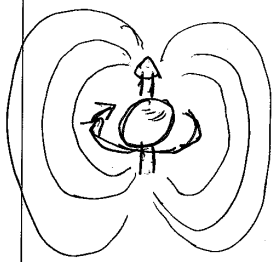
CLASSICAL DESCRIPTION OF MR

→ ODD #PROTONS/NEUTRONS have Nuclear spin ANGULAR MOMENTUM

INTRINSIC QM PROPERTY!

MORE CH. 4



INTRINSIC MAGNETIC MOMENT

→ SPINING MAGNETIC DIPOLES

→ BIOLOGICAL TISSUE MOSTLY ^1H in H_2O
SOMETIMES ^{31}P, ^{13}C, ^{23}Na exotic

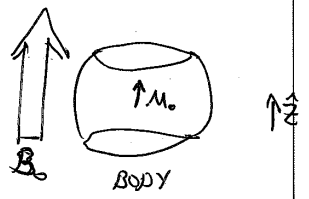
→ MR IS ABOUT INTERACTIONS WITH THREE FIELDS:

B_0 - MAIN FIELD
⇒ POLARIZATION

B_1 - RF FIELD
⇒ SIGNAL PRODUCTION/RECEPTION

$\vec{G}$ - GRADIEN FIELDS
⇒ SPATIAL ENCODING

B_0 - MAIN FIELD



→ PRODUCES POLARIZATION OF SAMPLE M_0

→ SPINS EXHIBIT RESONANCE AT LARMOR FREQUENCY

$$\omega = -\gamma B_0$$

↑ Gyromagnetic Ratio

γ DEPENDS ON NUCLEUS

FOR PROTONS!

$$\frac{\gamma}{2\pi} = 4.257 \text{ MHz/G}$$

$$\gamma = 2\pi\, 4.257 \text{ krad/G}$$

WORTH REMEMBERING!

OTHERS!

^{23}Na	$\frac{\gamma}{2\pi} = 1.127$ kHz/G	
^{13}C	$\frac{\gamma}{2\pi} = 1.071$ kHz/G	
^{15}N	$\frac{\gamma}{2\pi} = -0.43$ kHz/G	Negative!

TYPICAL B_0's

0.1 T	4.2 MHz	VERY LOW!
0.5 T	21 MHz	LOW (Permanent Resistive)
1 T	42 MHz	MILD (superconducting)
1.5 T	63 MHz	"HIGH" Diagnostic
3 T	126 MHz	"HIGH" fMRI Neuro
4 T	170 MHz	RARE (Decommissioned @ Berkeley)
7/9.4T		VERY HIGH Research ONLY!

FOR SPECTRAL/SPATIAL LOCALIZATION

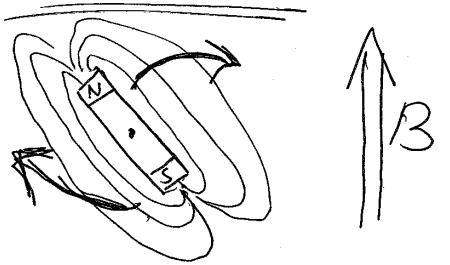
We REQUIRE HOMOGENEITY

$\Delta B_0 \sim 1PPM$ 40 cm^3 FOV

64 Hz @ 1.5T

PRETTY REMARKABLE!

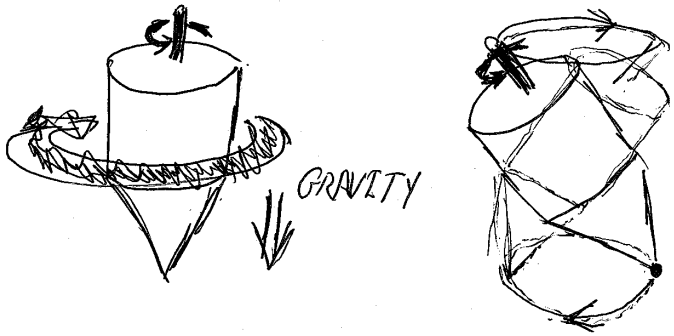
WHY RESONANCE?



→ TORQUE, BUT NO RESONANCE.

→ ~~MISSING~~ BAR MAGNET IS MISSING ANGULAR MOMENTUM.

LIKE A SPINNING TOP!



B_1 - RF FIELD

CAN'T DIRECTLY DETECT M_0.

WHY? HUGE FIELD!

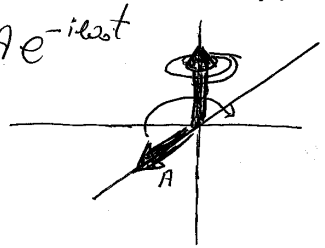
RESONANCE IS THE BIG DEAL...

B_0 IS DC SPIN RESONATES $\Rightarrow$ DETECTION!

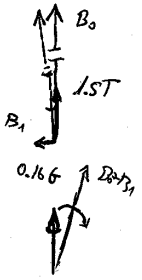
↪ SAMPLE RESONATES AT $\omega_0 = \gamma B_0$

↪ APPLY ROTATING RF FIELD AT $\omega_0 = \gamma B_0$ IN THE TRANSVERSE PLANE

$B_1(t) = A e^{-i\omega_0 t}$



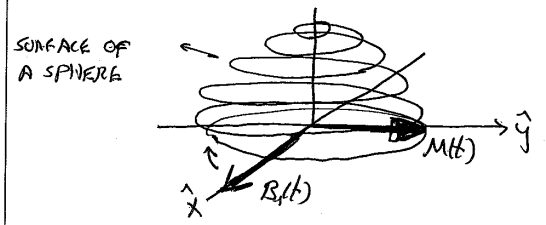
AT TIME $t=0$

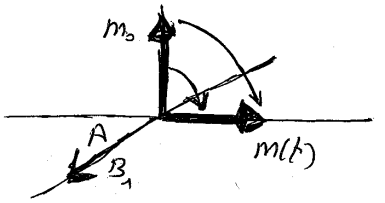


AT TIME $t = \frac{1}{\omega_0/2}$

HAS TO BE ON RESONANCE TO DO SOMETHING

ILLUSTRATE BY ROTATING A STRING





ROTATING FRAME @ ω_0

↪ IN ROTATING FRAME: PRECESSION ABOUT B_1

$\omega_e = \gamma B_1 = 4.257 \cdot 0.16 = 0.68$ kHz

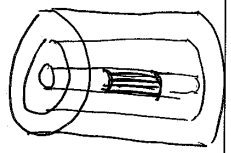
0.367 ms 90° $\Rightarrow$ 23000 ROTATIONS

TYPICAL B_1'S 0.14–0.35G

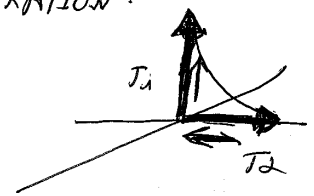
DURATION 1–3 ms LONG TIME AT 64MHz

PEAK POWER 20kW!

HEATING FROM THIS (SAR)



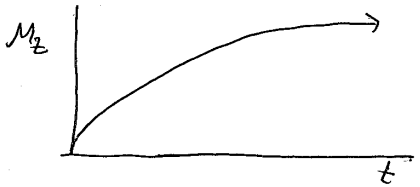
(-) MAGNETIZATION EXHIBITS RELAXATION:

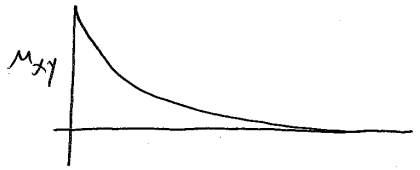


T_1 ~ LONGITUDAL 100 → 2000 ms In Tissue

T_2 ~ TRANSVERSE 10 → 300

MAIN SOURCE OF TISSUE CONTRAST (MORE LATER)

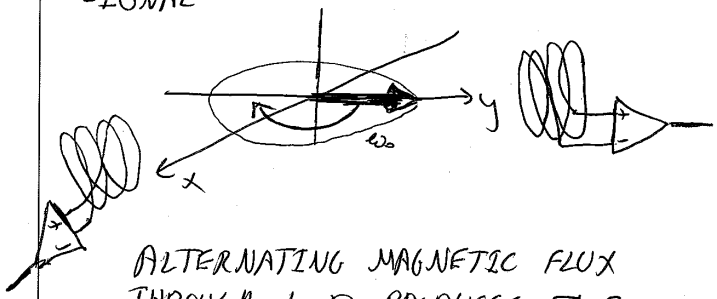




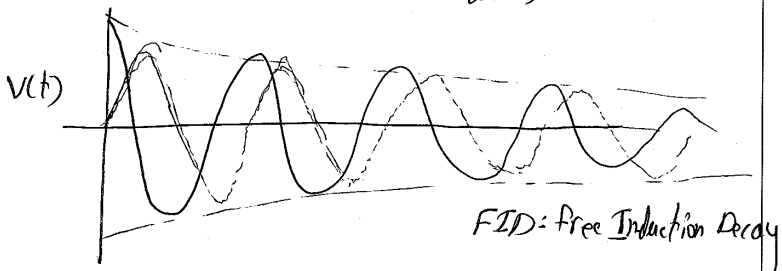
$T_2 < T_1$ ALWAYS

B_1 RECEPTION

ONCE EXCITED, WE CAN PICK UP SIGNAL



ALTERNATING MAGNETIC FLUX THROUGH A LOOP PRODUCES EMF (FARADAYS LAW $\mathcal{E} = \frac{d\Phi_B}{dt}$)



THIS IS WHAT CHEMISTS USE

CHEMICAL SHIFT ~1 PPM
WANT B_0 FLATTER.

(-) Body → TINY OSCILATORS
MRI: IMAGE OSCILATORS.

G - GRADIENT FIELDS

SPATIAL LOCALIZATION

B_1 HAS POOR LOCALIZATION $\lambda @ 64\,MHz \sim \frac{1}{3}m$ in Tissue

INSTEAD, CODE POSITION IN FREQUENCY

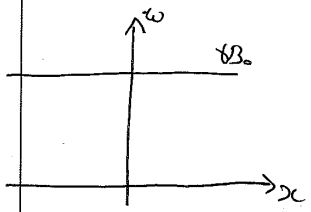
$$\omega(x) = \gamma(B_0 + G_x x)$$

↑ gradient in x

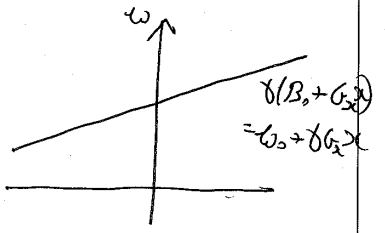
(*) CHANGE IN $\underline{Z}$ COMPONENT W/ x

$$G_x = \frac{\partial B_z}{\partial x}$$

(B_x, B_y DO NOT MATTER MUCH IN HIGH FIELD)



NO GRADIENT



LINEAR RELATION BETWEEN POSITION & FREQUENCY

TYPICAL #

$G = 1-10$ G/cm

$= 10-100$ mT/m

$= 4.2-42$ kHz/cm

GRADIENT WAVEFORMS IN AUDIO RANGE

SR = 15 G/cm/ms

SAFETY CONCERN is $\frac{dB}{dt}$

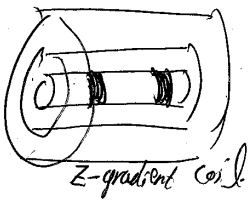
$$\left(\frac{d G_x(t)}{dt}\right) x$$

PERIFERAL NERVE STIMULATION

BIG AMPS: 1200 VOLTS
200 AMPS

GRADIENTS DO NOT SATISFY MAXWELL EQN.

NOT AN ISSUE IN HIGH FIELD



Z-gradient coil

B_1 – RF FIELD ± 10% ACCURACY

- MAGNETIZATION PRECESSES AROUND ROTATING FIELD AND IS TIPPED AWAY
- VERY SMALL FIELD (MAX 0.25[G])
- RESONANCE IS ESSENTIAL

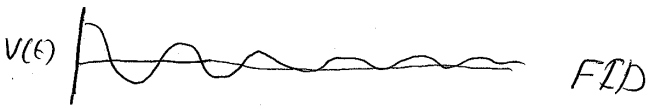
EASY TO DESCRIBE IN ROTATING FRAME.

RELAXATION

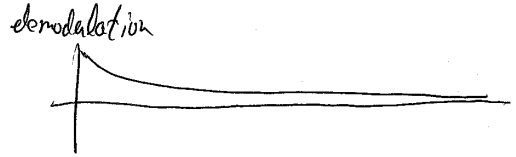
- LONGITUDAL T_1
- TRANSVERSE T_2

RECEPTION

- EMF $\mathcal{E} = \frac{d\Phi_B}{dt}$



FID



- ±10% ACCURACY LEADS TO DIFFERENT FLIPS IN SPACE

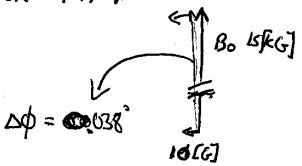
GRADIENT FIELDS

ENCODE POSITION ONTO FREQUENCY

$$G_x = \frac{\partial B_z}{\partial x}$$

(SMALL CONCAMITANT FIELDS B_x, B_y DO NOT CONTRIBUTE TO PRECESSION SINCE $\ll B_0$ AND NOT OSCILATING)

HIGH FIELD: 1.5T ASSUME $1\left[\frac{G}{cm}\right]$



$\|B_0\hat{z} + B_x\hat{x}\| = 15000.0333 \Rightarrow$ ~~$\Delta f = 0.1419$ Hz~~

$\Delta f = 14$ Hz

FOR $G_x \sim 1\frac{G}{cm} \Rightarrow 0.42\frac{kHz}{mm} = 420\left[\frac{Hz}{mm}\right]$

SHIFT OF 0.03[mm]

FOR ACCURACY OF 1mm NEED

$$\frac{\Delta B_0}{B_0} \approx \frac{0.42}{63{,}000\ kHz} \sim 6.5\ PPM$$

USUALLY 1PPM ?

WITH $G = \frac{4G}{cm} \sim 26\ PPM$

GRADIENT FIELDS

$\pm \omega_0 x$

ENCODE POSITION ONTO FREQUENCY

$$G_x = \frac{\partial B_z}{\partial x}$$

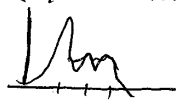
(SMALL CONCOMITANT FIELDS B_x, B_y DO NOT CONTRIBUTE TO PRECESSION ABOUT B_0 - Not OSCILLATING!)

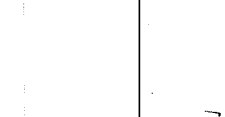
ANECDOTE

1973 Lauterbur

"Zeugmatography" - clever!

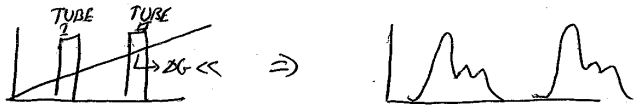
PAUL SOMEWHAT LAZY (AKA EFFICIENT) YOUNG PROFF AT STONYBROOK CHEMISTRY. LOTS OF MEASUREMENTS!

MEASURED 



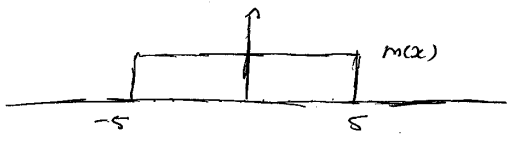
H H H DIFFERENT e^- SHIELDING

TO SPEED THINGS UP, HE PUT 2 TEST TUBES AND ADDED LINEAR GRADIENT



WE REALIZED THIS WAS IMAGING!

EXAMPLE SQUARE RECT



$$S(t) = \int_x m(x)\, e^{-i(\omega_0 + \gamma G_x x)t}\, dx =$$

$$= e^{-i\omega_0 t} \int_x m(x)\, e^{-i\gamma G_x x t}\, dx =$$

$$= e^{-i\omega_0 t} \int m(x)\, e^{-i2\pi \left(\frac{\gamma}{2\pi} G_x t\right) x}\, dx =$$

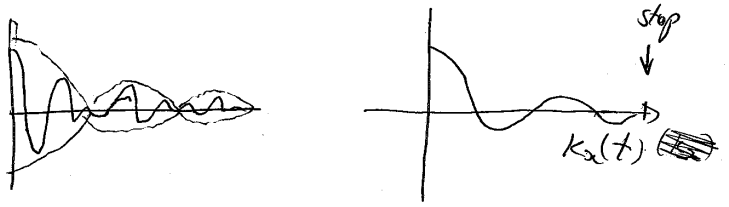
$$= e^{-i\omega_0 t}\, \mathcal{F}\{m(x)\}\Big|_{k_x = \frac{\gamma}{2\pi} G_x t}$$

$m(x) = \text{rect}(\cdot)$

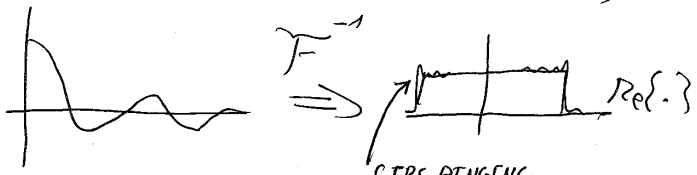
$\mathcal{F}(m(x)) \sim \text{SINC}(\cdot)$

$$s(t) = e^{-i\omega_0 t}\, \mathcal{F}\{m(x)\}\Big|_{k_x = \frac{\gamma}{2\pi} G_x t}$$

CARRIER BaseBand



INVERSE FT OF BASEBAND SIGNAL GIVES 1D PROJECTION (ALMOST.)



BEFORE TO SPECIFIC ISSUES: COMPLEX VALUE & DEMODULATION

FOR ANALYSIS IT IS CONVINIENT TO REPRESENT M_x, M_y AS A COMLEX NUMBER

RECIEVE SIGNAL (analysis):

$$S_r(t) = A(t)\, e^{-i(\omega_0 t + \phi(t))}$$

($S_r(t)$: complex; $A(t)$: Real; $\phi(t)$: real)

PHYSICAL SIGNAL:

$$S_p(t) = \mathrm{Re}\left\{ S_r(t) \right\} = A(t)\cos(\omega_0 t + \phi(t)) =$$

Real

$$\Rightarrow \underbrace{A(t)\cos(\phi(t))}_{I(t)\text{ in phase}} \cos\omega_0 t + \underbrace{\left[-A(t)\sin\phi(t)\right]}_{Q(t)\text{ quadrature}} \sin\omega_0 t$$

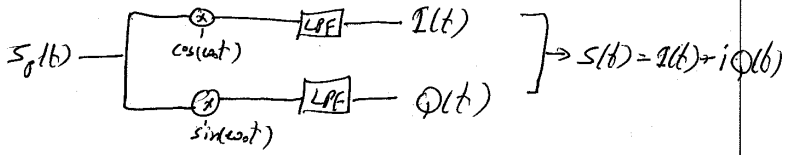
(Time)

BaseBand SIGNAL (analysis):

$$s(t) = S_r(t)\, e^{+i\omega_0 t} = A(t)\, e^{-i\phi(t)} =$$

$$= I(t) + iQ(t)$$

QUADRATURE PHASE-SENSITIVE DETECTION:



$\mathrm{Re}\{s(t)\} \rightarrow M_x$ in Rot Frame

SO FAR

(1) PLACE SAMPLE IN B_0
- M_z DEVELOPS ~ $5T_1$

(2) EXCITE USING $B_1(t)$ TIP AWAY FROM $\hat{z}$

(3) INSTANTANIOUS PRECESSION OF M_{xy}
PICK UP INDUCED EMF IN RF COIL

(4) ENCODE POSITION IN FREQENCY USING GRADIENTS

⇒ 1D PROJECTION.

(5) HOW DO WE GET AN IMAGE?

SEVERAL KEY COMPONENTS:

(1) SELECTIVE EXCITATION (dimension reduction)

(2) SPATIAL ENCODING

(3) LIMITATIONS:

GRADIENT STRENGTH + DURATION ⟶ RESOLUTION
↑
limited!

SIGNAL DECAY (T_2), FIELD INHOMOGENEITY (T_2^*)
DIFFUSION

TYPICAL RES: <1mm ~30µM IN SMALL ANIMAL SCANNERS