

Imaging:

- (1) place sample in B_0
 M_z Develops $\sim 5T_1$
- (2) Excite using $B_1(t)$
creates transverse magt.
- (3) Instantaneous Precession of M_{xy}
Induces EMF in coil
- (4) Encode position in Freq. using gradients
1-D "Projection"

some limitations:

Gradient strength } Resolution
Gradient duration }



signal decay (T_2)
Field Inhomogeneity (T_2^*)
DIFFUSION

TYPICAL RESOLUTION: $\leq 1\text{mm}$
($\sim 20\mu\text{m}$ in small
animal scanners)

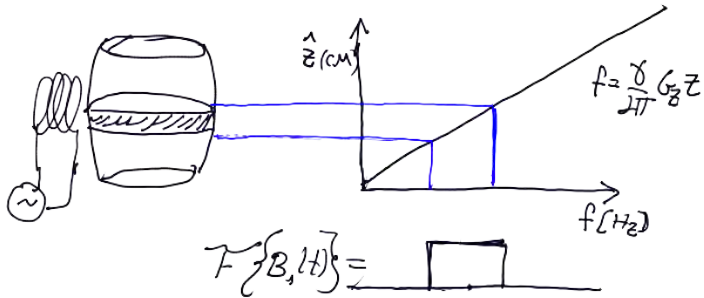
Q. How do we get an image?
several key components:

- ① Selective excitation (dimension reduction)
- ② spatial encoding (Freq/phase) ← more later
- ③ signal decay, M_0 recovery
- ④ Repeat N Times
- ⑤ Image Reconstruction

Selective excitation

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want to excite a slice



Gradient (G_z) maps position to bandwidth
Bandlimited $B_1(t) \Rightarrow$ Excites a slab in space

Excitation field:

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$$e^{-i\omega_0 t} B_1(t)$$

carrier (center of slice) envelope (shape of slice)

choose $B_1(t)$

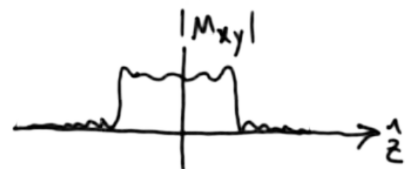
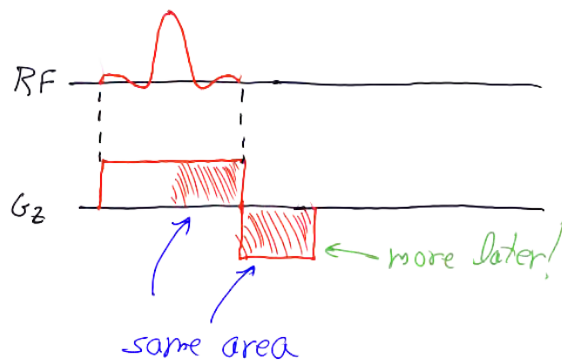
$$\mathcal{F}\{B_1(t)\} \approx \text{rect}\left(\frac{f}{\Delta}\right)$$

$$B_1(t) \approx A \text{sinc}(At)$$

finite in time
ripple in slice

we draw RF excitation as a pulse sequence:

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Quiz

(7a)

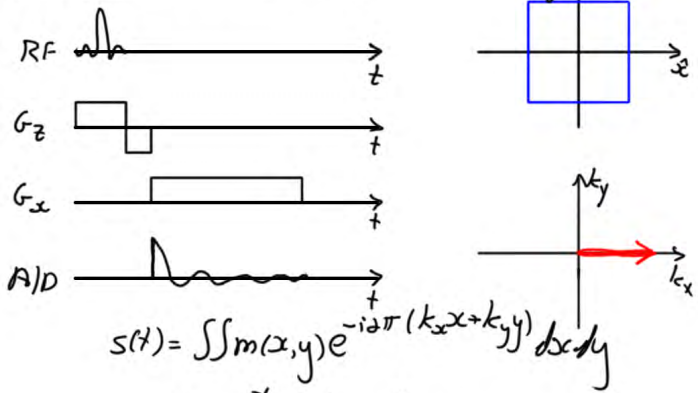
$$B_1(t) = \text{sinc}(5000t)$$

envelope

$$G_z = 1 \text{ G/cm}$$

what is the excited slice thickness?

2D SPATIAL ENCODING



$$s(t) = \iint m(x,y) e^{-i2\pi(k_x x + k_y y)} dx dy$$

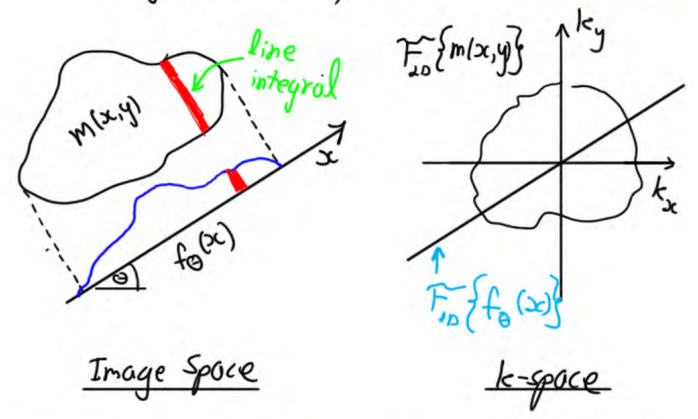
$$k_x = \frac{\gamma}{2\pi} G_x t \quad k_y = 0$$

Need k_x - k_y coverage:

Try: $G_x = G \cos \theta_n$ $\theta_n = \frac{\pi}{N} n$ $n \in [0, N-1]$
 $G_y = G \sin \theta_n$

The Central Section Theorem (Projection slice)

R. Bracewell



Transform (1D) of projection of an object
 \Rightarrow Diameter through transform (2D) of object
 Very cool! ... and useful...

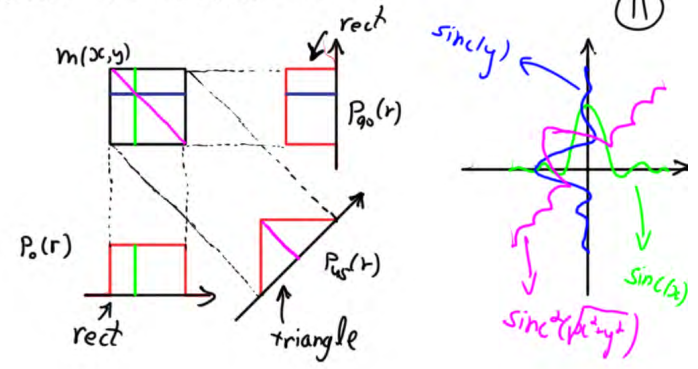
Projection slice \Rightarrow basis for CT (10)

Proof (for $\theta=0$)

$$p(x) = \int_{-\infty}^{\infty} m(x,y) dy$$

$$\begin{aligned} M(k_x, 0) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-i2\pi(k_x x + k_y y)} dx dy = \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-i2\pi k_x x} dx dy = \\ &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} m(x,y) dy \right] e^{-i2\pi k_x x} dx = \\ &= \int_{-\infty}^{\infty} p(x) e^{-i2\pi k_x x} dx = \mathcal{F}\{p(x)\} \end{aligned}$$

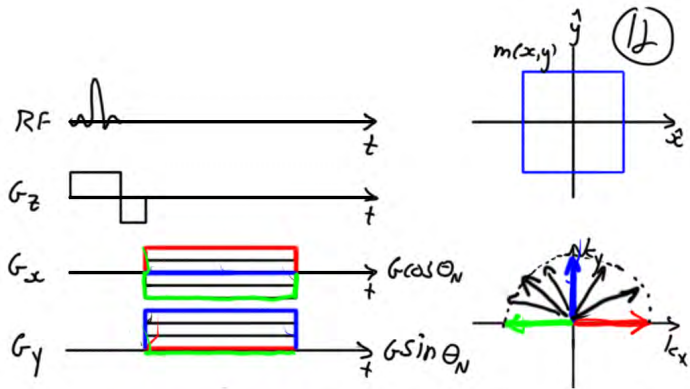
Let's see if it makes sense...



How to reconstruct?

- \rightarrow filtered back-projection
- \rightarrow Interpolation (in k-space)

Good class project

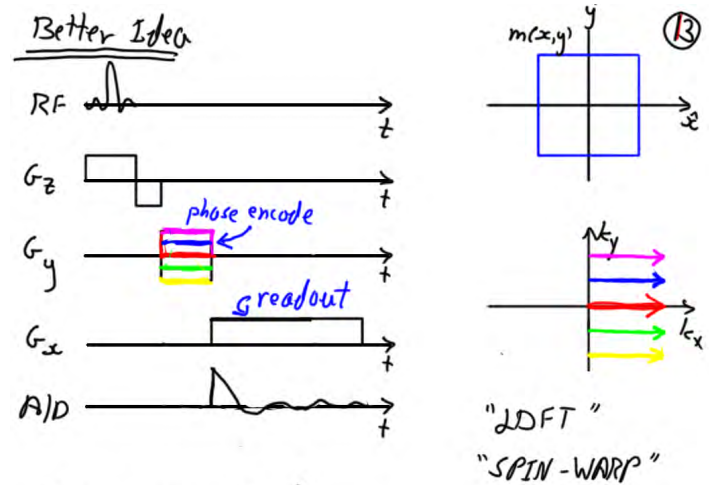


Projection Reconstruction (Lauterbur)

$$F_{1D}^{-1} \{ \text{Diameter} \} \Rightarrow \text{Projection}$$

However... not quite diameter yet...

Better Idea



(still not perfect)

- (-) Make "horizontal projections"
- (-) Encode phase in y prior to readout in x

Other approaches to Imaging

(14)

(-) Localization is the key!

(1) B₀ localization

- (-) saddle point
- (-) local magnet

(2) RF

- (-) Local coils
- (-) Local Excitation



(3) Non linear gradients

Complex value & Demodulation

(15)

Physical signal:

$$S_p(t) = A(t) \cos(\omega_0 t + \phi(t)) = \underbrace{A(t) \cos(\phi(t))}_{I(t) \text{ in phase}} \underbrace{[\cos(\omega_0 t) + \underbrace{-\sin(\phi(t))}_{\phi(t) \text{ quadrature}}]}_{\sin(\omega_0 t)}$$

Analysis signal (convenient)

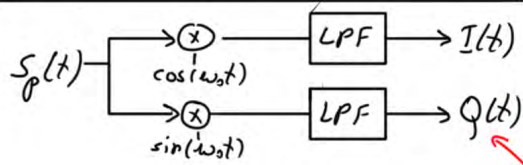
$$S_r = A(t) e^{-i\omega_0 t + \phi(t)} \quad S_p = \text{Re}\{S_r\}$$

Baseband signal

$$s(t) = S_r(t) e^{+i\omega_0 t} = A(t) e^{-i\phi(t)} = I(t) + i\phi(t)$$

Quadrature Phase-Sensitive Detection:

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$s(t) = I(t) + iQ(t)$ *often used in communication*

$I(t) \rightarrow M_x$ in rotating frame

$Q(t) \rightarrow M_y$ in rotating frame