

Imaging:

- (1) place sample in B_0
 M_z Develops $\sim 5T_1$
- (2) Excite using $B_1(t)$
creates transverse magt.
- (3) Instantaneous Precession of M_{xy}
Induces EMF in coil
- (4) Encode Position in Freq. using gradients
1-D "Projection"

some limitations:

Gradient strength } Resolution
Gradient duration }



signal decay (T_2)
Field Inhomogeneity (T_2^*)
DIFFUSION

TYPICAL RESOLUTION: $\leq 1\text{mm}$
($\sim 20\mu\text{m}$ in small
animal scanners)

④

Q. How do we get an image?

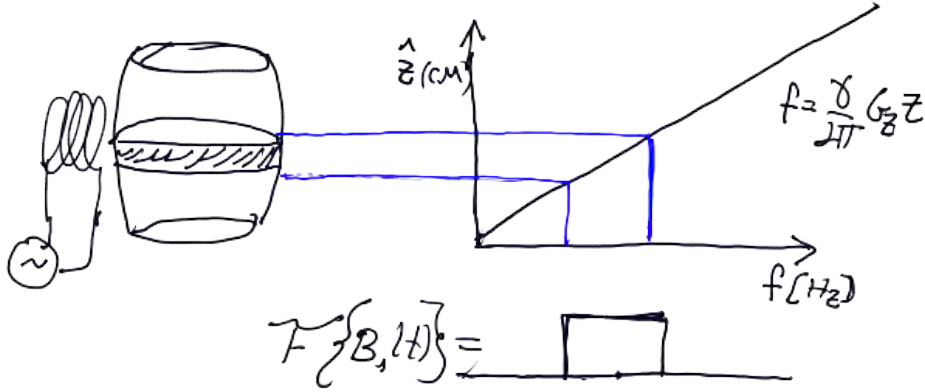
several key components:

- ① Selective excitation (dimension reduction)
- ② spatial encoding (freq/phase) ← more later
- ③ signal decay, M_0 recovery
- ④ Repeat N Times
- ⑤ Image Reconstruction

Selective excitation

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want to excite a slice



Gradient (G_z) maps position to bandwidth

Bandlimited $B_1(t) \Rightarrow$ Excites a slab in space

Excitation field:

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$$e^{-i\omega_0 t} \underbrace{B_1(t)}_{\substack{\text{envelope} \\ \text{(shape of slice)}}}$$

carrier (center of slice)

choose $B_1(t)$

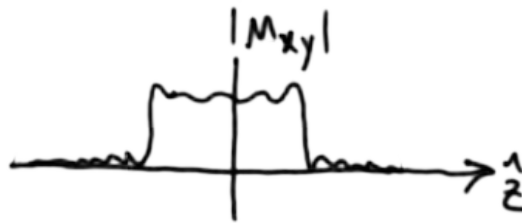
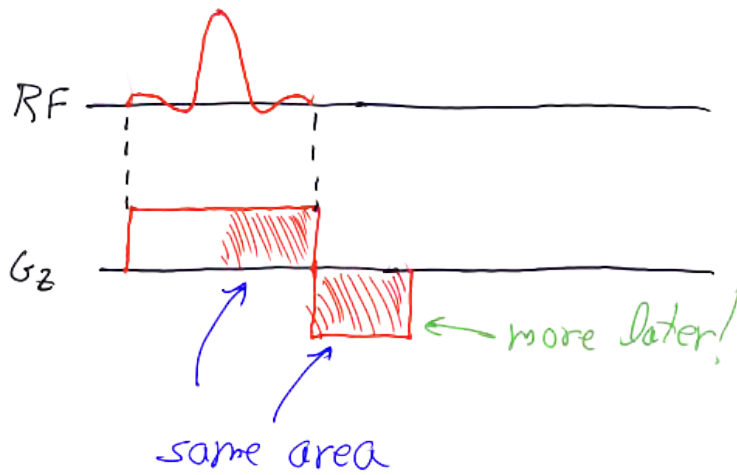
$$\mathcal{F}\{B_1(t)\} \approx \text{rect}\left(\frac{f}{A}\right)$$

$$B_1(t) \approx A \text{sinc}(At)$$

finite in time
ripple in slice

we draw RF excitation as a pulse sequence:

(7)



Quiz

(7a)

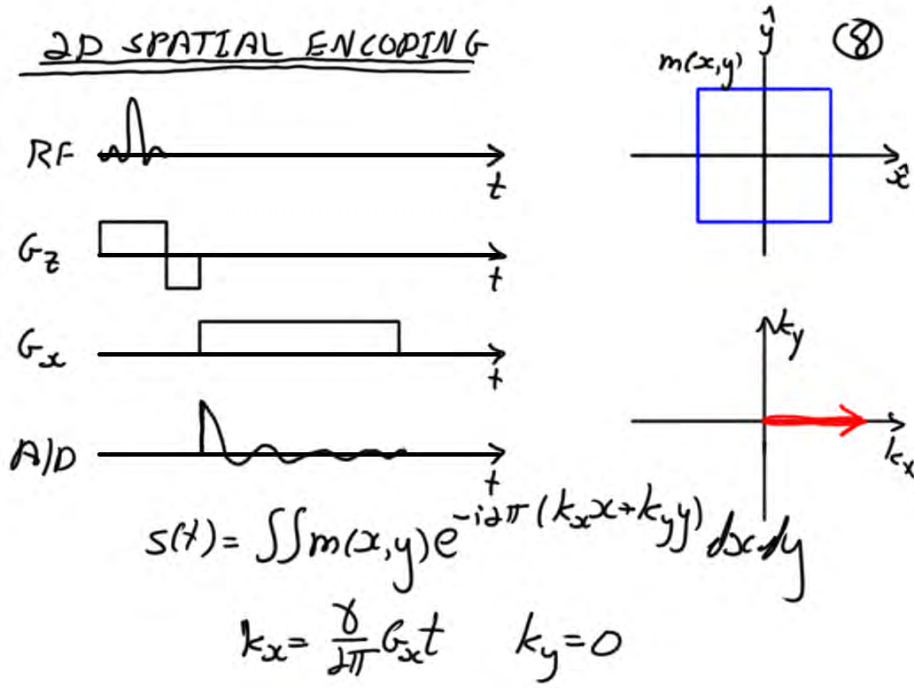
$$B_1(t) = \text{sinc}(5000t)$$

envelope

$$G_z = 1 \text{ G/cm}$$

what is the excited slice thickness?

2D SPATIAL ENCODING



Need k_x - k_y coverage:

Try: $G_x = G \cos \theta_n$ $\theta_n = \frac{\pi}{N} n \quad n = [0, N-1]$
 $G_y = G \sin \theta_n$

The Central Section Theorem
 (Projection slice)

R. Bracewell

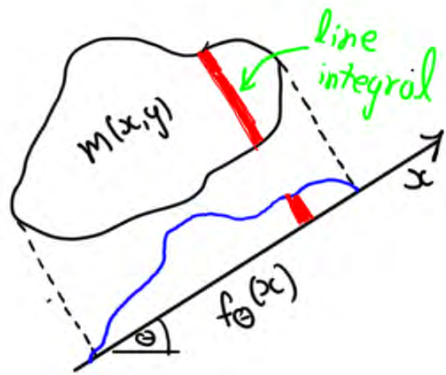


Image Space



k-space

Transform (1D) of projection of an object
 \Rightarrow Diameter through transform (2D) of object
 Very cool! ... and useful...

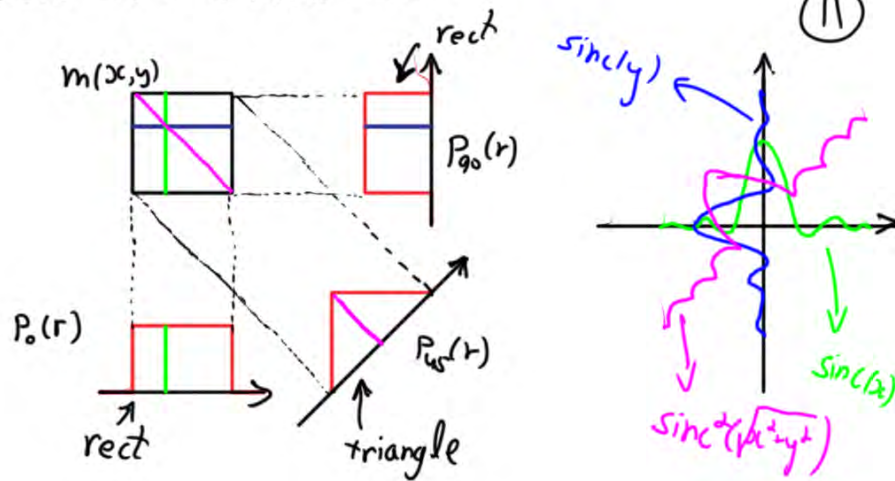
Projection slice \Rightarrow basis for CT

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Proof (for $\theta=0$)

$$\begin{aligned}
 p(x) &= \int_{-\infty}^{\infty} m(x,y) dy \\
 M(k_x, 0) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-i2\pi(k_x x + k_y y)} dx dy = \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-i2\pi k_x x} dx dy = \\
 &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} m(x,y) dy \right] e^{-i2\pi k_x x} dx = \\
 &= \int_{-\infty}^{\infty} p(x) e^{-i2\pi k_x x} dx = \mathcal{F}\{p(x)\}
 \end{aligned}$$

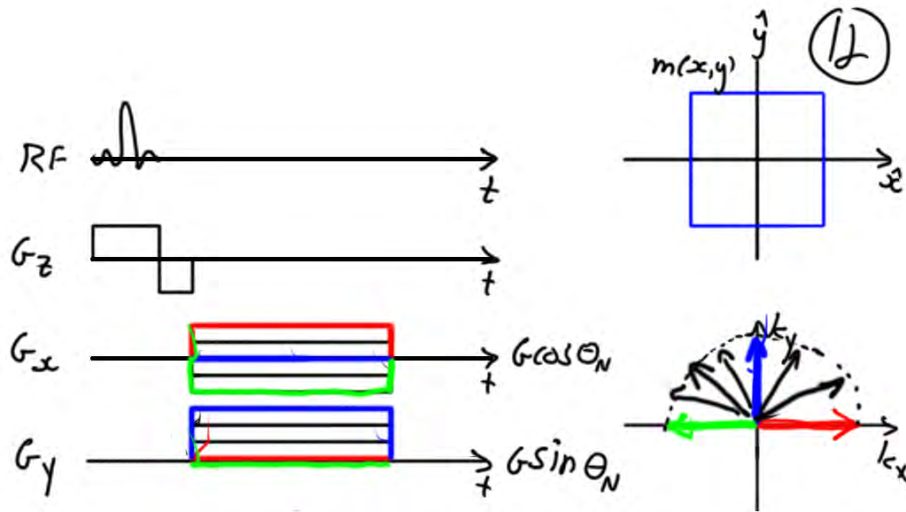
Let's see if it makes sense...



How to reconstruct?

- \rightarrow filtered back-projection
- \rightarrow Interpolation (in k -space)

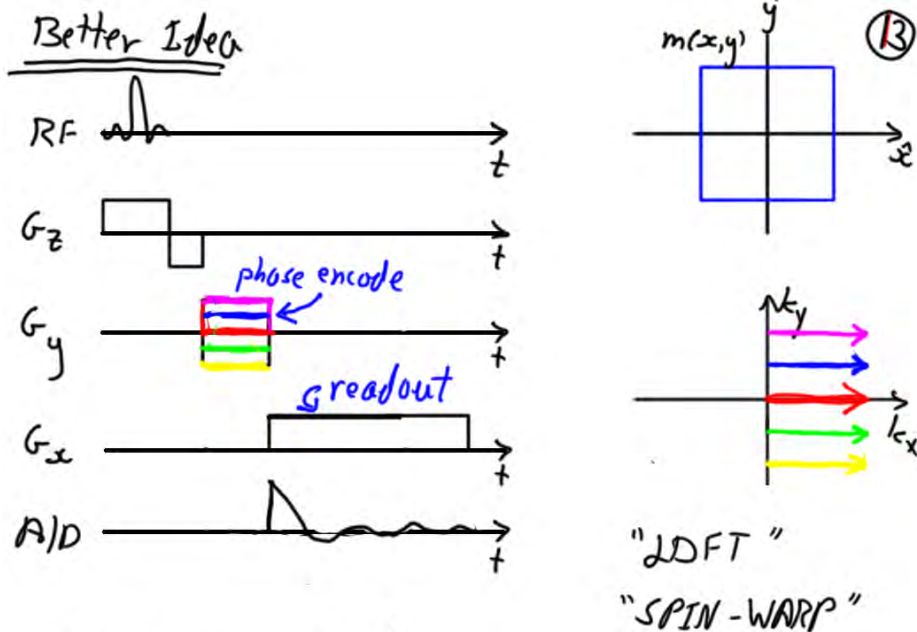
Good class project



Projection Reconstruction (Lauterbur)

$\mathcal{F}_{1D}^{-1} \{ \text{Diameter} \} \Rightarrow \text{Projection}$

However... not quite diameter yet...



(still not perfect)

(-) Make "horizontal projections"

(-) Encode phase in y prior to readout in x

Other approaches to Imaging

(14)

→ Localization is the key!

(1) B₀ localization

→ saddle point

→ local magnet

(2) RF

→ Local coils

→ Local Excitation



(3) Non linear gradients

Complex value & Demodulation

(15)

Physical signal:

$$S_p(t) = A(t) \cos(\omega_0 t + \phi(t)) =$$

real \Rightarrow $A(t) \cos(\phi(t)) \cos(\omega_0 t) + [-A(t) \sin(\phi(t))] \sin(\omega_0 t)$
trigo \uparrow

$I(t)$ in phase $Q(t)$ quadrature

Analysis signal (convenient)

$$S_r = A(t) e^{-i\omega_0 t + \phi(t)}$$

complex \leftarrow \leftarrow real \leftarrow

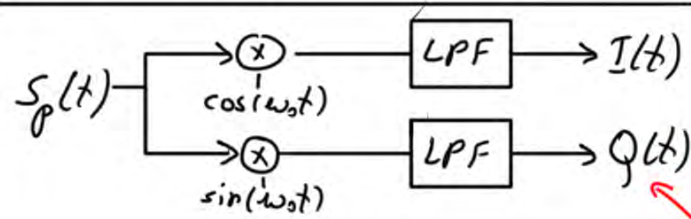
$S_p = \text{Re}\{S_r\}$

Baseband signal

$$s(t) = S_r(t) e^{+i\omega_0 t} = A(t) e^{-i\phi(t)} = I(t) + iQ(t)$$

Quadrature Phase-Sensitive Detection:

(16)



$$s(t) = I(t) + iQ(t)$$

*often used
in communication*

$I(t) \rightarrow M_x$ in rotating frame

$Q(t) \rightarrow M_y$ in rotating frame