LECTURE 05  1/31/12

CLASSICAL DESCRIPTION OF MR

- Odd # protons/neutrons have nuclear spin angular momentum

INTRINSIC ANGULAR MOMENTUM

INTRINSIC MAGNETIC MOMENT

- Spinning magnetic dipoles

- Biological tissue mostly \(^1\text{H}\) in H\(_2\)O
  - Sometimes \(^2\text{H}, ^3\text{C}, ^15\text{N}\) and \(^1\text{H}\) and \(^15\text{N}\)

- MR IS MOST INTERACTIONS WITH THREE FIELDS:

  - \(B_0\) - MAIN FIELD
    - \(\mathcal{H}\) - POLARIZATION
  - \(B_1\) - RF FIELD
    - SIGNAL PRODUCTION/RECEPTION
  - \(B\) - GRADIENT FIELDS
    - SPATIAL ENCODING

\(B_0\) - MAIN FIELD

- Produces polarization of sample \(M_s\)

- Spins exhibit resonance at Larmor frequency

\[ \omega = \gamma B_0 \]

\(\gamma\) - Gyromagnetic ratio

\(\nu\) depends on nucleus

- For protons:

\[ \frac{1}{\nu} = 4.857 \text{ MHz/}\text{G} \]

\[ \gamma = 2\pi \times 4.257 \text{ kHz/}\text{G} \]

WORTH REMEMBERING

OTHERS:

- \(^{13}\text{C}\) \[ \frac{1}{\nu} = 1.074 \text{ kHz/}\text{G} \]
- \(^{15}\text{N}\) \[ \frac{1}{\nu} = 0.473 \text{ kHz/}\text{G} \]

TYPICAL \(B_0\)'s

- 0.1 T \( \approx 4.2\) MHz - VERY LOW!
- 0.5 T \( \approx 21\) MHz - LOW (Proton Magnetic Resonance)
- 1 T \( \approx 41\) MHz - MILD (Carbon-13 Nuclear Magnetic Resonance)
- 1.5 T \( \approx 63\) MHz - "HIGH" MAGNETIC
- 3 T \( \approx 186\) MHz - "HIGH" NMR
- 4 T \( \approx 255\) MHz - PRECISION NMR
- 7/9.4 T VERY HIGH RESEARCH ONLY!

FOR SPECTRAL/SPATIAL LOCALIZATION WE REQUIRE HOMOGENEITY

\[ \Delta B_0 \leq \pm 1 \text{ ppm} \] 40 cm for

- 64 kHz @ 1.5 T

PRETTY REMARKABLE?

WHY RESONANCE?

- Torque, but no resonance.

- Bar magnet is missing angular momentum.

LIKE A SPINNING TOP

GRANITY
\[ B_1 = \text{RF FIELD} \]

*Can't directly detect M.*

Why? Huge field?

Resonance is the big deal...

\[ B_0 \text{ is DC spin resonates} \Rightarrow \text{detection} \]

\[ \Rightarrow \text{Sample resonates at } \omega = \gamma B_0 \]

\[ \Rightarrow \text{Apply rotating RF field at } \omega = 2\omega_0 \text{ in the transverse plane} \]

\[ B_1(t) = Ae^{-\gamma t} \]

\[ \text{At time } t = \frac{1}{\omega} \]

\[ \text{At time } t = \frac{T}{2} \]

\[ \text{Has to be on resonance to do something} \]

\[ \text{Illustrate by rotating a string} \]

\[ \text{Frame of reference} \]

\[ \text{Rotating frame} \]

\[ \text{In rotating frame: precession around } B_1 \]

\[ \omega = \gamma B_1 = 4.257 \times 10^6 \text{ rad/s} \]

\[ T_1 = 0.367 \text{ ms} \]

Typical \( B_1 \)'s 0.14-0.55 T

Duration 1-3 ms

Low time at base

Peak power 2 kW

HEATING FROM THIS (SAR)

\[ T_1 < T_2 \]

\[ \text{MAGNETIZATION EXHIBITS RELAXATION:} \]

\[ T_2 \text{ - Longitudinal } \lambda_0 \rightarrow 3000 \text{ ms in tissue} \]

\[ T_1 \text{ - Transverse } \lambda_0 \rightarrow 300 \]

Main source of tissue contrast (more later)

\[ M_L \]

\[ e \]

\[ M_T \]

\[ M_0 \]

\[ T_2 < T_1 \] Always

\[ B_2 \text{ RECEPTION} \]

\[ \text{Once excited, we can pick up signal} \]

\[ \text{Alternating magnetic flux through loop produces EMF (Faraday's Law } E = \frac{\partial}{} \]

\[ V(t) \]

\[ \text{This is what chemists use} \]

\[ \text{Chemical shift } \sim 1 \text{ ppm} \]

\[ \text{Want } B_1, \text{ flatter} \]

\[ \Rightarrow \text{Body \rightarrow Tiny oscillators} \]

\[ N_{MC} \text{ - Image oscillators}. \]
**Gradient Fields**

**Spatial Localization**

B_z has poor localization \( @ \text{high frequencies} \)

Instead, code position in frequency

\[ \omega(x) = \gamma (B_x + B_z x) \]

\( x \)-gradient in \( x \)

\( \omega(x) = \frac{\partial B_z}{\partial x} \)

\((B_x, B_z) \text{ do not matter much in high field}\)

**Peripheral Nerve Stimulation**

Big Amps: 1000 Volts

200 Amps

Graduates do not satisfy Maxwell Eqn.

Not an issue in high field

**B_1 - RF Field**

- Precession precesses around rotating field and is tuned away
- Very small field \( \varphi (\text{and often}) \)
- Resonance is essential
- Easy to describe in rotating frame

**Reception**

- E x F = \( \frac{\partial F}{\partial t} \)

**Relaxation**

- Longitudinal \( T_1 \)
- Transverse \( T_2 \)

**Gradient Fields**

Encode position onto frequency

\[ G_z = \frac{\partial B_z}{\partial x} \]

Small component fields \( B_x, B_y \) do not contribute to precession since \( \ll B_z \) and \( \omega \) oscillating

High Field: 1st

Assume \( \frac{3}{4} \)

\( \Delta B = \frac{B_r - B_i}{2} \)

\( 15000 \cdot 0.033 = 505 \)

\( \Delta B = 14 \) Hz

For \( G_z \sim \frac{1}{4} \) cm \( \Rightarrow 0.42 \text{ kHz} \) per mm

Shear of 0.03 ppm

For accuracy of 1 ppm need

\[ \frac{0.42}{13} \approx 0.4 \text{ Hz} \]

\[ \approx 6.5 \text{ ppm} \] (usually 1 ppm)

With \( G = \frac{1 \text{ cm}}{2} \approx 2.6 \text{ ppm} \)
GRAVITÉ FIELDS

encode position onto frequency

\[ G \xrightarrow{\text{obs.}} \frac{\text{dx}}{\text{dx}} \]

(smaller constant field does not contribute to precision, rough estimate)

ANEMOTHER

"Deoxograpy" - clever!

Paul Sommertag (name not clear) makes precise stoichiometric chemistry. 
Uses of measurements:

MEASURED

\[ \text{H} - \text{C} - \text{C} - \text{O} - \text{H} \]

\( H \text{H} \) is different "endo-

To speed things up, he put a test tube and added linear gradient

\[ \text{H} \text{H} \]

HE REMARKED THIS WAS IMAGINARY

EXAMPLE SQUARE RECT

\[ s(t) = \int_{-\infty}^{\infty} e^{-i(\omega_0 + \delta \omega) t} \, dt \]

\[ = e^{-i\omega_0 t} \int_{-\infty}^{\infty} e^{-i\delta \omega t} \, dt \]

\[ = e^{-i\omega_0 t} \left[ \frac{1}{\delta} \right] \]

\[ \text{m}(t) = \text{rect}(\cdot) \]

\[ f(t) \text{rect}(\cdot) \sim \text{square}(\cdot) \]

INVARIANTS TO SPECIFIC ISSUES: COMPLEX VALUE REPRESENTATION

For analysis, it is convenient to represent \( M_x, M_y \) as a complex number.

RECEIVED SIGNAL (complex):

\[ S_x(t) = R \{ f(t) \} e^{-i \delta \omega t} \]

PHYSICAL SIGNAL:

\[ S_y(t) = \text{Re} \left\{ S_x(t) \right\} = \text{Re} \left\{ A(t) e^{i \delta \omega t} \right\} \]

\[ = \text{Re} \left\{ \text{Re} \left\{ A(t) e^{i \frac{\delta \omega t}{2}} \right\} \cos \left\{ \frac{\delta \omega t}{2} \right\} \right\} \cos \left\{ \frac{\delta \omega t}{2} \right\} \]

\[ \text{in phase} \]

\[ \text{Q(t)} \]

\[ \text{quadrature} \]

Baseband SIGNAL (analytic):

\[ s(t) = s_x(t) e^{-i\delta \omega t} = A(t) e^{-i\delta \omega t} \]

\[ = s(t) + i Q(t) \]

QUADRATURE PHASE-SENSITIVE DETECTION:

\[ s_y(t) = \left[ \begin{array}{c} \text{I(t)} \\ \text{Q(t)} \end{array} \right] \]

\[ \Rightarrow s_y(t) = i \text{I(t)} + Q(t) \]

\[ \text{Re} \{ s_y(t) \} \rightarrow M_x \text{ in Rf Frame} \]
So far:

1. Place sample in \( B_0 \)
   - \( M \) develops \( -ST \)

2. Excite using \( B_x \) (Tip away from \( Z \))

3. Instantaneous precession of \( M \)
   - Pick up induced EMF in RF coil

4. Encode position in frequency using gradients
   - 1D projection

Q. How do we get an image?

Several key components:

10. Selective excitation (Dimension reduction)

11. Spatial encoding

3. Limitations:
   - Contrast + duration \( \rightarrow \) Resolution

   Limited, signal decay \( (T_2) \), field enhancement \( (T_1) \)

   Diffusion

   Typical res: \( 1 \text{mm} \)
   - 30-50 \( \mu \text{m} \) small anat. features