Assignment 10

Due April 20th, 2012

This is the last homework in class. Enjoy it.

1) MRI Artifacts and Debugging

In this question you will be given several pairs of images. The images may exhibit artifacts which will be indicated by arrows/circles. Answer the questions as best as you can. First describe the artifact and then provide as much information as possible on the source/s of the artifact and why it appears this way. There might be several possible right answers. A final answer without explanation will not be credited.

a) Example: What is the source of the artifact in the left image and what is the difference in the acquisition/processing of the two?

Solution:

The artifact in the image appears as the back of the brain aliasing over and wrapping to the front. This is typical when the prescribed FOV in the phase encode is smaller than the FOV of the object. In that case the sampling density does not meet the Nyquist rate. The phase encode is obviously Anterior-Posterior.

The image on the right does not exhibit aliasing. One way to avoid aliasing is to increase the number of phase encodes while keeping the resolution the same. The other way is to swap the readout and phase-encode directions. It seems that the latter approach of swapping the readout direction was used since the back of the skull as well as the nose are cropped. Choosing a larger FOV in the phase encode would have shown the entire nose and skull.

Source of Artifact: aliasing in the phase-encode direction (A/P)

Difference: readout and phase encode directions swapped
b) What is the source of the artifact in the left image and what is the difference in the acquisition/processing of the two?

![Left Image](image1)

**Solutions:**

Something has gone wrong in the geometry of the left guy’s head! It has been compressed in space with respect to the rest of the FOV. This is a sagital scan with a very large FOV. For such large of a FOV, the gradients non-linearity start to show up. The gradients will be much weaker at the top and bottom and will result in a geometric distortion. This is fixed in postprocessing by applying interpolation to correct for the distortion in the right image.

**Source of artifact:** Gradient non-linearity  
**Difference:** The right image was grad-warped and interpolated to correct for the non-linearities.

c) What is the source of the artifact in the right image and what is the difference in the acquisition/processing of the two?

![Right Image](image2)

**Source of artifact:** The fat in the right image is shifted right with respect to water. This is a result of a low-bandwidth/pixel readout. The readout direction is RL.  
**Difference:** The readout bandwidth in the left image is higher (shorter readout) and the fat does not have time to accrue much phase and appears to not shift by much. (There’s still a slight shift though!)

d) What is the source of the artifact in the two images and what is the difference in the acquisition/processing of the two?
Solutions:
There’s appear to be ghosting artifacts that seem to be originating from the eyes. This is an artifact of eye motion during a scan. Motion during an acquisition will create inconsistent data in the slow-direction (the phase encode). Therefore the phase encode direction in the left image is A/P and R/L in the right image.

Source of artifact: Eye motion during the scan
Difference: Readout-phase encode directions are swapped.

e) What is the source of the artifact in the left image and what is the difference in the acquisition/processing of the two? (Ignore the contrast difference between the two)

Solutions:
The right image exhibits loss of signal next to air-tissue interfaces near the sinuses and the ear canal. This is a result of $T_2^*$ decay typical for a gradient echo sequence. The right image does not exhibits the loss of signal. This can be due to much shorter echo time, but there’s significant T2 weighting in the image that makes this infeasible. The other explanation is a Spin-echo sequence!

Source of artifact: Loss of signal due to $T_2^*$ intravoxel dephasing in a gradient echo sequence
Difference: The right image does not exhibits loss of signal and is most likely a spin-echo sequence
2) In class when we presented the spin-echo saturation recovery pulse sequence, we assumed that the echo time \( T_E \) was short relative to the repetition time \( T_R \), so we could ignore it. This is not always a good approximation, particularly if we want to measure \( T_1 \).

In fact, after the 90 there will be some \( M_z \) recovery. The 180 will invert this, and \( M_z \) will continue to recover until the next 90. Let \( M_z^- \) be the magnetization immediately before the 90.

Assume that \( T_E \) is small compared to \( T_1 \), but not negligible, so that we can approximate

\[
(1 - e^{-T_E/T_1}) \approx T_E/T_1.
\]

This means that the \( M_z \) recovery curves are approximately linear within a time \( T_E \) after the 90.

a) Find an approximate expression for the time when the magnetization cross the \( M_z = 0 \) axis.

Solution:

If \( T_E \) is short relative to \( T_1 \), we can linearize the recovery curves near \( M_z = 0 \). As long as \( M_z \) does get too big (i.e. \( T_E \) is short) the recovery curves are lines with a slope \( 1/T_1 \). Following the 90 at \( t = 0 \), the \( z \) magnetization evolves as

\[
M_z = M_0 \left( \frac{t}{T_1} \right)
\]

where the constant scale factor \( M_0 \) has been suppressed in the plot.

The \( z \) magnetization increases with a slope \( M_0/T_1 \) until \( T_E/2 \), when it is \( M_0 \left( \frac{T_E}{2T_1} \right) \). This is inverted by the 180 to \(-M_0 \left( \frac{T_E}{2T_1} \right) \), which again recovers linearly with a slope \( M_0/T_1 \). The equation for this part of the recovery curve is

\[
M_z = M_0 \left( \frac{t - T_E}{T_1} \right)
\]

This goes through zero at \( t = T_E \).
b) This might be called a $T_1$ echo. Why?

**Solution:**
A spin echo refocuses spins no matter what their resonant frequencies are. In this case $M_z$ goes through zero at $T_E$ no matter what the $T_1$ is. This is illustrated below for the case of three different $T_1$'s.

![Graph](image)

The 180° refocuses the different decay rates of the different $T_1$'s, just as the 180° refocuses different precession rates for a spin echo.

b) Find an expression for $M_z^-$ using what you found in (a). **Solution:**
The $z$ magnetization is zero at time $T_E$, and then recovers for the remainder of the $T_R$ interval.
The $z$ magnetization just before the next RF pulse is then

$$M_z^- = (1 - e^{-\frac{T_R-T_E}{T_1}})M_0$$

c) Compare $M_z^-$ calculated using your expression and the expression ignoring $T_E$ for the case where $T_R = 200$ ms, $T_E = 20$ ms, and $T_1 = 800$ ms. What is the percent error produced by neglecting $T_E$ in this case?

**Solution:** If we ignore $T_E$,

$$M_z^- = M_0(1 - e^{-\frac{T_R}{T_1}}) = M_0(1 - e^{-\frac{200}{800}}) = M_0(0.2212).$$

If we include the effect of $T_E$,

$$M_z^- = M_0(1 - e^{-\frac{(T_R-T_E)}{T_1}}) = M_0(1 - e^{-\frac{180}{800}}) = M_0(0.2015).$$

The percent error is then

$$\frac{0.2212 - 0.2015}{0.2015} \times 100 = 9.8\%$$

3) Choosing Scan Parameters (We will cover material for this on Tuesday.)

You are designing a pulse sequence to image the upper abdomen. The tissues of interest are

- Liver, $T_1 = 600$ ms, $T_2 = 50$ ms.
- Spleen, $T_1 = 1000$ ms, $T_2 = 80$ ms.
- Fat, $T_1 = 350$ ms, $T_2 = 60$ ms.
- Gall Bladder, $T_1 = 2000$ ms, $T_2 = 300$ ms.

This will be a spin echo acquisition, where you have to choose the repetition time $T_R$, the echo time $T_E$. In addition, you can also add an inversion recovery pulse *if you need it*, and specify the inversion time $T_I$. Otherwise leave the $T_I$ blank. The minimum $T_E$ is 15 ms, and the minimum $T_R$ is 20 ms.

In each case you only need to specify the scan parameters *approximately*. You only need reasonable values, not necessarily the optimum values. However, you need to describe the reasoning behind your choices.
a) Choose the scan parameters so that you have good contrast between liver and spleen, and low signal from the gall bladder.

**Solution:**
The liver and spleen differ in both $T_1$ and $T_2$ so either of these contrasts might work.
If we think about $T_2$ contrast (long $T_R$, long $T_E$), the spleen would be brighter than the liver. However we also see that the gall bladder would also be brighter still, since it has a $T_2 = 300$ ms. This doesn’t work unless we null it out with an inversion pulse. We only want to do this if we have to.
Next, if we think about $T_1$ contrast ($T_R \leq T_1$, short $T_E$) we see that the liver would be brighter than the spleen. In addition, the gall bladder is has a low signal since it has a very long $T_1$.
This is the answer.
A reasonable choice of paramters might be $T_R = 800$ (the average of the $T_1$’s for liver and spleen), and a minimum $T_E$ to maximize signal, and minimize $T_2$ contrast. Any shorter $T_R$ would also work, provided it is longer than the $T_E$.
Note that a sequence with short $T_R$ and long $T_E$ doesn’t work, because this produces both $T_1$ and $T_2$ weighting, and these go in opposite directions. The result is no contrast.

\[
\begin{array}{ccc}
T_R = 800 \text{ ms} & T_E = 15 \text{ ms} & T_I = \text{none} \\
\end{array}
\]

b) Choose the scan parameters so that you see only the gall bladder, with little signal from the other tissues.

**Solution:**
The gall bladder has a long $T_1$ and a long $T_2$.
$T_1$ contrast by itself won’t work, long $T_1$’s produce low signal, and that isn’t what we want. We could use an inversion pulse, but we don’t want to if we don’t have to.
Heavy $T_2$ contrast will make the gall bladder the brighter than everything else, since it has the longest $T_2$ by far. This is the answer.
For $T_2$ contrast we want a long $T_R$ and a long $T_E$. If we choose $T_R = T_1 = 2000$ ms we will be reasonably SNR efficient (at the cost of some $T_1$ weighting). Then we choose $T_E = T_2 = 300$ ms, which is much longer that the $T_2$’s of all of the other tissues, resulting in almost no signal from these tissues.
Other values of $T_R$ would also work, provided it is bigger than $T_E$.

\[
\begin{array}{ccc}
T_R = 2000 \text{ ms} & T_E = 300 \text{ ms} & T_I = \text{none} \\
\end{array}
\]

c) Choose the scan parameters so that the liver produces no signal at all, but the spleen is bright.

**Solution:**
In this case we want to completely eliminate the signal from liver, while maintaining some signal from the spleen. The $T_1$’s and $T_2$’s of these tissues don’t differ enough to do this with either simple $T_1$ contrast or $T_2$ contrast alone.
In this case we need to use an inversion-recovery pulse to null out the signal from liver. Many different $T_R$’s and $T_E$’s can be used, provided they don’t lose the spleen signal. To maximize the spleen signal, a good choice is the minimum $T_E = 15$ ms. The $T_R$ should be
significantly longer than this to allow time for the recovery, and the inversion pulse. A reasonable choice might be $T_R = 1000$ ms.

Once these are decided, we need to figure out what the inversion time should be. This is in the book, and was one of the cases we worked through in class. The result is

$$T_I = -T_1 \log \left( \frac{1 + e^{-T_R/T_1}}{2} \right)$$

Substituting $T_R = 1000$ ms, and $T_1 = 600$ ms, we get $T_I = 312$ ms.

There were many other acceptable answers, depending on what you chose for $T_R$.

| $T_R$ = 1000 ms | $T_E$ = 15 ms | $T_I$ = 312 ms |

4) Contrast Preparation:

Consider the following sequence of two $\theta$ degrees tip-angle RF pulses separated by $T$ seconds.

\[\Theta \quad T \quad \Theta\]

a) Given that the equilibrium magnetization is $M_0 \hat{z}$, derive an expression for the $M_z$ component of the magnetization immediately following the second RF as a function of $T_2$, $T$, and $\theta$. You can neglect $T_1$ recovery (since $T_1 \gg T$) and off-resonance.

**Solutions:**

For easy notation we will define $E_2 = e^{-T/T_2}$, $C_\theta = \cos(\theta)$ and $S_\theta = \sin(\theta)$

\[
M(T_+) = \begin{bmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} E_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} 0 \\ M_0 \end{bmatrix} = \begin{bmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} E_2 S_\theta M_0 \\ C_\theta M_0 \end{bmatrix}
\]

So, the $M_z(T_+)$ component is,

$$M_z(T_+) = (C_\theta^2 - E_2 S_\theta^2) M_0$$

\[
M_z = \left( \cos^2(\theta) - e^{-T/T_2} \sin^2(\theta) \right) M_0
\]
b) Given $T = T_0$, find the flip angle $\theta$ for which the $M_z$ component is zero for spins with a desired $T_2$ transverse relaxation value.

_Solutions:_

\[
\begin{align*}
0 &= C_\theta^2 - E_2 S_\theta^2 \\
\frac{C_\theta^2}{S_\theta^2} &= E_2 \\
\tan(\theta) &= E_2^{-1/2} \\
\theta &= \text{atan}(E_2^{-1/2})
\end{align*}
\]

\[
\theta = \text{atan}\left(\frac{T}{e^{\frac{T_2}{T_1}}}\right)
\]

MR Angiography is an important tool in assessing vascular diseases in patients. Often, $T_1$ shortening Gadolinium contrast agents are used in combination with short TR sequences to increase the blood-muscle contrast. However, using Gadolinium based contrast agents can result in a life threatening syndrome, called NSF, in patients that have renal disease.

Consider the contrast preparation imaging paradigm below. For each preparation sequence a single phase encode is collected. Also, assume $TR \gg T_1$.

![Preparation Sequence Diagram](image)

The $T_1/T_2$ of blood are 1000/220ms and the $T_1/T_2$ of muscle are 870/50ms. We would like to design a non-contrast enhanced preparation pulse that will ideally have good blood signal and no muscle signal at all. In addition, we would like the preparation pulse to not be much longer than 50ms.

c) Based on your previous derivations, design a preparation sequence that nulls the muscle signal while producing signal from blood. Draw the sequence pointing out the relevant parameters. (Extra points will be given for those coming with solutions that are insensitive to off-resonance). What is the blood $M_z$ magnetization after the prep-pulse?

_Solutions:_

The previous questions have taught us that we can null the signal of a particular $T_2$ specie. In this part we need to design a preparation sequence that will null the muscle signal while still producing blood signal. Unfortunately, we can not use inversion recovery based on the $T_1$ since that would take much longer than than 50ms. Using an excitation recovery would probably not leave much blood signal. Therefore we are going to use the $T_2$ prep-pulse from before.

For this preparation sequence we need to choose the flip angle $\theta$ and $T$ such that the muscle signal is nulled and in addition the blood signal is as high as possible. $T$ is a free parameter, but it can not be longer than 50ms. Calculating the $\theta$ for $T = 10ms, 25ms, 35ms, 50ms$ we
get, $\theta = 47.86^\circ$, $52.09^\circ$, $54.83^\circ$, $58.76^\circ$ with corresponding $M_z = -0.075 M_0$, $-0.18 M_0$, $-0.24 M_0$, $-0.31 M_0$. So, $T = 50\text{ms}$ gives the highest blood signal with $\theta = 58.76^\circ$.

Of course this pulse is VERY sensitive to $B_0$ inhomogeneity because spins would be precessing during the waiting time and the second RF pulse will have different effect on different resonances. This can be mitigated by applying a spin-echo 180 pulse such that all the spins will refocus exactly at the time the second RF pulse is applied. However, if we use a single refocusing, we need to change the sign of the second RF so it will have the same effect. Alternatively we can use two refocusing RF pulses. Each 180 refocusing pulses should have crushers gradients on both sides to prevent a parasitic FID.

A very important part of the question is to realize that at the end of this preparation sequence there remains is a significant transverse magnetization. To remove it, we need to apply a large gradient crusher after the second RF.

Here is an acceptable solution:

\[
\begin{array}{c|c}
\text{Prep Sequence} & \text{Imaging Sequence} \\
\hline
\text{RF} & 90 \\
\Theta_x & \\
T & \\
\hline
\text{Gz} & \\
\text{Gy} & \\
\text{Gx} & \\
\end{array}
\]

Here are two acceptable $B_0$ insensitive solutions:

\[
\begin{array}{c|c}
\text{Prep Sequence} & \text{Imaging Sequence} \\
\hline
\text{RF} & 90 \\
\Theta_x & \\
T/2 & \\
180 & \\
T/2 & \\
\hline
\text{Gz} & \\
\text{Gy} & \\
\text{Gx} & \\
\end{array}
\]

\[
\begin{array}{c|c}
\text{Prep Sequence} & \text{Imaging Sequence} \\
\hline
\text{RF} & 90 \\
\Theta_x & \\
T/4 & \\
180 & \\
T/4 & \\
\hline
\text{Gz} & \\
\text{Gy} & \\
\text{Gx} & \\
\end{array}
\]

Blood $|M_{z\text{prep}}| = 0.31 M_0$
5) Nishimura 7.3

Solutions:

a.

\[ I(x, y) = K \rho(x, y) \left[ 1 - e^{-\frac{TR}{T_1}(x, y)} \right] e^{-\frac{TE}{T_2}(x, y)} \]

Let \( C(x, y) = I_A(x, y) - I_B(x, y) \) then,

\[ \frac{d}{TR} C(x, y) = 0 \]

\[ = K \rho \frac{d}{TR} \left[ (1 - e^{-\frac{TR}{T_1}A}) e^{-\frac{TE}{T_2}A} - \left( 1 - e^{-\frac{TR}{T_1}B} \right) e^{-\frac{TE}{T_2}B} \right] \]

\[ = K \rho \frac{d}{TR} \left[ e^{-\frac{TE}{T_2}A} - e^{-\frac{TE}{T_2}A} e^{-\frac{TR}{T_1}A} - \left( e^{-\frac{TE}{T_2}A} - e^{-\frac{TR}{T_1}B} e^{-\frac{TE}{T_2}B} \right) \right] \]

\[ = K \rho \left[ \frac{1}{T_1} A e^{-\frac{TE}{T_2}A} e^{-\frac{TR}{T_1}A} - \frac{1}{T_1 B} e^{-\frac{TR}{T_1 B} e^{-\frac{TE}{T_2}B}} \right] \]

\[ = K \rho \left[ \frac{1}{T_1 B} e^{-\frac{TR}{T_1 B} e^{-\frac{TE}{T_2}B}} - \frac{1}{T_1 A} e^{-\frac{TR}{T_1 A} e^{-\frac{TE}{T_2}A}} \right] \]

\[ = K \rho \left[ \frac{1}{T_1 B} e^{-\frac{TR}{T_1 B} e^{-\frac{TE}{T_2}B}} - \frac{1}{T_1 A} e^{-\frac{TR}{T_1 A} e^{-\frac{TE}{T_2}A}} \right] \]

\[ = K \rho \left[ \frac{1}{T_1 B} e^{-\frac{TR}{T_1 B} e^{-\frac{TE}{T_2}B}} - \frac{1}{T_1 A} e^{-\frac{TR}{T_1 A} e^{-\frac{TE}{T_2}A}} \right] \]

\[ = e^{\frac{TR}{T_1 A} e^{-\frac{TE}{T_2}A}} - e^{\frac{TR}{T_1 B} e^{-\frac{TE}{T_2}B}} \]

\[ = e^{\frac{TR}{T_1 A} e^{-\frac{TE}{T_2}A}} - e^{\frac{TR}{T_1 B} e^{-\frac{TE}{T_2}B}} \]

\[ = \frac{ln(T_1 B / T_1 A) + TE(1/T_2 B - 1/T_2 A)}{1/T_1 A - 1/T_1 B} = TR \]

b.

\[ TR = \frac{ln(920/780) + 20(1/100 - 1/92)}{1/780 - 1/920} = 757ms \]

c. Using the same \( C(x, y) \) we take \( \frac{d}{TE} \) and get after some algebra:

\[ TE = \frac{ln(T_2 B / T_2 A) + ln \left( \frac{1 - e^{-TR/T_1 A}}{1 - e^{-TR/T_1 B}} \right)}{1/T_2 A - 1/T_2 B} \]

d.

\[ TE = \frac{ln(100/92) + ln \left( \frac{1 - e^{-3000/780}}{1 - e^{-3000/920}} \right)}{1/92 - 1/100} = 116ms \]

6) In order to produce a very high resolution image of the brain, you specify a scan that provides 0.25 mm resolution over a 25.6 cm FOV. Unfortunately, the SNR is very low.

a) You lowpass filter the image to increase the SNR, using an ideal lowpass filter in both \( x \) and \( y \).

If the resulting resolution is 0.5 mm in both \( x \) and \( y \), what is the final SNR compared to the original image?

Solution:
The voxel volume has been double in both dimensions. However, the ideal lowpass filter throws away 3/4 of the data. The effective A/D time is now 1/4 of what it originally was. The new SNR is then

\[ \text{SNR}_2 = f(T_1, T_2, \rho) \sqrt{\frac{T_{A/D,2}}{T_{A/D,1}}} \delta_x \delta_y \delta_z \]

\[ = f(T_1, T_2, \rho) \sqrt{\frac{T_{A/D,1}}{4}} (2\delta_x, 1)(2\delta_y, 1) \delta_z, 1 \]

\[ = 2 f(T_1, T_2, \rho) \sqrt{\frac{T_{A/D,1}}{2}} \delta_x, 1 \delta_y, 1 \delta_z, 1 \]

\[ = 2 \text{SNR}_1 \]

b) How does this SNR compare to an image with 0.5 mm resolution, and the same total imaging time as the 0.25 mm resolution image? Assume the same A/D duration for each readout interval, and that the 0.5 mm scan averages to provide the same number of readouts.

**Solution:**

In this case the voxel volume doubles, but we get the benefit of the full scan time

\[ \text{SNR}_3 = f(T_1, T_2, \rho) \sqrt{\frac{T_{A/D,3}}{T_{A/D,1}}} \delta_x, 3 \delta_y, 3 \delta_z, 3 \]

\[ = f(T_1, T_2, \rho) \sqrt{\frac{T_{A/D,1}}{2}} (2\delta_x, 1)(2\delta_y, 1) \delta_z, 1 \]

\[ = 4 f(T_1, T_2, \rho) \sqrt{\frac{T_{A/D,1}}{2}} \delta_x, 1 \delta_y, 1 \delta_z, 1 \]

\[ = 4 \text{SNR}_1 \]

The conclusion from (a) and (b) is that while we can recover somewhat in SNR by lowpass filtering, we can never do as well as acquiring the data at the final resolution directly. There is an opportunity cost is having collected high-spatial frequency data that you won’t ultimately use.

7) Image SNR

You have a working 2DFT gradient-recalled echo pulse sequence that produces an image with an \( \text{SNR}_1 \) of 100. What is the new \( \text{SNR}_2 \) after you make one of the following changes.

a) Double the TBW of the slice selective RF pulse, while the RF pulse duration and the slice select gradient remain the same.

**Solution**

Doubling the TBW while keeping the pulse the same length (the gradient is the same length) doubles the bandwidth of the RF. The gradient is the same strength, so the slice thickness is doubled,

\[ \delta_z, 2 = 2\delta_z, 1 \]

The voxel volume has doubled, and every thing else has remained the same. SNR is proportional to voxel volume, so

\[ \text{SNR}_2 = 2 \text{SNR}_1 = 2(100) = 200 \]

\[ \text{SNR}_2 = 200 \]
b) Double the readout gradient strength, while keeping the A/D duration and sampling rate the same.

**Solution**
Doubling the readout gradient strength while keeping the duration the same doubles the extent in k-space. The halves the resolution in $x$, and halves the voxel volume.
Since SNR is proportional to voxel volume

$$SNR_2 = \frac{1}{2}SNR_1 = \frac{1}{2}(100) = 50.$$ 

\[SNR_2 = 50\]

c) Double both the readout gradient strength and the sampling rate, while halving the duration of the A/D window. Assume the anti-aliasing filter bandwidth matches the sampling rate.

**Solution**
In this case we cover the same extent in k-space in half the time. The resolution remains the same, but the A/D time is halved.
Since SNR is proportional to the square root of the total A/D time,

$$SNR_2 = \frac{1}{\sqrt{2}}SNR_1 = \frac{1}{\sqrt{2}}(100) = 70.7$$

\[SNR_2 = 70.7\]

d) Double the number of phase encodes, while keeping the maximum phase-encode gradient amplitude the same.

**Solution**
The maximum phase encode gradient remains the same, so the resolution remains the same. The total A/D time is doubled, since there are twice as many phase encodes.
Since SNR is proportional to the square root of the total A/D time,

$$SNR_2 = \sqrt{2}SNR_1 = \sqrt{2}(100) = 141$$

\[SNR_2 = 141\]
8) Nishimura 7.10

*Solutions:*

Reference 2DFT has:
256 Phase encodes
10ms readout (256 points)
Total imaging time $T_0$
FOV: $FOV_x \delta_y$  
Resolution: $\delta_x, \delta_y$
SNR=$\sqrt{13}$

a) In this case, we have 128 phase encodes $\rightarrow$ Imaging time cut by half. BW is the same. FOV the same $\delta_y$ is doubled.

$$SNR \propto \delta_x \delta_y \sqrt{T}$$
$$= \delta_x (2\delta_y) \sqrt{T_0/2}$$
$$= \sqrt{2} \text{SNR}_0 = \sqrt{2}$$

b) 512 phase encodes (same y-resolution, double $FOV_y$). Imaging time is doubled, resolution the same, BW is the same.

$$SNR \propto \delta_x \delta_y \sqrt{T}$$
$$= \delta_x \delta_y \sqrt{2T_0}$$
$$= \sqrt{2} \text{SNR}_0 = \sqrt{2}$$

c) Here the resolution in both dimension is improved, so $\delta_x$ and $\delta_y$ are doubled. Same number of phase encodes and frequency encodes, so $FOV_x$ and $FOV_y$ are halved.

We have the same number of frequency encoding points. The gradient amplitude is unchanged, but the k-space coverage is doubled. This means that the BW is halved (twice as much time spent on each sample).

$$SNR \propto \delta_x \delta_y \sqrt{T}$$
$$= (\delta_x/2)(\delta_y/2) \sqrt{2T_0}$$
$$= \sqrt{1/8} \text{SNR}_0 = \sqrt{1/8}$$