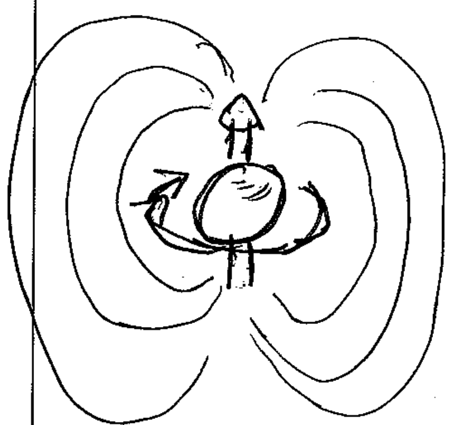
CLASSICAL DESCRIPTION OF MR

(→) ODD # PROTONS / NEUTRONS have Neuclear Spin ANGULAR MOMENTUM



INTRINSIC QM PROPERTY!
MORE CH. 4

INTRINSIC MAGNETIC MOMENT

(→) SPINING MAGNETIC DIPOLES

(→) BIOLOGICAL TISSUE MOSTLY ^1H in H_2O
SOMETIMES ^{31}P, ^{13}C, ^{23}Na exotic

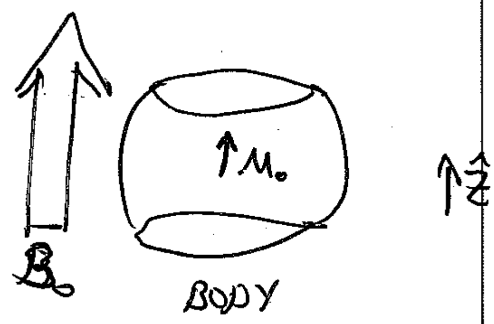
(→) MR IS ABOUT INTERACTIONS WITH THREE FIELDS:

B_0 - MAIN FIELD
⇒ POLARIZATION

B_1 - RF FIELD
⇒ SIGNAL PRODUCTION / RECEPTION

$\vec{G}$ - GRADIEN FIELDS
⇒ SPATIAL ENCODING

B_0 - MAIN FIELD



(→) PRODUCES POLARIZATION OF SAMPLE M_0

(→) SPINS EXHIBIT RESONANCE AT LARMOR FREQUENCY

$$\omega = -\gamma B_0$$

↑ Gyromagnetic Ratio

γ DEPENDS ON NUCLEUS

FOR PROTONS:

$$\frac{\gamma}{2\pi} = 4.257 \text{ KHZ/G}$$

$$\gamma = 2\pi \, 4.257 \text{ kRad/G}$$

WORTH REMEMBERING!

OTHERS: ^{23}Na $\quad \frac{\gamma}{2\pi} = 1.127 \text{ kHz/G}$

^{13}C $\quad \frac{\gamma}{2\pi} = 1.071 \text{ kHz/G}$

^{15}N $\quad \frac{\gamma}{2\pi} = -0.43 \text{ kHz/G}$ Negative!

TYPICAL B_0'S

0.1 T	4.2 MHz	VERY LOW!
0.5 T	21 MHz	LOW (Permanent Resistive)
1 T	42 MHz	MILD (super conducting)
1.5 T	63 MHz	"HIGH" Diagnostic
3 T	126 MHz	"HIGH" fMRI Neuro
4 T	170 MHz	RARE (Decomissioned @ Berkeley)
7/9.4 T		VERY HIGH Reasearch ONLY!

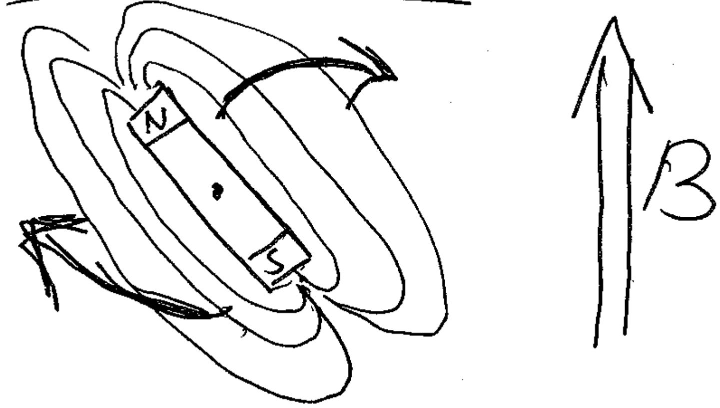
FOR SPECTRAL/SPATIAL LOCALIZATION We REQUIRE HOMOGENEITY

$\Delta B_0 \sim 1 PPM$ $40 cm^3$ FOV

64 Hz @ 1.5T

PRETTY REMARKABLE!

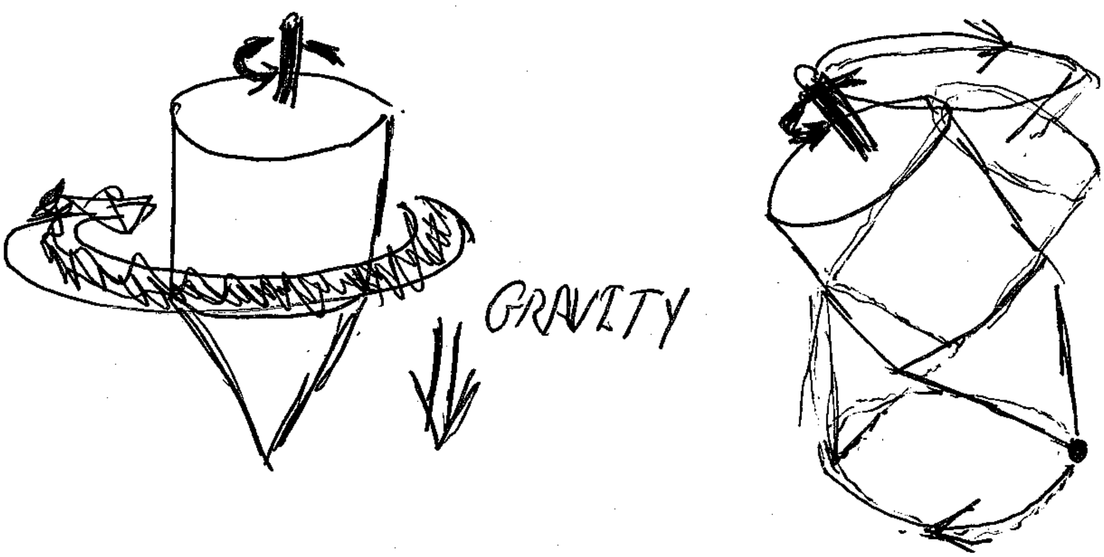
WHY RESONANCE?



→ TORQUE, BUT NO RESONANCE.

→ ~~MISSING~~ BAR MAGNET IS MISSING ANGULAR MOMENTUM.

LIKE A SPINNING TOP



B_1 – RF FIELD

CANT DIRECTLY DETECT M_0

WHY? HUGE FIELD!

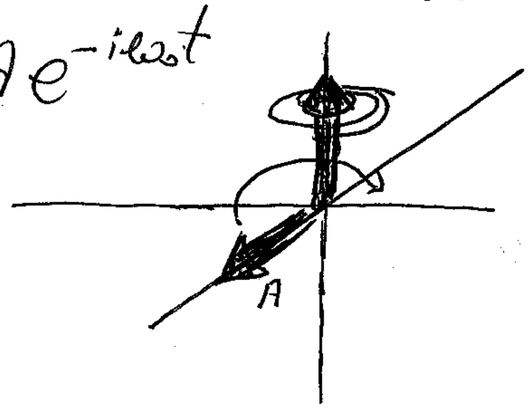
RESONANCE IS THE BIG DEAL...

B_0 IS DC SPIN RESONATE $\Rightarrow$ DETECTION!

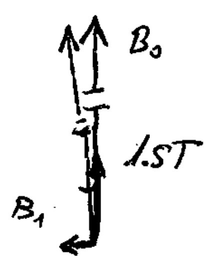
→ SAMPLE RESONATES AT $\omega_0 = \gamma B_0$

→ APPLY ROTATING RF FIELD AT $\omega_0 = \gamma B_0$ IN THE TRANSVERSE PLANE

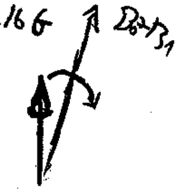
$$B_1(t) = A e^{-i\omega_0 t}$$



AT TIME $t=0$

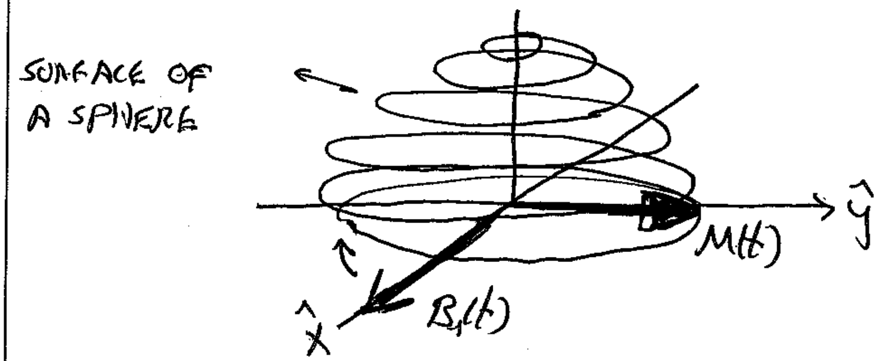


AT TIME $t = \frac{1}{\omega_2}$

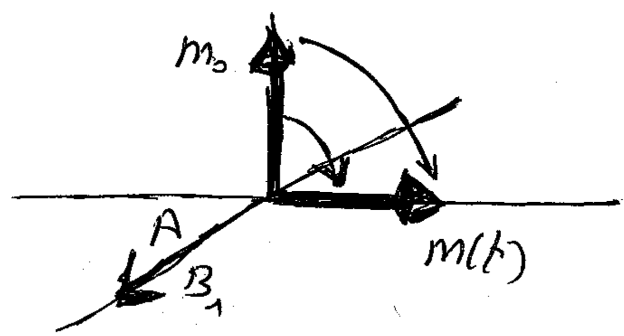


HAS TO BE ON RESONANCE TO DO SOMETHING

ILLUSTRATE BY ROTATIN A STRING



"LAB" FRAME



ROTATING FRAME @ ω_0

→ IN ROTATING FRAME: PRECESSION ABOUT B_1

$$\omega_e = \gamma B_1 = 4.257 \cdot 0.16 = 0.68 \text{ kHz}$$

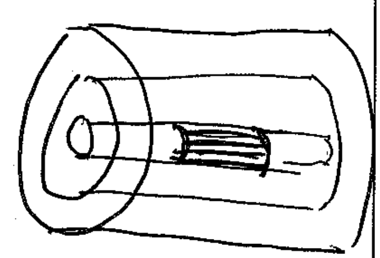
0.367 ms 90° ⇒ 23000 ROTATIONS

TYPICAL B_1'S 0.14–0.35G

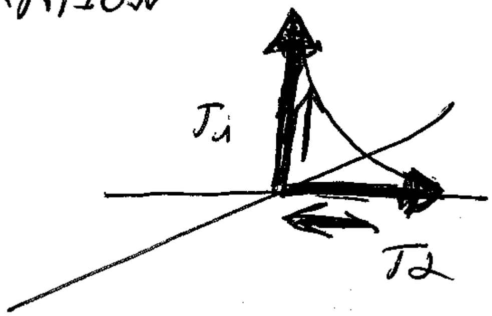
DURATION 1–3 ms LONG TIME AT 64MHz

PEAK POWER 20kW !

HEATING FROM THIS (SAR)



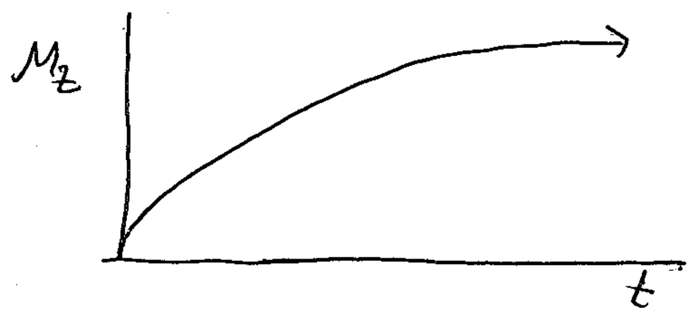
(–) MAGNETIZATION EXHIBITS RELAXATION:

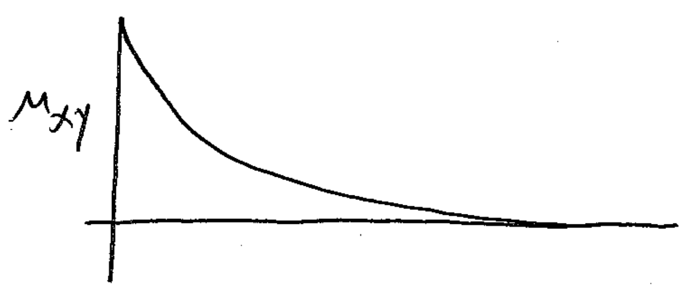


T_1 ~ LONGITUDAL 100 → 2000 ms in Tissue

T_2 ~ TRANSVERSE 10 → 300

MAIN SOURCE OF TISSUE CONTRAST (MORE LATER)

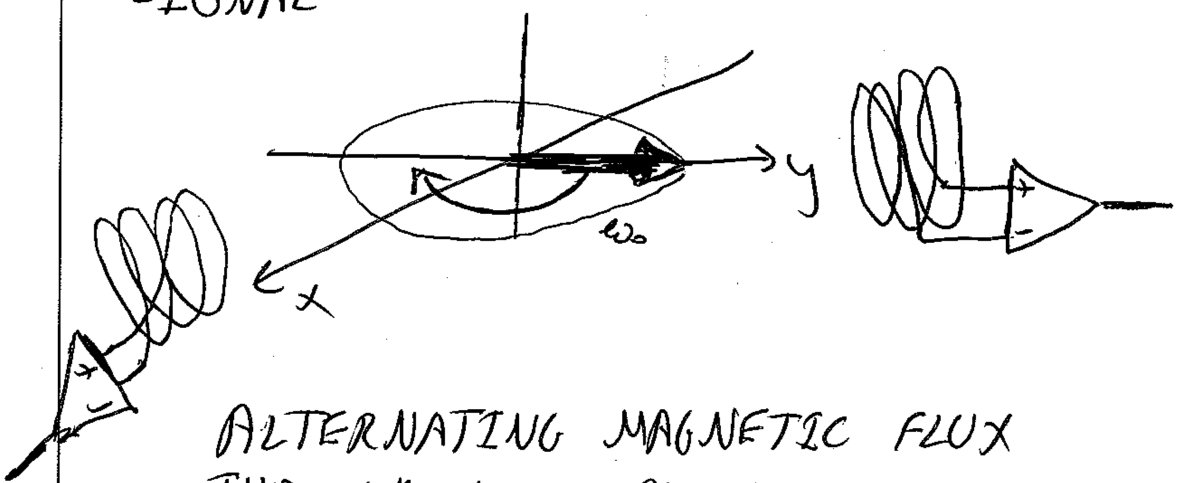




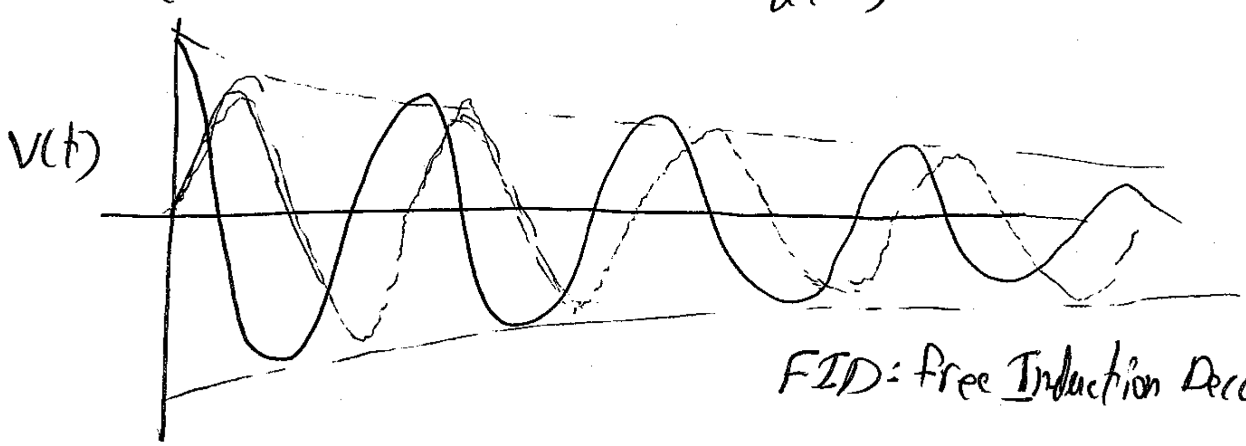
$T_2 < T_1$ ALWAYS

B. RECEPTION

ONCE EXCITED, WE CAN PICK UP SIGNAL



ALTERNATING MAGNETIC FLUX THROUGH LOOP PRODUCES EMF (FARADAYS LAW $\mathcal{E} = \frac{d\Phi_B}{dt}$)



FID: Free Induction Decay

THIS IS WHAT CHEMISTS USE

CHEMICAL SHIFT ~ 1 PPM
WANT B_0 FLATTER.

(-) Body → TINY OSCILATORS
MRI : IMAGE OSCILATORS.

G- GRADIENT FIELDS

SPATIAL LOCALIZATION

B_1 HAS POOR LOCALIZATION λ @ 64MHz ~ 1/2m in Tissue

INSTEAD, CODE POSITION IN FREQUENCY

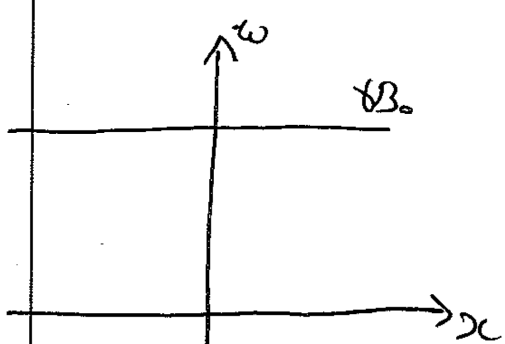
$$\omega(x) = \gamma(B_0 + G_x x)$$

↑ gradient in x

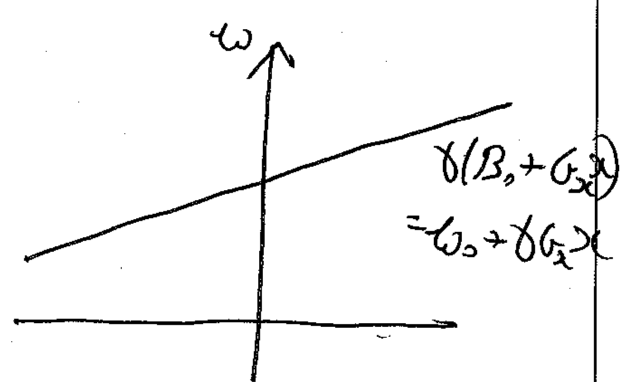
(*) CHANGE IN Z COMPONENT W/ x

$$G_x = \frac{\partial B_z}{\partial x}$$

(B_x, B_y DO NOT MATTER MUCH IN HIGH FIELD)



NO GRADIENT



LINEAR RELATION BETWEEN POSITION & FREQUENCY

TYPICAL #

$G = 1 - 10$ G/CM

$= 10 - 100$ mT/m

$= 4.2 - 42$ KHz/CM

GRADIENT WAVEFORMS IN AUDIO RANGE.

SR = 15 G/cm/ms

SAFETY CONCERN is $\frac{dB}{dt}$

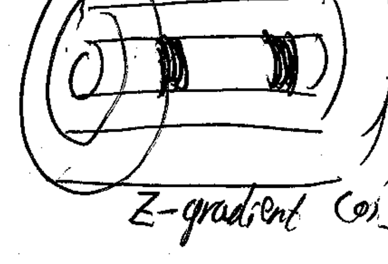
$$\left(\frac{d G_x(t)}{dt} \right) x$$

PERIFERAL NERVE STIMULATION

BIG AMPS: 1200 VOLTS
200 AMPS

GRADIENTS DO NOT SATISFY MAXWELL EQN.

NOT AN ISSUE IN HIGH FIELD



Z-gradient coil

⊛ B_1 - RF FIELD ± 10% ACCURACY

(+) MAGNETIZATION PRECESSES AROUND ROTATING FIELD AND IS TIPPED AWAY
(-) VERY SMALL FIELD (MAX 0.35[G])
(-) RESONANCE IS ESSENTIAL

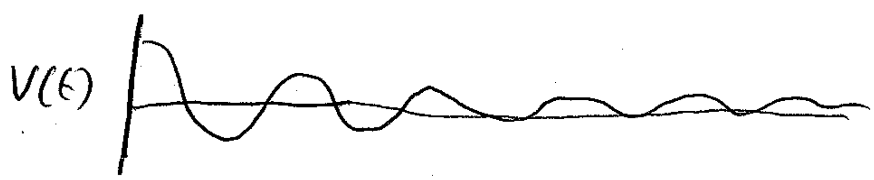
EASY TO DESCRIBE IN ROTATING FRAME.

RELAXATION

(-) LONGITUDAL T_1
(-) TRANSVERSE T_2

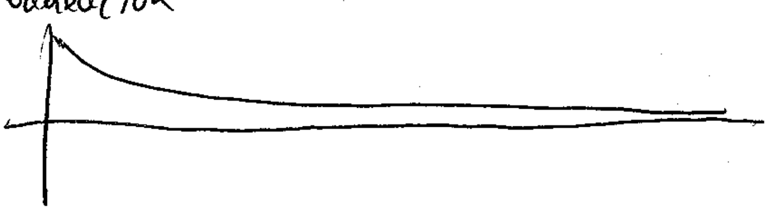
RECEPTION

(+) EMF $\mathcal{E} = \frac{d\phi_B}{dt}$

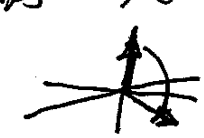


FID

demodulation



(*) ±10% ACCURACY LEADS TO DIFFERENT FLIPS IN SPACE



GRADIENT FIELDS

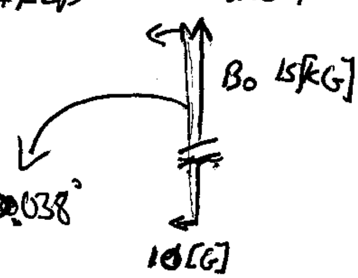
ENCODE POSITION ONTO FREQUENCY

$$G_x = \frac{\partial B_t}{\partial x}$$

(SMALL CONCAMITANT FIELDS B_x, B_y DO NOT CONTRIBUTE TO PRECESSION SINCE $\ll B_0$ AND NOT OSCILATING)

HIGH FIELD: 1.5T

ASSUME $1\left[\frac{G}{cm}\right]$



$\Delta\phi = .038^\circ$

$\|B_0\hat{z} + B_x\hat{x}\| = 15000.\,333 \Rightarrow$ ~~Δf = 0.1...Hz~~

$\Delta f = 14$ Hz

FOR $G_x \sim 1\frac{G}{cm} \Rightarrow 0.42\frac{kHz}{mm} = 420\left[\frac{Hz}{mm}\right]$

SHIFT OF 0.03 [mm]

FOR ACCURACY OF 1mm NEED

$$\frac{\Delta B_0}{B_0} \approx \frac{0.42}{63{,}000\text{ kHz}} \sim 6.5\text{ PPM}$$

USUALLY 1PPM ?

WITH $G = \frac{4G}{cm} \sim 26$ PPM

GRADIENT FIELDS ±20X

ENCODE POSITION ONTO FREQUENCY

$$G_x = \frac{\partial B_z}{\partial x}$$

(SMALL CONCAMITANT FIELDS B_x, B_y DO NOT CONTRIBUTE TO PRECESSION ABOUT B_0 - Not oscillating)

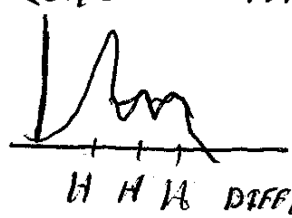
ANECDOTE

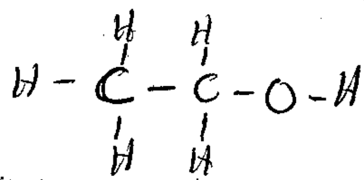
1973 Lauterbur

"Zeugmatography" - clever!

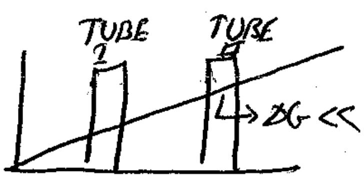
PAUL SOMEWHAT LAZY (AKA EFFICIENT) YOUNG PROFF AT STONYBROOK CHEMISTRY. LOTS OF MEASUREMENTS!

MEASURED

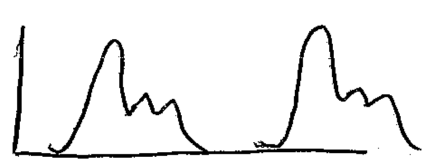




TO SPEED THINGS UP, HE PUT 2 TEST TUBES AND ADDED LINEAR GRADIENT

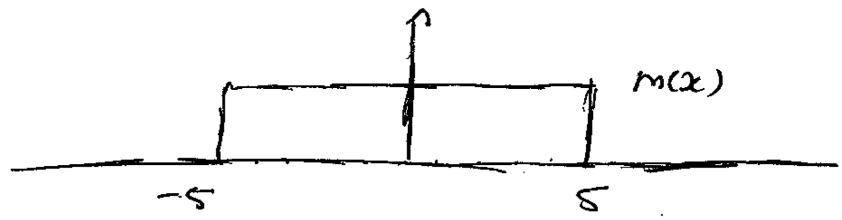


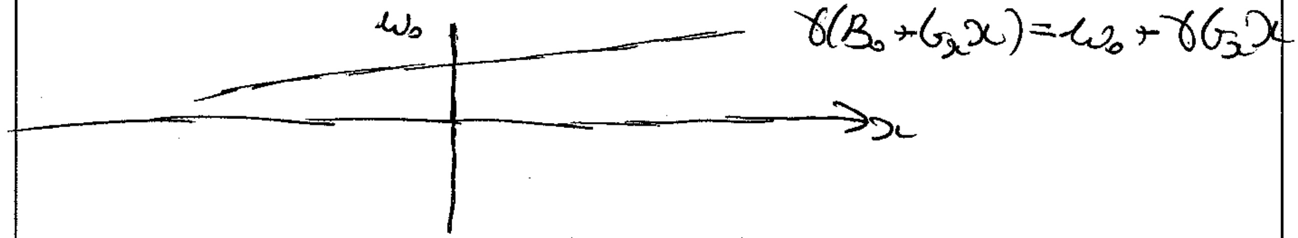
⇒



HE REALIZED THIS WAS IMAGING!

EXAMPLE SQUARE RECT





$$s(t) = \int_x m(x)\, e^{-i(\omega_0 + \gamma G_x x)t}\, dx =$$

$$= e^{-i\omega_0 t} \int_x m(x)\, e^{-i\gamma G_x x t}\, dx =$$

$$= e^{-i\omega_0 t} \int m(x)\, e^{-i2\pi\left(\overset{k_x}{\frac{\gamma}{2\pi} G_x t}\right)x}\, dx =$$

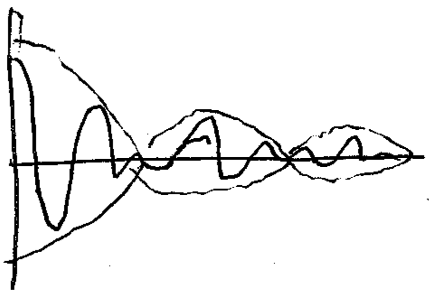
$$= e^{-i\omega_0 t}\, \mathcal{F}\{m(x)\}\Big|_{k_x = \frac{\gamma}{2\pi} G_x t}$$

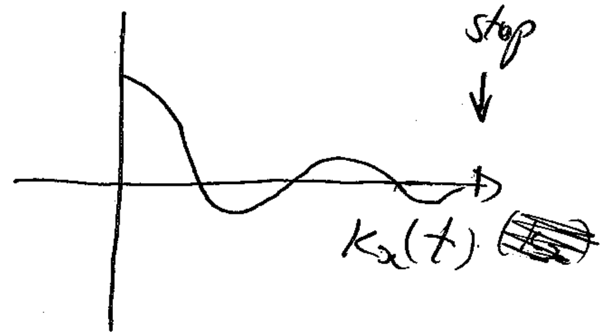
$m(x) = \mathrm{rect}(\cdot)$

$\mathcal{F}(m(x)) \sim \mathrm{SINC}(\cdot)$

$$s(t) = e^{-i\omega_0 t}\, \mathcal{F}\{m(x)\}\Big|_{k_x = \frac{\gamma}{2\pi} G_x t}$$

CARRIER BaseBand





INVERSE FT OF BASEBAND SIGNAL GIVES 1D PROJECTION (ALMOST..)

[illegible] TO SPECIFIC ISSUES: COMPLEX VALUE & DEMODULATION

FOR ANALYSIS IT IS CONVINIENT TO REPRESENT M_x, M_y AS A COMLEX NUMBER

RECIEVE SIGNAL (analysis):

$$S_r(t) = A(t)\, e^{-i(\omega_0 t + \phi(t))}$$

($S_r(t)$ ← complex; $A(t)$ ← Real; $\phi(t)$ ← real)

PHYSICAL SIGNAL:

$$S_p(t) = Re\{S_r(t)\} = A(t)\cos(\omega_0 t + \phi(t)) =$$

(Real ↙)

$$\Rightarrow \underbrace{A(t)\cos(\phi(t))}_{I(t)\ \text{in phase}} \cos\omega_0 t + \underbrace{\left[-A(t)\sin\phi(t)\right]}_{Q(t)\ \text{quadrature}} \sin\omega_0 t$$

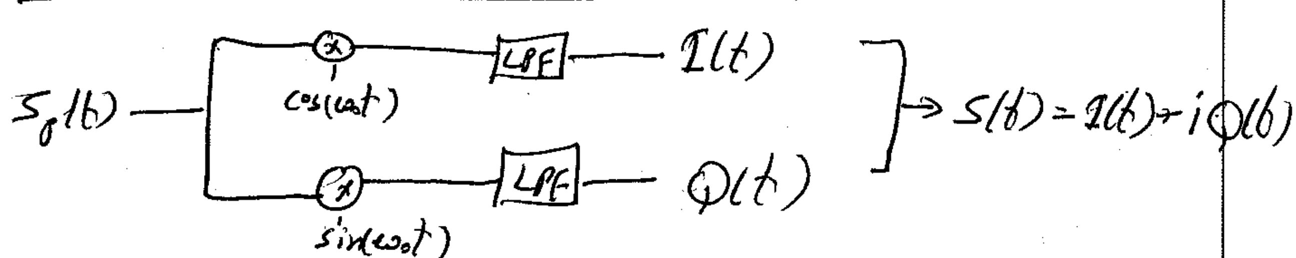
(↑ TRIG)

BaseBand SIGNAL (analysis):

$$S(t) = S_r(t)e^{+i\omega_0 t} = A(t)e^{-i\phi(t)} =$$

$$= I(t) + iQ(t)$$

QUADRATURE PHASE-SENSITIVE DETECTION:



$Re\{S(t)\} \rightarrow M_x$ in Rot Frame

SO FAR

(1) PLACE SAMPLE IN B_0
- M_z DEVELOPS ~ $5T_1$

(2) EXCITE USING $B_1(t)$ TIP AWAY FROM $\hat{z}$

(3) INSTANTANIOUS PRECESSION OF M_{xy}
PICK UP INDUCED EMF IN RF COIL

(4) ENCODE POSITION IN FREGENCY USING GRADIENTS

⇒ 1D PROJECTION.

~~(5) HOW DO WE GET AN IMAGE?~~

~~SEVERAL KEY COMPONENTS:~~

~~(1) SELECTIVE EXCITATION (dimension reduction)~~

~~(2) SPATIAL ENCODING~~

(3) LIMITATIONS!

GRADIENT STRENGTH + DURATION ⟶ RESOLUTION
↗ limited!

SIGNAL DECAY (T_2), FIELD INHOMOGENEITY (T_2^*)
DIFFUSSION

TYPICAL RES: < 1 mm $\approx 10\mu m$ IN SMALL ANIMAL SCANNERS