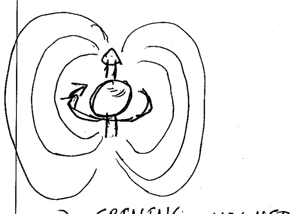
Lecture 05 1/31/12

CLASSICAL DESCRIPTION OF MR

(→) ODD #PROTONS/NEUTRONS have Nuclear Spin ANGULAR MOMENTUM



INTRINSIC QM PROPERTY!
~~MORE CH. 4~~

INTRINSIC MAGNETIC MOMENT

(→) SPINING MAGNETIC DIPOLES

(→) BIOLOGICAL TISSUE MOSTLY ^1H in H_2O
SOMETIMES ^{31}P, ^{13}C, ^{23}Na exotic

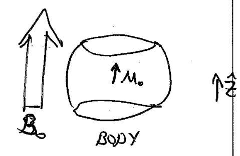
(→) MR IS ABOUT INTERACTIONS WITH THREE FIELDS:

B_0 – MAIN FIELD
⇒ POLARIZATION

B_1 – RF FIELD
⇒ SIGNAL PRODUCTION/RECEPTION

$\vec{G}$ – GRADIEN FIELDS
⇒ SPATIAL ENCODING

B_0 – MAIN FIELD



(→) PRODUCES POLARIZATION OF SAMPLE M_0

(→) SPINS EXHIBIT RESONANCE AT LARMOR FREQUENCY

$$\omega = -\gamma B_0$$

↑ Gyromagnetic Ratio

γ DEPENDS ON NUCLEUS

FOR PROTONS:

$$\frac{\gamma}{2\pi} = 4.257 \text{ KHz/G}$$

$$\gamma = 2\pi\, 4.257 \text{ kRad/G}$$

WORTH REMEMBERING!

OTHERS: ^{23}Na $\frac{\gamma}{2\pi} = 1.127$ kHz/G

^{13}C $\frac{\gamma}{2\pi} = 1.071$ kHz/G

^{15}N $\frac{\gamma}{2\pi} = -0.43$ kHz/G Negative!

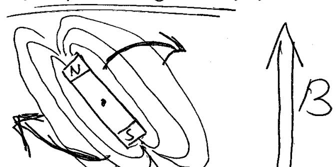
TYPICAL B_0's

0.1 T	4.2 MHz	VERY LOW!
0.5 T	21 MHz	LOW (Permanent Resistive)
1 T	42 MHz	MILD (super conducting)
1.5 T	63 MHz	"HIGH" Diagnostic
3 T	126 MHz	"HIGH" fMRI Neuro
4 T	170 MHz	RARE (Decomissioned @ Berkeley)
7/9.4 T		VERY HIGH Reasearch ONLY!

FOR SPECTRAL/SPATIAL LOCALIZATION
We REQUIRE HOMOGENEITY

$\Delta B_0 \sim 1$ PPM   40 cm³ FOV
64 Hz @ 1.5T
PRETTY REMARKABLE!

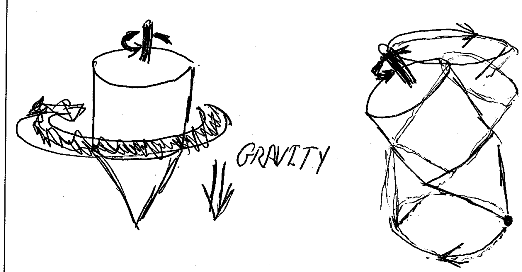
WHY RESONANCE?



(→) TORQUE, BUT NO RESONANCE.

(→) ~~MISSING~~ BAR MAGNET IS MISSING ANGULAR MOMENTUM.

$\boxed{\text{LIKE A SPINNING TOP!}}$



B_1 – RF FIELD

CANT DIRECTLY DETECT M_0

WHY? HUGE FIELD!

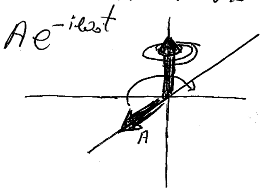
RESONANCE IS THE BIG DEAL...

B_0 IS DC SPIN RESONATE ⇒ DETECTION!

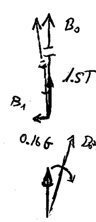
(–) SAMPLE RESONATES AT $\omega_0 = \gamma B_0$

(–) APPLY ROTATING RF FIELD AT $\omega_0 = \gamma B_0$ IN THE TRANSVERSE PLANE

$B_1(t) = A e^{-i\omega_0 t}$



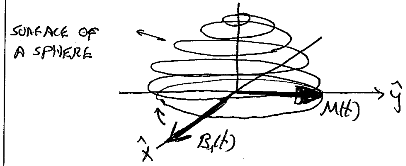
AT TIME $t=0$

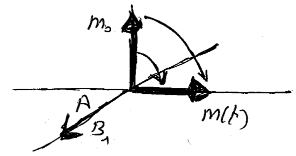


AT TIME $t = \frac{1}{\omega_0}$

HAS TO BE ON RESONANCE TO DO SOMETHING

ILLUSTRATE BY ROTATIN A STRING





(–) IN ROTATING FRAME: PRECESSION ABOUT B_1

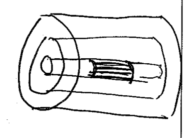
$$\boxed{\begin{array}{l}\omega_e = \gamma B_1 = 4.257 \cdot 0.16 = 0.68\ \text{kHz} \\ 0.367\ \text{ms}\ \ 90^\circ \Rightarrow 23000\ \text{ROTATIONS}\end{array}}$$

TYPICAL B_1'S 0.14 – 0.35G

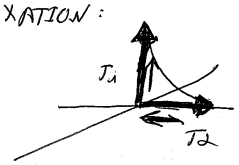
DURATION 1–3 ms LONG TIME AT 64MHz

PEAK POWER 20kW!

HEATING FROM THIS (SAR)



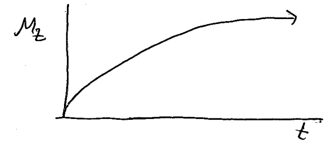
(–) MAGNETIZATION EXHIBITS RELAXATION:

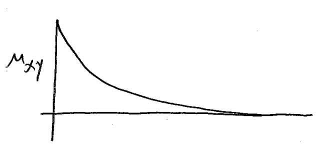


T_1 ~ LONGITUDAL 100 → 2000 ms in Tissue

T_2 ~ TRANSVERSE 10 → 300

MAIN SOURCE OF TISSUE CONTRAST (MORE LATER)

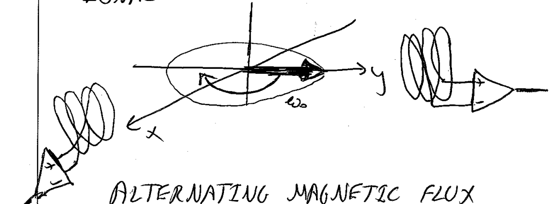




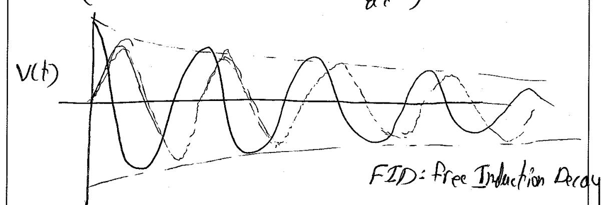
$T_2 < T_1$ ALWAYS

B_1 RECEPTION

ONCE EXCITED, WE CAN PICK UP SIGNAL



ALTERNATING MAGNETIC FLUX THROUGH LOOP PRODUCES EMF (FARADAYS LAW $\mathcal{E} = \frac{d\Phi_B}{dt}$)



THIS IS WHAT CHEMISTS USE

CHEMICAL SHIFT ~ 1PPM

WANT B_0 FLATTER.

(–) Body → TINY OSCILATORS

MRI: IMAGE OSCILATORS.

G- GRADIENT FIELDS

SPATIAL LOCALIZATION

B_1 HAS POOR LOCALIZATION λ @ 64MHz ~ ½m in Tissue

INSTEAD, CODE POSITION IN FREQUENCY

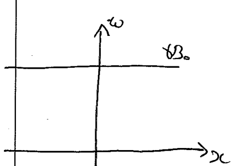
$$\omega(x) = \gamma(B_0 + G_x x)$$

gradient in x

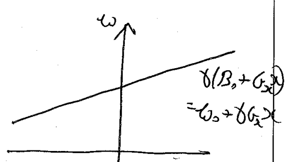
(*) CHANGE IN Z COMPONENT W/ x

$$G_x = \frac{\partial B_z}{\partial x}$$

(B_x, B_y DO NOT MATTER MUCH IN HIGH FIELD)



NO GRADIENT



LINEAR RELATION BETWEEN POSITION & FREQUENCY

TYPICAL #

$G = 1–10$ G/cm

$= 10–100$ mT/m

$= 4.2–42$ kHz/cm

GRADIENT WAVEFORMS IN AUDIO RANGE

SR = 15 G/cm/ms

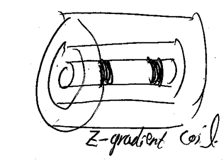
SAFETY CONCERN is $\frac{dB}{dt}$

$$\left(\frac{dG_x(t)}{dt}\right)x$$

PERIFERAL NERVE STIMULATION

BIG AMPS: 1200 VOLTS
200 AMPS

GRADIENTS DO NOT SATISFY MAXWELL EQN.
NOT AN ISSUE IN HIGH FIELD



Z-gradient coil

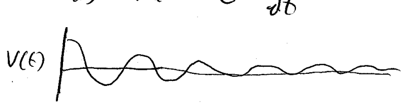
B_1 – RF FIELD ± 10% ACCURACY

(→) MAGNETIZATION PRECESSES AROUND ROTATING FIELD AND IS TIPPED AWAY
(−) VERY SMALL FIELD (MAX 0.35[G])
(−) RESONANCE IS ESSENTIAL

EASY TO DESCRIBE IN ROTATING FRAME.

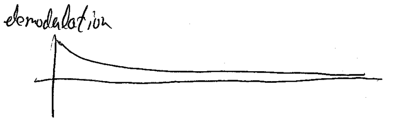
RELAXATION
(−) LONGITUDINAL T_1
(−) TRANSVERSE T_2

RECEPTION
(+) EMF $\varepsilon = \frac{d\Phi_B}{dt}$



FID

demodulation



(*) ±10% ACCURACY LEADS TO DIFFERENT FLIPS IN SPACE

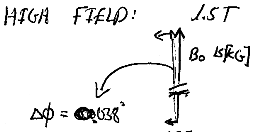
GRADIENT FIELDS

ENCODE POSITION ONTO FREQUENCY

$$G_x = \frac{\partial B_z}{\partial x}$$

(SMALL CONCAMITANT FIELDS B_x, B_y DO NOT CONTRIBUTE TO PRECESSION SINCE $<< B_0$ AND NOT OSCILLATING)

HIGH FIELD: 1.5T ASSUME $1\left[\frac{G}{cm}\right]$



$\|B_0\hat{z} + B_x\hat{x}\| = 15000.$[illegible]$333 \Rightarrow \Delta f = 14$ Hz

FOR $G_x \sim 1\frac{G}{cm} \Rightarrow 0.42\frac{kHz}{mm} = 420\frac{Hz}{mm}$]

SHIFT OF 0.03 [mm]

FOR ACCURACY OF 1mm NEED

$$\frac{\Delta B_0}{B_0} \approx \frac{0.42}{63{,}000\ kHz} \sim 6.5\ PPM$$

USUALLY 1PPM ?

WITH $G = \frac{4G}{cm} \sim 26$ PPM

GRADIEN FIELDS $\pm 20X$

ENCODE POSITION ONTO FREQUENCY

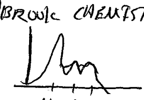
$$G_x = \frac{\partial B_z}{\partial x}$$

(SMALL CONCAMETANT FIELDS B_x, B_y DO NOT CONTRIBUTE TO PRECSSION ABOUT B_0 - Not oscillating)

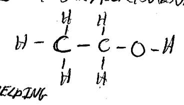
ANEKDOTE 1973 Lauterbur

"Zeugmatography" - clever!

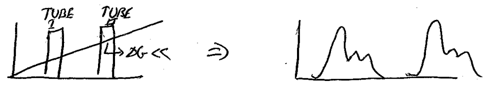
PAUL SOMEWHAT LAZY (AKA EFFICENT) YOUNG PROFF AT STONYBROOK CHEMISTRY. LOTS OF MEASUREMENTS!

MEASURED 

DIFFERENT e^- SHIELDING

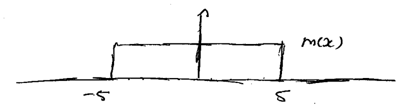


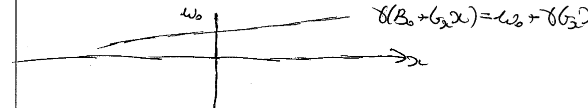
TO SPEED THINGS UP, HE PUT 2 TEST TUBES AND ADDED LINEAR GRADIENT



HE REALIZED THIS WAS IMAGING!

EXAMPLE SQUARE RECT





$$s(t) = \int_x m(x)\, e^{-i(\omega_0 + \gamma G_x x)t}\, dx =$$

$$= e^{-i\omega_0 t} \int_x m(x)\, e^{-i\gamma G_x x t}\, dx =$$

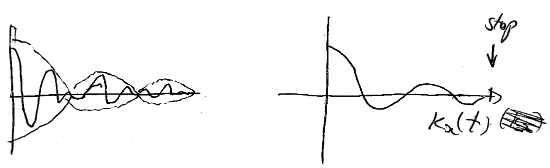
$$= e^{-i\omega_0 t} \int m(x)\, e^{-i2\pi(\overbrace{\tfrac{\gamma}{2\pi} G_x t}^{k_x})x}\, dx =$$

$$= e^{-i\omega_0 t}\, \mathcal{F}\{m(x)\}\Big|_{k_x = \frac{\gamma}{2\pi} G_x t}$$

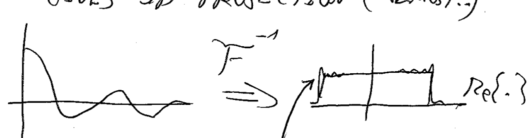
$m(x) = \mathrm{rect}(\cdot)$

$\mathcal{F}(m(x)) \sim \mathrm{SINC}(\cdot)$

$$s(t) = \underbrace{e^{-i\omega_0 t}}_{\text{CARRIER}}\, \underbrace{\mathcal{F}\{m(x)\}\Big|_{k_x = \frac{\gamma}{2\pi} G_x t}}_{\text{BaseBand}}$$



INVERSE FT OF BASEBAND SIGNAL GIVES 1D PROJECTION (ALMOST..)



BACK TO SPECIFIC ISSUES: COMPLEX VALUE & DEMODULATION

FOR ANALYSIS IT IS CONVINIENT TO REPRESENT M_x, M_y AS A COMPLEX NUMBER

RECIEVE SIGNAL (analysis):

$$\underset{\text{complex}}{S_r(t)} = \underset{\text{Real}}{A(t)}\, e^{-i(\omega_0 t + \underset{\text{real}}{\phi(t)})}$$

PHYSICAL SIGNAL:

$$\underset{\text{Real}}{S_p(t)} = Re\{S_r(t)\} = A(t)\cos(\omega_0 t + \phi(t)) =$$

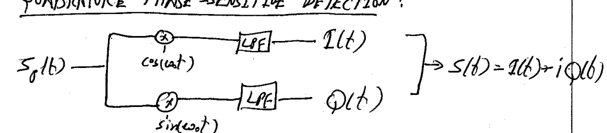
$$\underset{\text{Trig}}{\Rightarrow} \underbrace{A(t)\cos(\phi(t))}_{\substack{I(t) \\ \text{in phase}}} \cos\omega_0 t + \underbrace{[-A(t)\sin\phi(t)]}_{\substack{Q(t) \\ \text{quadrature}}} \sin\omega_0 t$$

BaseBand SIGNAL (analysis):

$$s(t) = S_r(t)\, e^{+i\omega_0 t} = A(t)\, e^{-i\phi(t)} =$$

$$= I(t) + iQ(t)$$

QUADRATURE PHASE-SENSITIVE DETECTION:



$Re\{s(t)\} \rightarrow M_x$ in Rot Frame

SO FAR

(1) PLACE SAMPLE IN B_0
- M_z DEVELOPS ~ $5T_1$

(2) EXCITE USING $B_1(t)$ TIP AWAY FROM $\hat{z}$

(3) INSTANTANIOUS PRECESSION OF M_{xy}
PICK UP INDUCED EMF IN RF COIL

(4) ENCODE POSITION IN FREQENCY USING GRADIENTS
$\Rightarrow$ 1D PROJECTION.

~~(5) HOW DO WE GET AN IMAGE?~~

~~SEVERAL KEY COMPONENTS:~~

~~(1) SELECTIVE EXCITATION (dimension reduction)~~

~~(2) SPATIAL ENCODING~~

(3) LIMITATIONS!

GRADIENT STRENGTH + DURATION $\longrightarrow$ RESOLUTION

limited!

SIGNAL DECAY (T_2), FIELD INHOMOGENEITY (T_2^*)
DIFFUSSION

TYPICAL RES: < 1 mm ~ 10 μm IN SMALL ANIMAL SCANNERS