Imaging:

(1) place sample in B_0

M_z Develops $\sim 5T_1$

(2) Excite using $B_1(t)$

creates transverse magn.

(3) Instantaniouse precession of M_{xy}

Induces EMF in coil

(4) Encode Position in Freq. using gradients

1-D "Projection"

some limitations:

Gradient strength
Gradient duration
} Resolution

$\Downarrow$

signal decay (T_2)
Field Inhomogeneity (T_2^*)
DIFFUSION

TYPICAL RESOLUTION : ≤ 1 mm
(~ 20 μm in small animal scanners)

Repeat Rect Example

1D

Q. How do we get an image?

several key components:

① Selective excitation (Dimension reduction)

② spatial encoding (freq/phase) ← more later

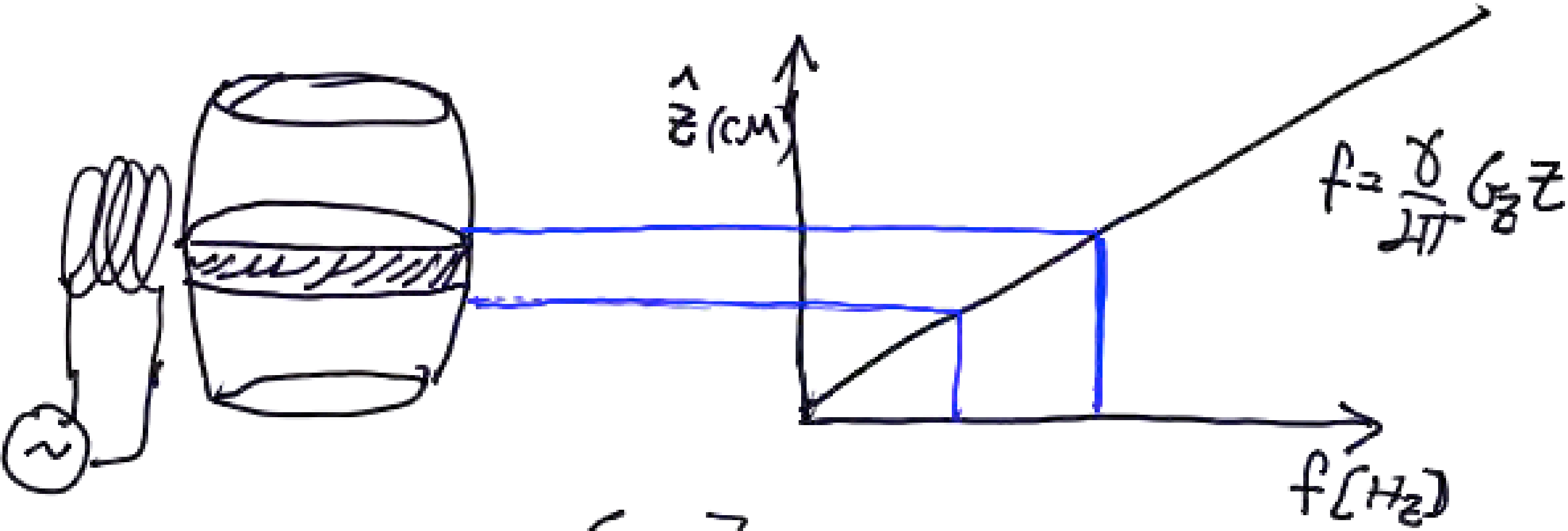
③ signal decay, M_0 recovery

④ Repeat N Times

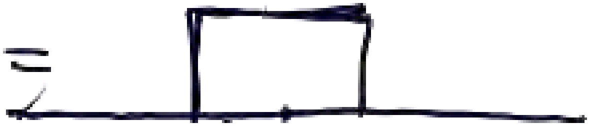
⑤ Image Reconstruction

Selective excitation

want to excite a slice



$\mathcal{F}\{B_1(t)\} =$



Gradient (G_z) maps position to bandwidth

Bandlimited $B_1(t) \Longrightarrow$ Excites a slab in space

Excitation field:

$$\underbrace{e^{-i\omega_0 t}}_{\substack{\text{carrier} \\ \text{(center of slice)}}} \; \underbrace{B_1(t)}_{\substack{\text{envelope} \\ \text{(shape of slice)}}}$$

choose $B_1(t)$

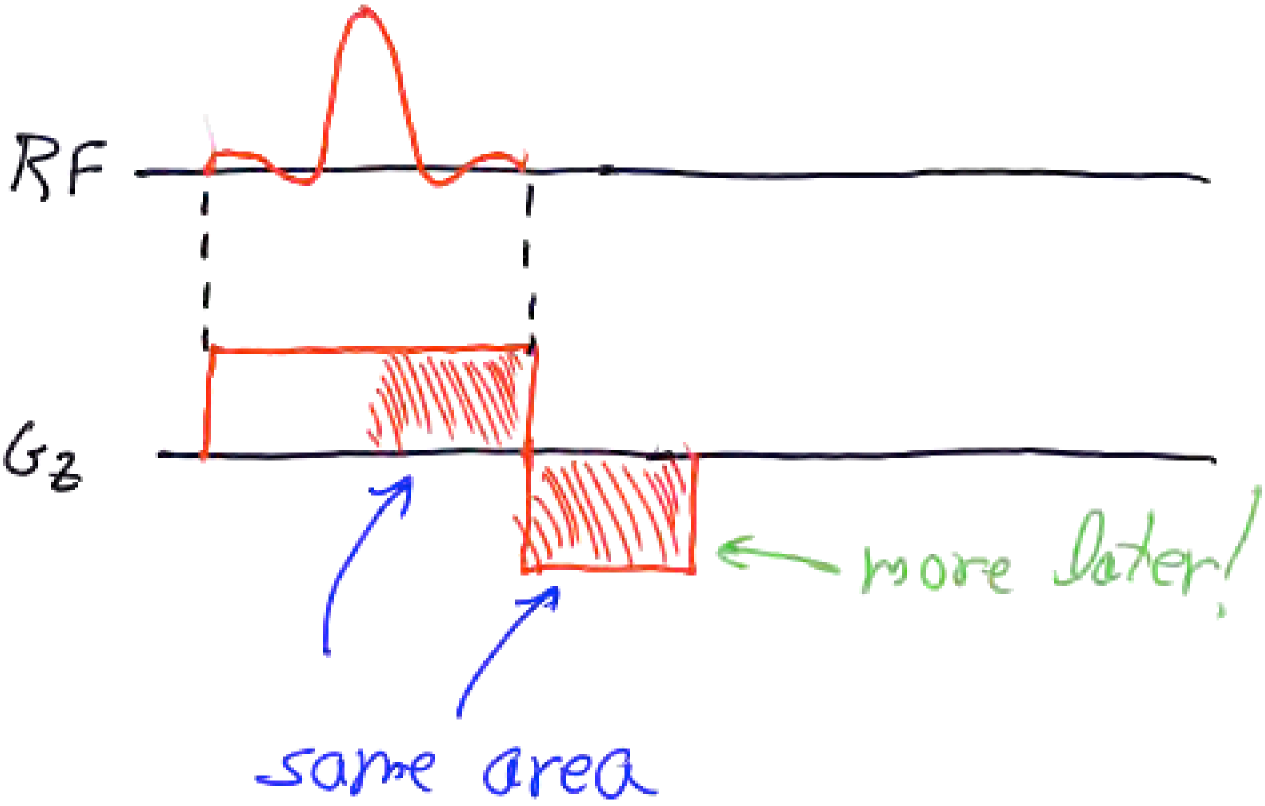
$$\mathcal{F}\{B_1(t)\} \simeq \mathrm{rect}\left(\frac{f}{A}\right)$$

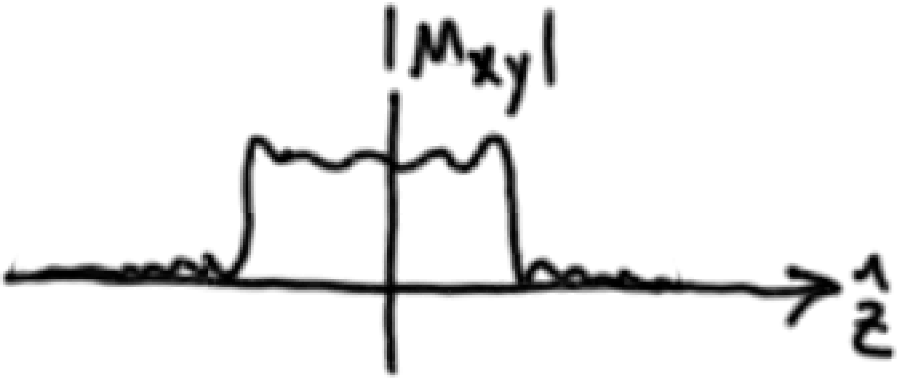
$$B_1(t) \simeq A\,\mathrm{sinc}(At)$$

finite in time

ripple in slice

we draw RF excitation as a pulse seqence:





Quiz

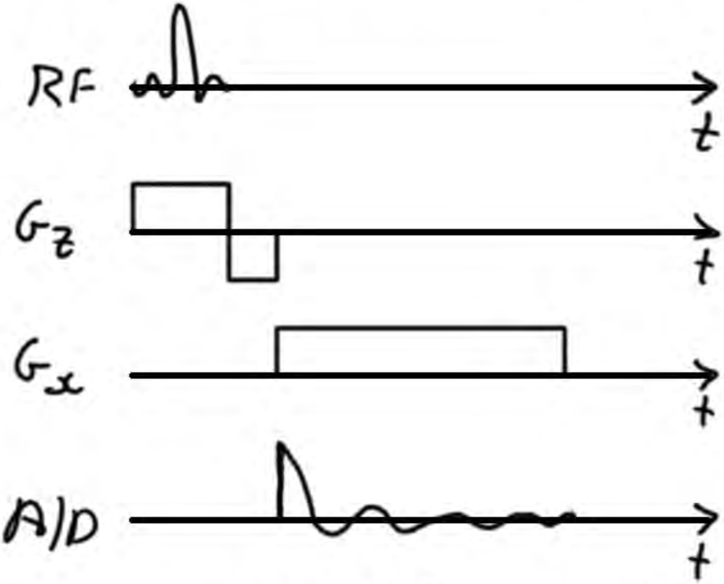
$$B_1(t) = \text{sinc}(5000t)$$

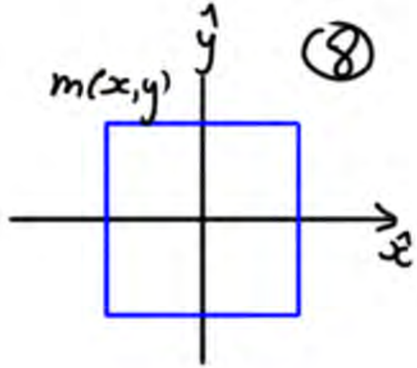
envelope

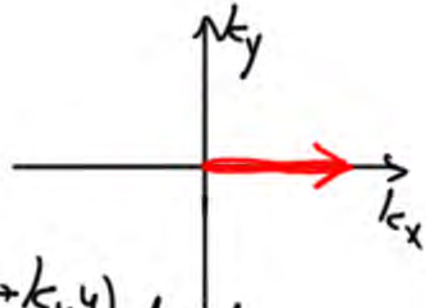
$$G_z = 1\ \text{G/cm}$$

what is the excited slice thickness?

2D SPATIAL ENCODING







$$s(t) = \iint m(x,y)\, e^{-i2\pi(k_x x + k_y y)}\, dx\, dy$$

$$k_x = \frac{\gamma}{2\pi} G_x t \qquad k_y = 0$$

Need k_x–k_y coverage:

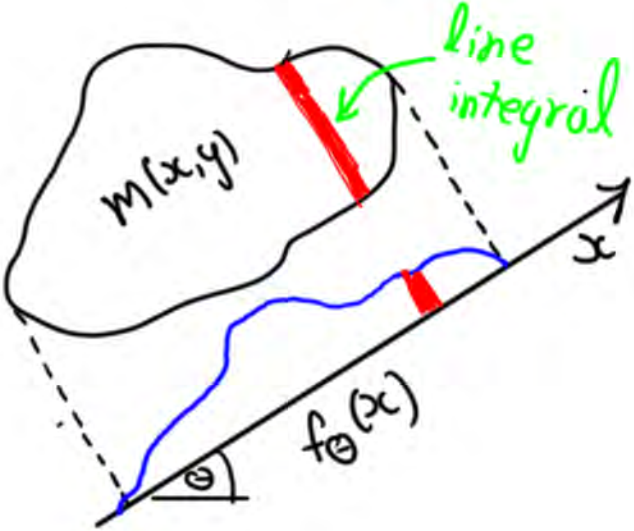
Try: $G_x = G \cos \theta_n \qquad \theta_n = \frac{\pi}{N} n \quad n = [0, N-1]$

$G_y = G \sin \theta_n$

The Central Section Theorem

(Projection slice)

R. Bracewell



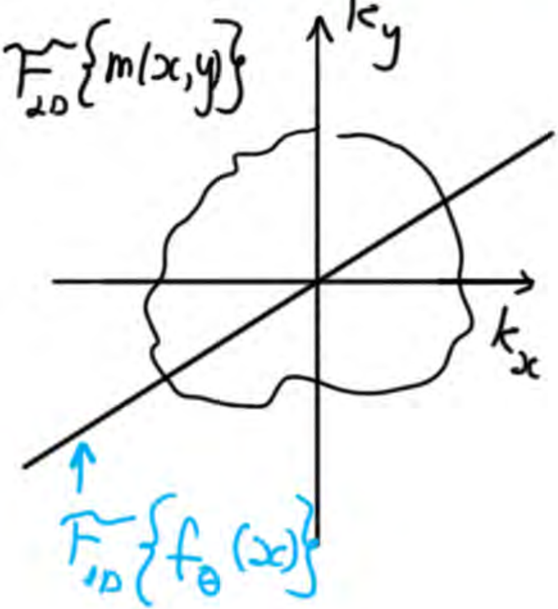


Image Space

k-space

Transform (1D) of projection of an object

$\Longrightarrow$ Diameter through transform (2D) of object

Very cool! ... and useful...

Projection slice $\Rightarrow$ basis for CT

Proof (for $\theta = 0$)

$$p(x) = \int_{-\infty}^{\infty} m(x,y)\, dy$$

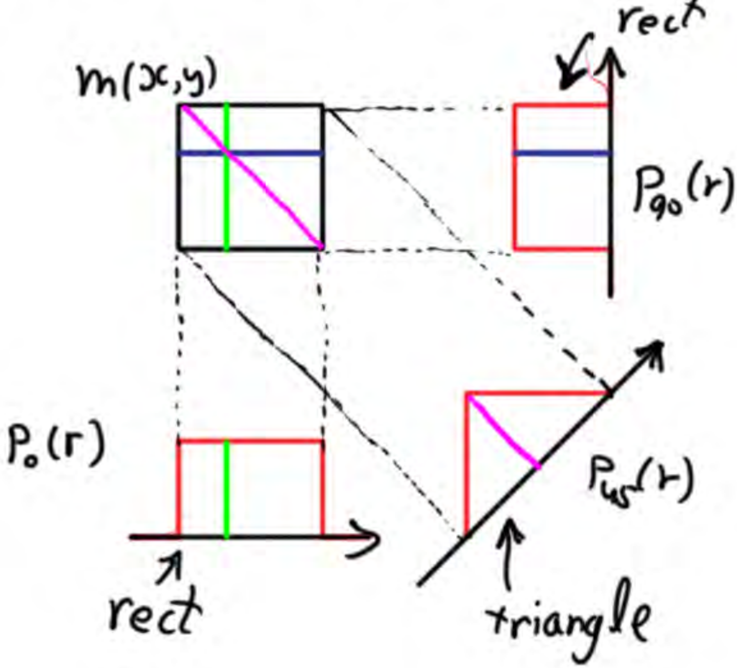
$$M(k_x, 0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\, e^{-i2\pi(k_x x + k_y y)}\, dx\, dy =$$

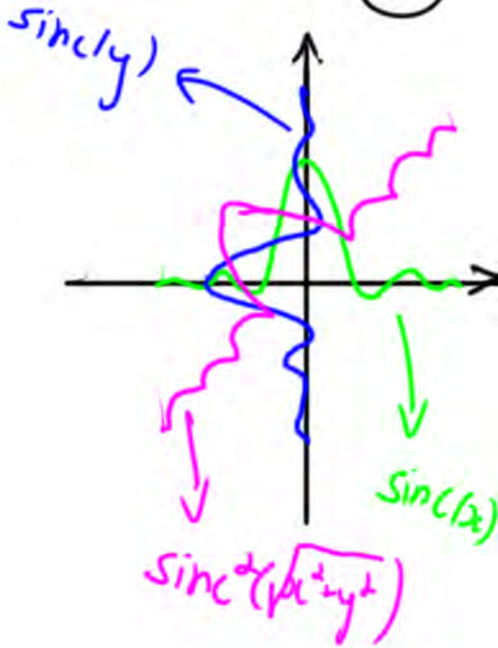
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\, e^{-i2\pi k_x x}\, dx\, dy =$$

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} m(x,y)\, dy \right] e^{-i2\pi k_x x}\, dx =$$

$$= \int_{-\infty}^{\infty} p(x)\, e^{-i2\pi k_x x}\, dx = \mathcal{F}\{p(x)\}$$

Let's see if it makes sense...





How to reconstruct?

(-) filtered back-projection

(-) Interpolation (in k-space)

Good class project

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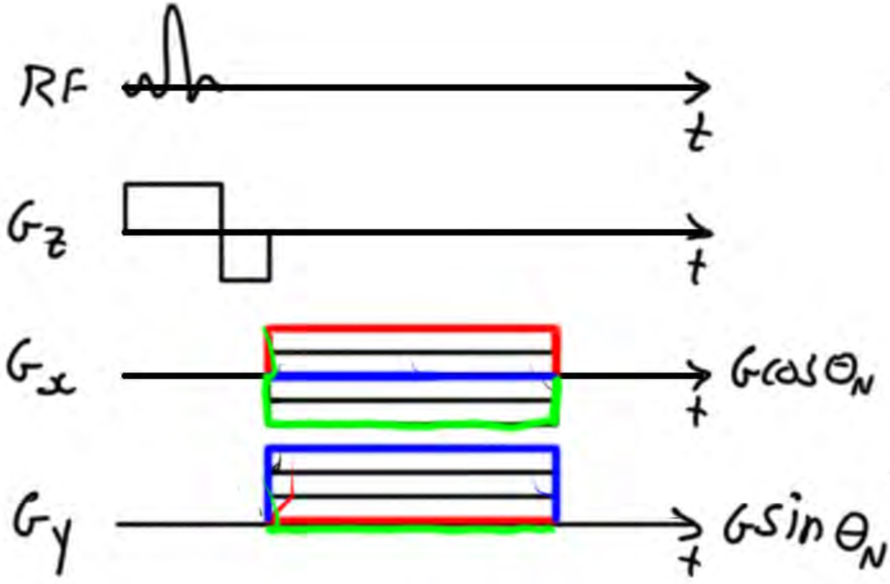
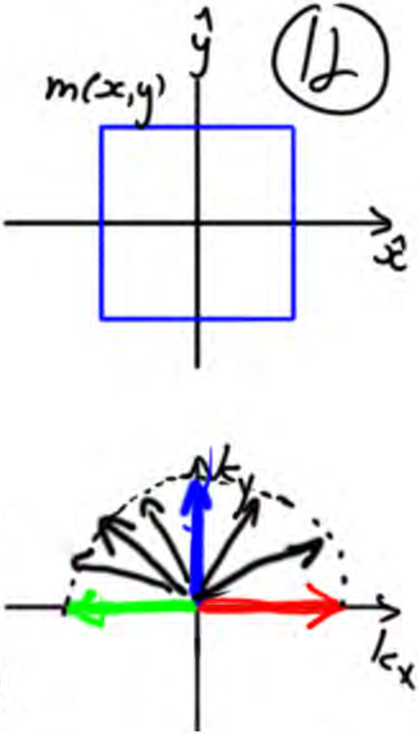


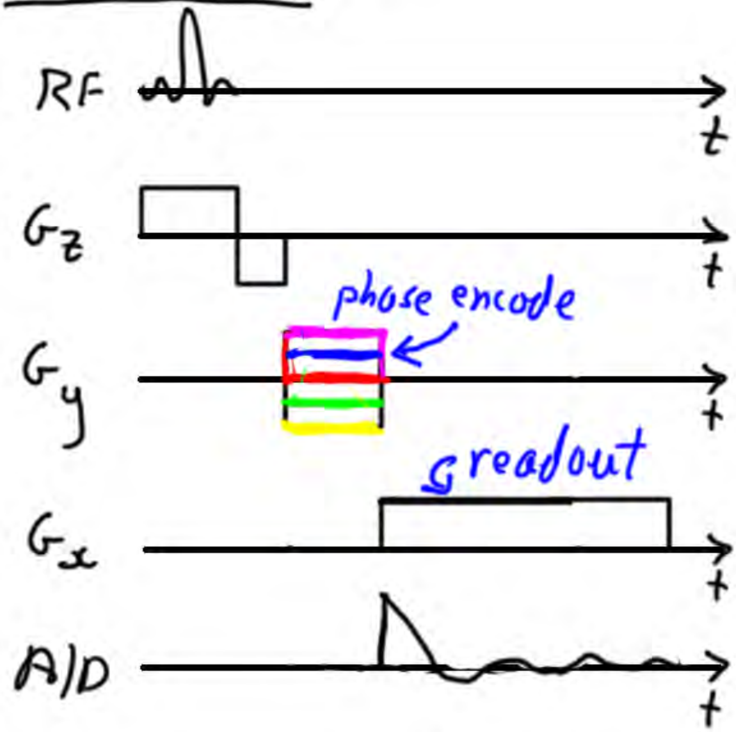


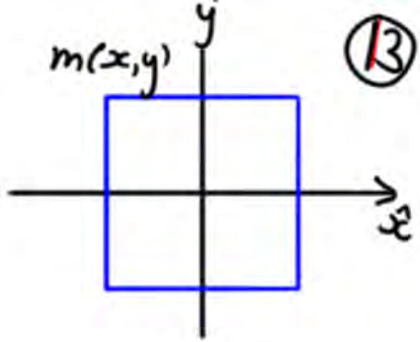
Projection Reconstruction (Lauterber)

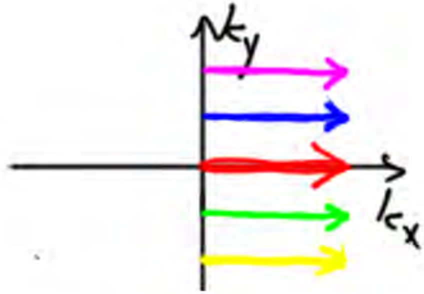
$\mathcal{F}_{1D}^{-1}\{\text{Diameter}\} \Rightarrow$ Projection

However... not quite diameter yet...

Better Idea







"2DFT"

"SPIN-WARP"

(still not perfect)

(-) Make "horizontal projections"

(-) Encode <u>phase</u> in y prior to <u>readout</u> in x

Other approaches to Imaging

(→) Localization is the key!

(1) B_0 localization

(→) saddle point

(→) local magnet

(2) RF

(→) Local coils

(→) Local Excitation

(3) Non linear gradients

Complex value & Demodulation

Physical signal:

$$S_p(t) = A(t)\cos(\omega_0 t + \phi(t)) =$$

real

$$\xRightarrow{\text{trigo}} \underbrace{A(t)\cos(\phi(t))}_{\substack{I(t) \\ \text{in phase}}}\cos(\omega_0 t) + \underbrace{\left[-A(t)\sin(\phi(t))\right]}_{\substack{Q(t) \\ \text{quadrature}}}\sin(\omega_0 t)$$

Analysis signal (convinient)

$$S_r = A(t)e^{-i\omega_0 t + \phi(t)}$$

complex

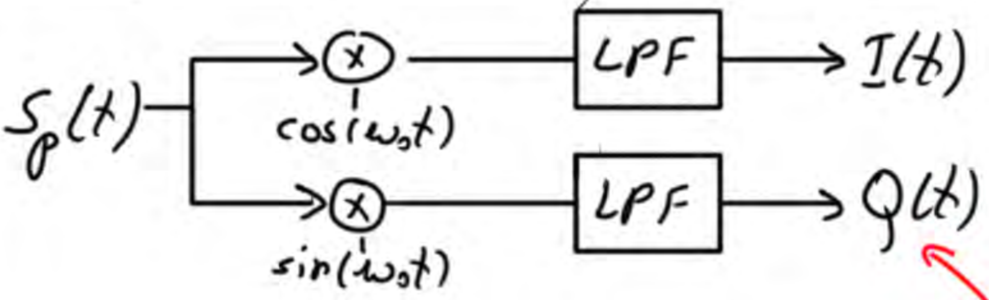
real

$$S_p = R_e\{S_r\}$$

Baseband signal

$$s(t) = S_r(t)e^{+i\omega_0 t} = A(t)e^{-i\phi(t)} = I(t) + iQ(t)$$

Quadrature Phase-Sensitive Detection:



often used in communication

$$s(t) = I(t) + i\,Q(t)$$

$I(t) \to M_x$ in rotating frame

$Q(t) \to M_y$ in rotating frame