( \mu \)
- Variance: \( \sigma^2 \)
- Standard deviation: \( \sigma \)

99.7% of values lie within 3 standard deviations of the mean:

\[ [\mu - 3\sigma, \mu + 3\sigma] \]

- Electronic noise
- Sensor noise due to poor illumination or high temperature
(2) Rayleigh:

\[ p(z) = \begin{cases} \frac{z}{b} (z - a) e^{-\frac{(z-a)^2}{b}} & \text{if } z \geq a \\ 0 & \text{if } z < a \end{cases} \]

\[ \mu = a + \sqrt{\frac{b\pi}{4}} \]

noise in range imaging apps.
3. Erlang (Gamma) noise:

\[ p(z) = \begin{cases} \frac{a^b z^{b-1} e^{-az}}{(b-1)!} & z > 0 \\ 0 & z < 0 \end{cases} \]

4. Exponential special case of Erlang:

\[ p(z) = \begin{cases} a e^{-az} & z > 0 \\ 0 & z < 0 \end{cases} \]

\[ \mu = \frac{b}{a} \]

\[ \sigma^2 = \frac{b}{a^2} \]

Laser imaging.

Networking.
3. Uniform: 
\[ p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \]

\[ \mu = \frac{a+b}{2} \]

\[ \text{var} = \frac{(b-a)^2}{12} \]

random # generator.

6. Impulse, salt/pepper noise.

\[ p(z) = \sum Pa \delta(z-a) \delta(z-b) \]

if \( b > a \), then show up as light dots; 
if \( b \leq a \), then show up as dark dot.
1. Arithmetic Mean.

\[ S_{x,y} = \text{window} \]

\[ f(\{x,y\}) = \frac{1}{mn} \sum_{(s,t) \in S_{x,y}} g(\{s,t\}) \]

2. Geometric Mean.

\[ f(\{x,y\}) = \prod_{(s,t) \in S_{x,y}} a(\{s,t\}) \]

other equations
3. Harmonic mean filter.

\[ \hat{f}(x, y) = \frac{\sum_{(s, t) \in S_{xy}} \frac{1}{g(s, t)}}{mn} \]

works for salt noise. fails for pepper

4. Contour Harmonic.

\[ \hat{f}(x, y) = \frac{\sum_{(s, t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s, t) \in S_{xy}} g(s, t)} \]

Q > 0 \rightarrow remove pepper

Q < 0 \rightarrow remove salt noise
Order Statistics 6.14

1. Median:
   - Search for $S_{xy}$. Find median replacement: $\hat{m}(x,y)$ with the median value.

2. Max:
   - Also removes some data points.

3. Min:
   - Also removes some white pixels.

4. Midpoint:
   - $f(x,y) = \max_{S_{xy}} \min_{S_{xy}} \{ \max + \min \}$. 

   $f(x,y) = \begin{cases} \max \ g(s,t) & \text{if } x < S_{xy} \\ \min \ g(s,t) & \text{if } x > S_{xy} \end{cases}$
Alpha Trimmed mean filter

\[ \hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s, t) \in S_{xy}} g(\Theta(x, y)) \]

\[ S_{xy} \text{ must} \]

\[ g_r(s, t) \rightarrow g(s, t) \text{ excluding} \]

\[ d/2 \text{ brightest graylevel} \]

\[ \text{and } d/2 \text{ darkest } \]

\[ \text{grayscale layout} \]
Adaptive local noise reduction

\[ \hat{f}(x, y) = g(x, y) - \frac{\sigma_n^2}{\sigma_L^2} (g(x, y) - m_L) \]

\[ m_L = \text{local mean} = \frac{1}{mn} \sum_{(x,t)eS_{xy}} g(x, t) \]

\[ \sigma_L^2 = \text{local variance} \]

\[ \sigma_n^2 = \text{noise variance} \]

if \( \sigma_n^2 \ll \sigma_L^2 \rightarrow \hat{f} \approx g \quad \text{good} \]

if \( \sigma_n^2 \ll \sigma_L^2 \rightarrow \hat{f} \ll m_L \)
Image Restoration

Spatial domain

- "additive Gaussian noise"

Frequency domain

- salt/pepper

- median

- Adaptive Median filters

- Adaptive Median Filter

- looking at neighborhood $S_{x,y}$ around pixel $(x,y)$

- $z_{xy}$ = pixel value at center of $S_{x,y}$

- $z_{max}$ = max value of intensity over $S_{x,y}$
- $Z_{\text{min}} = \text{min value in } S_{xy}$
- $Z_{\text{med}} = \text{median value in } S_{xy}$

Outline

- Keep increasing window size until $Z_{\text{med}}$ is not an impulse, $Z_{\text{min}} < Z_{\text{med}} < Z_{\text{max}}$

- When this happens, check $Z_{xy}$.
  - If $Z_{xy}$ is not an impulse $\rightarrow$ output $Z_{xy}$
  - If $Z_{xy}$ is an impulse $\rightarrow$ output $Z_{\text{med}}$

Since $Z_{\text{med}}$ is not an impulse.
Psuedo code

Part A

if 

\[ Z_{\text{min}} < Z_{\text{med}} < Z_{\text{max}} \]

Then go to part B. \( \Rightarrow Z_{\text{med}} \) is not an impulse.

else 

if windowsize < \( S_{\text{max}} \)

window \( \leftarrow \) window + 1, go to part A

else

output \( Z_{xy} \).

Part B.

If 

\[ Z_{\text{min}} < Z_{xy} < Z_{\text{max}} \]

\( \Rightarrow Z_{xy} \) is not an impulse

output \( Z_{xy} \).

else

output \( Z_{\text{med}} \).