CLASSICAL DESCRIPTION OF MR

- Odd # Protons/Neutrons have Nuclear Spin
  - Intrinsic Spin Property
  - Intrinsic Magnetic Moment

- Spinning Magnetic Dipoles

- Biological tissue mostly H in H₂O
  - Sometimes ³¹P, ¹³C, ⁵²Na (exotic)

- MR is about interactions with three fields:
  - \( B₀ \) - MAIN FIELD
    - Produces Polarization
  - \( B₁ \) - RF FIELD
    - Signal Production/Reception
  - \( G \) - Gradient Fields
    - Spatial Encoding

- Spins exhibit resonance at Larmor Frequency
  \( \omega = γ \cdot B₀ \)
  - Gyromagnetic Ratio

- \( γ \) depends on nucleus for PROTONS:
  \( \frac{γ}{MHz} = 4.257 \cdot kHz/γ \)

- Others:
  - \( ^{31}P \) \( \frac{γ}{MHz} = 1.197 \cdot kHz/γ \)
  - \( ^{13}C \) \( \frac{γ}{MHz} = 1.024 \cdot kHz/γ \)
  - \( ^{15}N \) \( \frac{γ}{MHz} = -0.83 \cdot kHz/γ \) Negative!
TYPICAL $B_0$'S

- 0.1 T  4.2 MHz  VERY LOW!
- 0.5 T  2.1 MHz  BASE (Permanent Magnetic)
- 1 T   4.2 MHz  MILD (Superconducting)
- 1.5 T  6.3 MHz  "HIGH" Diagnostic
- 2 T   12 MHz  "HIGH" MRI Quality
- 4 T   120 MHz  RARE (Reconceived/Prototype)
- 7/9.5 T

VEERY HIGH RESEARCH ONLY!

FOR SPECTRAL/SPATIAL LOCALIZATION
WE REQUIRE HOMOGENEITY

$B_0$ ~ 4 ppm  40 cm FOV
64 Hz @ 1.5 T
PRETTY REMARKABLE!!

WHY RESONANCE?

B

$\Rightarrow$ TORQUE, BUT NO RESONANCE.

$\Rightarrow$ MISSING BAR MAGNET IS MISSING ANGULAR MOMENTUM.

LIKE A SPINNING TOP

GRAVITY
$B_x$ - RF FIELD

CANT DIRECTLY DETECT $M_0$.

WHY? HUGE FIELD!

RESONANCE IS THE BIG DEAL...

$B_0$ IS DC SPIN RESONATE $\Rightarrow$ DETECTION!

- SAMPLE RESONATES AT $\omega = \frac{1}{2} B_0$.

- APPLY ROTATING RF FIELD AT $\omega = B_0$ IN THE TRANSVERSE PLANE.

$B_x(t) = A e^{-i\omega t}$

- AT TIME $t = 0$

- AT TIME $t = \tau$.

HAS TO BE ON RESONANCE TO DO SOMETHING

ILLUSTRATE BY ROTATING A SPHERE

"LAB" FRAME

SURFACE OF A SPHERE

"LAB" FRAME

"LAB" FRAME

"LAB" FRAME

"LAB" FRAME

ROTATING FRAME @ $\omega$.

- IN ROTATING FRAME: PRECESSION ABOUT $B_0$.

$\omega = \gamma B_0 = 4.257 \cdot 0.16 = 0.68 \text{ kHz}$

$0.367 \text{ ms} 90^\circ \Rightarrow \text{30 cm NMR}$

TYPICAL $B_0$'S 0.14 - 0.35 G.

DURATION 1 - 3 ms, LONG TIME AT 64Hz.

PEAK POWER 0.5 kW.

HEATING FROM THIS (SAR)
Magnitization Exhibits Relaxation:

\[ T_1 \sim \text{Longitudinal} \rightarrow \text{2000 ms in Tissue} \]

\[ T_2 \sim \text{Transverse} \rightarrow \text{300} \]

Main source of tissue contrast (more later)

\[ M_0 \]

\[ M_0(t) \]

\[ M_0(t) = k \]

\[ T_2 < T_1 \quad \text{Always} \]

**B_0 Reception**

Once excited, we can pick up signal

Alternating magnetic flux through loop produces EMF

(Faraday's Law \[ \mathbf{E} = \mathbf{V} \times \mathbf{B} \])

FID: Free Induction Decay

This is what chemists use

Chemical shift \( \approx \text{1 ppm} \)

Want \( B_0 \) flatter.

**(-) Body** \( \rightarrow \text{Tiny Oscillators} \)

MRE: Image oscillators.
**G - GRADIENT FIELDS**

**SPATIAL LOCALIZATION**

$B_0$ has poor localization $\propto \text{length}^{-1}$.

Instead, code position in frequency

$$\omega(x) = 6 (B_0 + G_x x)$$

$\omega^2$ = gradient in $x$

- Change in $z$ component with $x$

$$G_z = \frac{\partial B_z}{\partial x}$$

($B_x, B_0$ do not matter much in high field)

**Typical**

$$G = 1 - 10 \text{ G/cm}$$

$$= 10 - 100 \text{ mT/m}$$

$$= 42 - 42 \text{ kHz/cm}$$

**Gradient waveforms in audio range**

$$SR = 15 \text{ G/cm/ms}$$

**Safety concern** is

$$\frac{\partial B}{\partial t}$$

$$\left( \frac{\partial B_z(B)}{\partial t} \right) x$$

**Peripheral nerve stimulation**

Big amps: 1000 volts 300 amps

Gradients do not satisfy Maxwell eqn.

Not an issue in high field

Z-graded coil
$B_a$ - RF FIELD ± 10 V. Accuracy

- Magnetization processes around rotating field and is tipped away
- Very small field $\theta (\text{max} \ 0.3 \text{rad})$
- Resonance is essential
  - Easy to describe in rotating frame

Relaxation
- Longitudinal $T_1$
- Transverse $T_2$

Reception
- EMF $E = \frac{\Delta \phi}{\Delta t}$

FID

- 10 V. Accuracy leads to different flips in space $\Delta$

Gradient Fields

Encode position onto frequency

$G_x = \frac{\partial \phi}{\partial x}$

Small constant fields $B_x, B_y$ do not contribute to precession since $< B_0$ and $\Delta f$

Accumulating

High field: 1.5T

Assume $J_{CM}$

$11.28 \Delta \phi = 15000 \Delta t \Rightarrow 1.033 \Rightarrow 14 \text{ Hz}$

For $G_x \approx 1 \text{ G}_\text{cm} \Rightarrow 0.42 \text{ kHz} = 0.01 \text{ cm}$

Shift of 0.03 (ppm)

For accuracy of 1 ppm need

$\frac{\Delta B_0}{B_0} \approx 0.42 \text{ kHz} \approx 6.5 \text{ ppm}$ Usually 1 ppm?

With $G = \frac{4 \Delta}{\text{G}_\text{cm}} \approx 26 \text{ ppm}$
GRADIENT FIELDS

Encode position onto frequency

\[ G_x = \frac{\partial \phi}{\partial x} \]

Small constant fields Rx, Ry do not contribute to precision about B0 - ML oscillation

1973 Lauderber

"Zeugmatography" - clever!

Paul somewhat lazy (aka efficient) young prok at Stonybrook chemistry. Lots of measurements!

Measured

H H H different e situations

To speed things up, he put 2 test tubes and added linear gradient

He realized this was imaging!

EXAMPLE SQUARE RECT

\[
F(\max) \approx 52\text{vol}() 
\]

\[
\mathcal{F}(m(x)) = \int_{-\infty}^{\infty} e^{-i\omega x} m(x) dx
\]

\[
 \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega x} m(x) dx
\]

\[
 m(x) = \text{rect}(\cdot) 
\]

\[
 F(\max) = 52\text{vol}() 
\]
Inverse FT of baseband signal gives in-phase projection (almost).

In-phase projection:

\[ s(t) = e^{j\omega_0 t} \mathcal{F} \{ m(t) \} \]

\[ \mathcal{S}(\omega) \]

\[ k_0(t) \]

\[ \mathcal{K}(\omega) \]

Physical signal:

\[ S_R(\omega) = \text{Re} \left\{ S_r(\omega) \right\} = A(\omega) \cos(\omega t + \phi(\omega)) = \text{Re} \left[ A(\omega) \cos(\omega t) \cos(\Phi) + \sin(\Phi) \sin(\omega t) \right] \]

Baseband signal (analysis):

\[ s(t) = S_r(t) e^{j\omega t} = A(t) e^{j\phi(t)} = i(t) + jQ(t) \]

Quadrature phase-sensitive detection:

\[ S_0(t) = \begin{bmatrix} \cos(\omega t) & I(t) \\ \sin(\omega t) & Q(t) \end{bmatrix} \]

\[ S(t) = \Re \left\{ S_0(t) \right\} = M_0 \text{ in Ref Frame} \]
SO FAR

1. PLACE SAMPLE IN $B_0$
   - $M_z$ DEVELOPS $\approx ST_1$

2. EXCITE USING $B_1(t)$ TIP AWAY FROM $t$

3. INSTANTANEOUS PRECESSION OF $M_y$
   - PICK UP INDUCED EMF IN RF COIL

4. ENCODE POSITION IN FREQUENCY USING GRADIENTS
   $\Rightarrow 2D$ PROJECTION

5. HOW DO WE GET AN IMAGE?
   - SEVERAL KEY COMPONENTS:
     - SELECTIVE EXCITATION (DIMENSION REDUCTION)
     - STRUCTURAL ENCODING

6. LIMITATIONS!
   - GRADIENT STRENGTH + DURATION $\rightarrow$ RESOLUTION
     $\Rightarrow$ LIMITED

   - SIGNAL DECAY ($T_2$), FIELD INHOMOGENEITY ($\delta$)
   - DIFFUSION

   TYPICAL RES: $<1\,\mu m$ for small animal scanners