Imaging:
1. Place sample in $B_0$
   $M_z$ develops ~ $5T_1$
2. Excite using $B_{1+}$
   Creates transverse magnetization
3. Instantaneous precession of $M_{xy}$
   Induces EMF in coil
4. Encode position in freq. using gradients
   1-D "Projection"

Some limitations:
- Gradient strength vs Resolution
- Gradient duration vs Resolution
- Signal decay ($T_1$)
- Field inhomogeneity ($T_2^*$)
- Diffusion

Typical resolution: ≤ 1mm
(≈ 20 μm in small animal scanners)

Q. How do we get an image?
   Several key components:
   1. Selective excitation (dimension reduction)
   2. Spatial encoding (freq./phase)
   3. Signal decay, $M_0$ recovery
   4. Repeat $N$ times
   5. Image reconstruction
Selective excitation

- Want to excite a slice

- Gradient ($G_z$) maps position to bandwidth

- Bandlimited $B_x(t)$ excites a slab in space

Excitation field:

$$e^{-i\omega t} \frac{B_x(t)}{\text{carrier}} \frac{1}{\text{envelope}}$$

- $\omega$: carrier frequency (center of slice)
- $A$: shape of slice

Choose $B_x(t)$:

$$\mathcal{F}\{B_x(t)\} \approx \text{rect}(\frac{f}{A})$$

- $B_x(t) = A \sin(\pi t)$

- Finite in time, ripples in slice

Quiz:

$$B_x(t) = \text{sinc}(\frac{\omega_0 t}{2})$$

$G_z = 1 \text{ G/cm}$

- What is the excited slice thickness?
2D Spatial Encoding

\[ \begin{align*}
\text{RF} & \quad t \\
G_z & \quad t \\
G_x & \quad t \\
A/D & \quad t \\
\end{align*} \]

\[ s(t) = \sum_{m,n} m(x,y) e^{-j\omega(k_x x + k_y y)} dx dy \]

\[ k_x = \frac{\omega}{2\pi} G_x t \quad k_y = 0 \]

Need \( k_x - k_y \) coverage:
Try: \[ G_x = G \cos \Theta \quad G_y = G \sin \Theta \]

The Central Slice Theorem

(Projection Slice)

Image Space

k-space

Transform (1D) of projection of an object
\[ \Rightarrow \text{Diameter through transform(2D) of object} \]

Very cool! ... and useful...

Projection Slice \( \Rightarrow \) basis for CT

Proof (for \( \Theta = 0 \))

\[ p(x) = \int_{-\infty}^{\infty} m(x,y) dy \]

\[ M(k_x,0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-j\omega(k_x x + k_y y)} dx dy = \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-j\omega k_x x} dx dy = \]

\[ = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} m(x,y) dy \right] e^{-j\omega k_x x} dx = \]

\[ = \int_{-\infty}^{\infty} p(x) e^{-j\omega k_x x} dx \]

\[ = \hat{p}(k_x) \]

How to reconstruct?
1) Filtered back-projection
2) Interpolation (in k-space)

Good class project
Projection Reconstruction (Lauterberg)

$P_{\text{ID}} \{ \text{Diameter} \} \Rightarrow \text{Projection}$

However... not quite there yet...

Other approaches to imaging

1. Localization is the key!
   1. Be localization
      1. Saddle point
      1. Local magnet
   2. RF
   3. Local coils
   4. Local excitation
   5. Non-linear gradients

Complex value & Demodulation

Physical signal:

$S_p(t) = A(t)\cos(\omega_c t + \phi(t)) = \text{real} \rightarrow A(t)\cos(\phi(t))\cos(\omega_c f) + \left[-A(t)\sin(\phi(t))\sin(\omega_c f)\right]$  

In phase

Quadature

Analysis signal (convenient)

$S_r = A(t)e^{-i\omega_c t + \phi(t)}$  

$S_p \rightarrow \text{real}$

Baseband signal

$s(t) = S_r(t)e^{i\omega_c t} = A(t)e^{-i\phi(t)} = I(t) + iQ(t)$
Quadature Phase Sensitive Detection:

\[ s(t) = I(t) + i Q(t) \]

\[ I(t) \rightarrow M_x \text{ in rotating frame} \]
\[ Q(t) \rightarrow M_y \text{ in rotating frame} \]