

EE 225E/BIOE 265 LECTURE 3 01/23/12

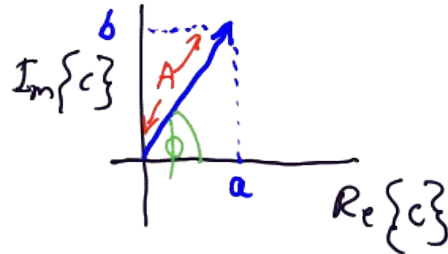
Preliminaries:

complex numbers:

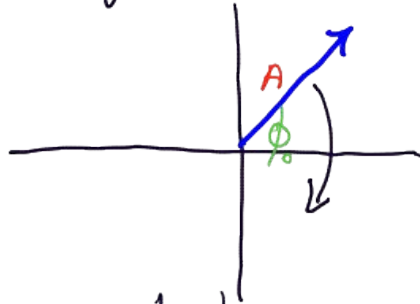
$$c = a + ib \quad i = \sqrt{-1}$$

or phasors:

$$c = Ae^{i\phi} = A(\cos\phi + i\sin\phi)$$



It is convenient to use complex # to describe a vector in a plane



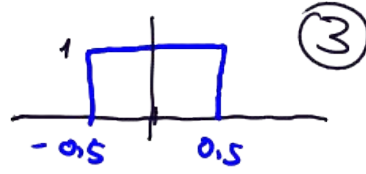
$$c(t) = Ae^{-i\omega t + i\phi_0} \Rightarrow \text{clock-wise rotation w/ angular freq. } \omega$$

VERY USEFUL TO DESCRIBE

(transverse) MAGNETIZATION

USEFUL FUNCTIONS:

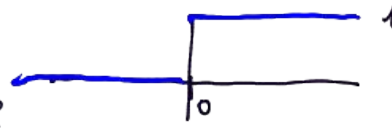
$$\Pi(x) = \begin{cases} 1 & |x| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



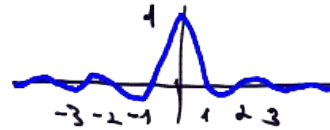
$$\Delta(x) = \begin{cases} 1 - |x| & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$



$$u(x) = \begin{cases} 1 & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$



The Impulse function $\delta(x)$

(4)

DEFINITION $\Rightarrow \int_{-\infty}^{\infty} \psi(x) \delta(x) dx = \psi(0)$

Properties:



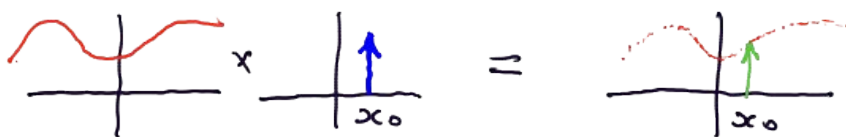
(1) $\delta(x) = 0 \quad \forall x \neq 0$

(2) $\delta(x) \rightarrow \infty \quad |x=0$

(3) $\int_{-\infty}^{\infty} \delta(x) dx = 1$

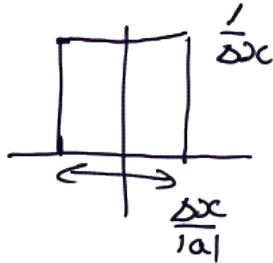
(4) $\delta(ax) = \frac{1}{|a|} \delta(x)$

(5) $\psi(x) \delta(x-x_0) = \psi(x_0) \delta(x-x_0)$



many realizations to $\delta(x)$: (5)
example: $\delta(x) = \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \Pi\left(\frac{x}{\Delta x}\right)$

$$\delta(x) = \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \Pi\left(\frac{x}{\Delta x}\right) =$$



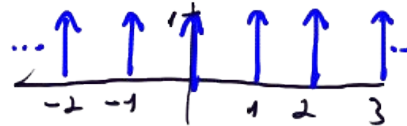
$$\text{Area} = \frac{\Delta x}{|a|} \cdot \frac{1}{\Delta x} = \frac{1}{|a|}$$

$$= \frac{1}{|a|} \delta(x)$$

Impulse Train $\mathcal{U}(x)$ (6)

(impulse train)
shah function
comb

$$\mathcal{U}(x) = \sum_{n=-\infty}^{\infty} \delta(x-n)$$



what about:



$$\sum_{n=-\infty}^{\infty} \delta(x - \Delta x n) = \sum \delta\left(\Delta x \left(\frac{x}{\Delta x} - n\right)\right) =$$

$$= \frac{1}{\Delta x} \sum \delta\left(\frac{x}{\Delta x} - n\right) = \frac{1}{\Delta x} \mathcal{U}\left(\frac{x}{\Delta x}\right)$$

$$\delta(x) = \frac{1}{|a|} \delta\left(\frac{x}{a}\right)$$

USEFUL TO MODEL SAMPLING!

SYSTEMS

(7)



Properties:

Linear

$$T\{q_1(x) + q_2(x)\} = y_1(x) + y_2(x)$$

$$T\{aq(x)\} = ay(x)$$

shift-invariant

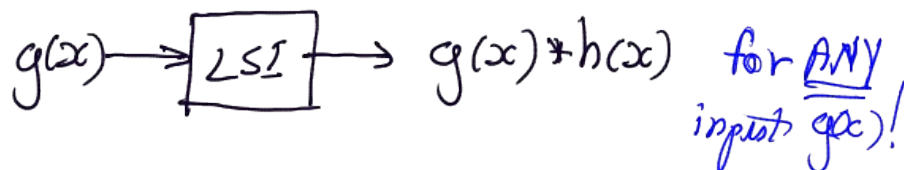
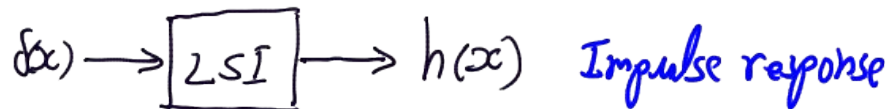
$$T\{q(x-x_0)\} = y(x-x_0)$$

convolution: important math. operator

(8)

$$g(x) * h(x) = \int_{-\infty}^{\infty} g(x-\tau)h(\tau) d\tau$$

why useful??



convolution \Rightarrow blur

Example

$$\Pi(x) * \Pi(x) = \Lambda(x)$$

Show example

FOURIER TRANSFORM (1D)

(9)

complex
val. \rightarrow

$$F(k_x) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi k_x x} dx$$

frequency
spectrum

HOW COME?

inverse xform:

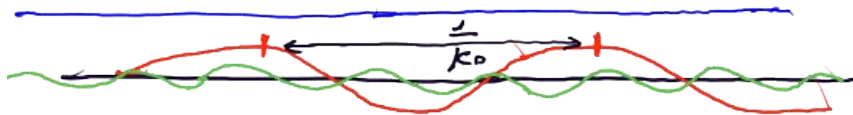
$$f(x) = \int_{-\infty}^{\infty} F(k_x) e^{+i2\pi k_x x} dk_x$$

units: If x in cm $\Rightarrow k$ is $1/cm$
 f in s $\Rightarrow f$ is Hz

Intuition for FT

(10)

$F(k_0)$ IS THE ENERGY OF $f(x)$ at freq. k_0



- (-) Amplitude \Leftrightarrow Height
- (-) PHASE \Leftrightarrow shift

Some FT

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① $\Pi(x) \Rightarrow \text{sinc}(k)$

② $1 \Rightarrow \delta(k)$

$e^{i2\pi k_0 x} \Rightarrow \delta(k - k_0)$

} Periodic \Rightarrow impulsive

③ $\delta(x - x_0) \Rightarrow e^{i2\pi x_0 k}$ impulsive \Rightarrow periodic

④ $\Lambda(x) \Rightarrow \Lambda(k)$

· self transform

impulsive & Periodic \Rightarrow impulsive & periodic

Properties of Fourier Transform

(12)

scaling

$f(x) \Rightarrow F(k)$

(1) $f(ax) \Rightarrow \frac{1}{|a|} F(\frac{k}{a})$

shrink \Leftrightarrow expand
expand \Leftrightarrow shrink

shift

(2) $f(x - x_0) \Rightarrow e^{-i2\pi x_0 k} F(k)$

shift \Leftrightarrow modulation

modulation

(3) $e^{-i2\pi k_0 x} f(x) \Rightarrow F(k - k_0)$

symmetry

④ real + even \Rightarrow real + even

real + odd \Rightarrow imag + odd

$f(x)$ real $\Rightarrow F(-k) = F^*(k)$
what is it good for?

convolution

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$$\textcircled{5} f(x) * g(x) \Rightarrow F(k) \cdot G(k)$$

$$f(x) \cdot g(x) \Rightarrow F(k) * G(k)$$

why useful?

$$\mathcal{F}\{f(x) * g(x)\} = F(k)G(k)$$

↑
hard

↑
easy

why powerful:

(14)

$$g(x) \longrightarrow \boxed{h(x)} \longrightarrow y(x)$$

often, want specific output so
need to design $y(x) \Rightarrow$ "deconvolution"

easy with FT

$$Y(k) = G(k)H(k)$$

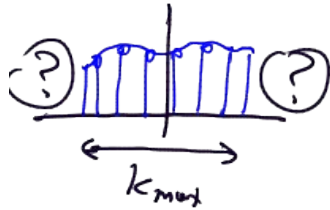
$$G(k) = \frac{Y(k)}{H(k)}$$

← what is
the problem?

EXAMPLE

(15)

suppose $F(k)$ known over limited range



problems: ① support

② sampling → later

how do we get back $f(x)$?

① we know $F(k) \Pi(\frac{k}{k_{max}})$

$FT^{-1} \Rightarrow f(x) * k_{max} \text{sinc}(k_{max}x)$ *blurr!*

② can you deblurr?

Less blurr $k_{max} \uparrow$

