Spin, Magnetic Moment and Magnetization

- Nuclei with Odd # protons/Neutrons possess spin angular momentum
  \[ S = \hbar \hat{I} \]

- Associated with \( S \) is a magnetic moment
  \[ \mu = \gamma \hat{S} = \gamma \hbar \hat{I} \]
Demonstration of Magnetic Resonance

Spin, Magnetic Moment and Magnetization

- In a strong magnetic field \( B_0 \), spins align with \( B_0 \) giving a net magnetization
- Magnetic Moment is produced

\[ M = \sum \mu \]
\[ E = -\mu \cdot B = -\mu_z B_0 \gamma L_z B_0. \]

Potential energy with no inverse component.

\[ S_z \text{ is quantized to } \hbar I_z \Rightarrow L_z = \pm \frac{1}{2} \]

(1) \(-6\hbar B_0 \frac{1}{2}\)

(2) \(6\hbar B_0 (\frac{1}{2})\)

\[ \Delta E = \frac{6}{\hbar} \hbar B_0. \]

**Zeeman Splitting**

No field singlet state

Two populations \( N_+ / N \)

\[ N_- / N_+ \]

Jump between states due to thermal energy

\[ \frac{N_-}{N_+} = e^{-\Delta E / kT} \approx 0.999993 \]

**Boltzmann Distribution**
Can be shown:

\[ M_0 = \frac{N \gamma^2 h I_z (I_z n)}{3kT} \]

1. Linear with field
2. Quadratic with \( \gamma \)
3. Linear with # Protons

Increase in signal-to-noise ratio of >10,000 times in liquid-state NMR

Solid material doped with unpaired electrons

 MW

P_e = 94% and P_C = 0.086%

Microwaves transfer the polarization from electrons to nuclei

3.35T, ~1.2K

Polarization

Temperature (K)

Proton

Electron

Carbon

Slide: Simon Hu, UCSF
Magnetism

Most objects display induced magnetism.

Apply field $\Rightarrow$ magnetic moment

$M_{\text{ind}} = \mu_0 V \times B$

- magnetic permeability
- volume
- magnetic susceptibility

$(\text{paramagnetism})$

$x > 0$

$(\text{diamagnetism})$

$ x < 0$

WHERE DOES IT COME FROM?

(i) Circulation of electric currents

(ii) Magnetic moment of $\mathcal{F}$

(iii) Magnetic moment of nucleii

$(i) + (ii) \gg (iii)$
(i) EXPLAINED BY CLASSICAL PHYSICS

CONTRIBUTES TO DIAMAGNETISM.

(ii) and (iii) INTRINSIC MAGNETISM

\[ \mathbf{p} = -\mathbf{S} \]

\[ I(\mathbf{e}) \gg I(\mathbf{p}) \]

EFFECT (ii) CANCELS IN MOST MATERIALS DUE TO PAIRED ELECTRONS
EFFECT (I) IS HUGE!
MACROSCOPICALLY, JUST CHANGES BULK MAGNETIC
MOMENT OR FIELD

\[ x_{\text{water}} = 9 \times 10^{-6} \quad x_{\text{iron}} = 4 \times 10^{-9} \]

No Precession / Precession

Diamagnetic Levitation example:

CHEMICAL SHIFT

BUT (I) HAS MICROSCOPIC EFFECT

\[ B_{\text{eff}} = B_0 - B_{\text{ax}} = B_0 (1 - x) \]

\[ \omega_{\text{eff}} = \omega_0 (1 - x) \]

\[ \delta \to \text{CHEMICAL SHIFT} = \frac{\Delta \omega - \Delta \omega_{\text{physical}}}{\Delta \omega_{\text{physical}}} \times 10^6 \]

FOR EXAMPLE:
ACETIC ACID \( CH_3COOH \)

\[ \begin{array}{c}
\text{O} \quad \text{C} \\
\text{H} - \text{C} - \text{O} - \text{H}
\end{array} \]

\[ \begin{array}{c}
\text{CH}_3 \\
\text{O} \quad \text{C} \\
\text{H} - \text{C} - \text{O} - \text{H}
\end{array} \]

DIRECTION IS HISTORICAL

OXGEN ATTRACTION ELECTRONS
PRECESSION

\[ \tau = I \times \alpha \]

\[ \text{Torque} = \frac{d\alpha}{dt} \]

\[ \text{change of angular momentum.} \]

\[ \frac{d\mathbf{L}}{dt} = \mathbf{F} \times \mathbf{L} \]

\[ \frac{d\mathbf{L}}{dt} = \mathbf{F} \times \mathbf{L} \]

Unit vector

\[ \frac{d\mathbf{L}}{dt} = \mathbf{F} \times \mathbf{L} \]

or

\[ \frac{d\mathbf{L}}{dt} = -\mathbf{B} \times \mathbf{I} \]

SOLUTION TO PRECESSION & \[ \theta = \frac{1}{2} \beta t \]

\[ \frac{d\mathbf{L}}{dt} = \mathbf{F} \times \mathbf{L} \]

\[ \frac{d\mathbf{L}}{dt} = \mathbf{F} \times \mathbf{L} \]

\[ \text{special distribution} \]

\[ \text{we have control!} \]