Followup on Homework

- Gradient non-Linearities

![Graph showing Actual Gradient and Linear Model]

- Frequency, kHz
- Position, z, cm
Followup on Homework

- Gradient non-Linearities

Before correction  Grad-warp correction
Followup on Last time

• Magnetization:

\[ M_0 = \frac{N \gamma^2 \hbar^2 I_z (I_z + 1) B_0}{3kT} \]
Increase in signal-to-noise ratio of >10,000 times in liquid-state NMR

Jan H. Ardenkjær-Larsen*, Björn Fridlund, Andreas Gram, Georg Hansson, Lennart Hansson, Mathilde H. Lerche, Rolf Servin, Mikkel Thaning, and Klaes Golman

Amersham Health Research and Development AB, Medeon, SE-205 12 Malmö, Sweden

Communicated by Albert W. Overhauser, Purdue University, West Lafayette, IN, June 20, 2003 (received for review April 16, 2003)

PNAS September 2, 2003 vol. 100 no. 18 10158-10163

Solid material doped with unpaired electrons

P₀ = 94% and Pₐ = 0.086%

Microwaves transfer the polarization from electrons to nuclei

3.35T, ~1.2K

electron

proton

carbon

Slide: Simon Hu, UCSF
Precession

• Recall from Last time:

\[ \vec{S} + \vec{\mu} + \vec{B} \Rightarrow \text{Precession} \]

\[ \frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \]

Solution: \( \omega = \gamma |B| \)
Principles of MRI
EE225E / BIO265

Lecture 09

Instructor: Miki Lustig
UC Berkeley, EECS
Mathematical Description of MRI

• Three Elements:
  - Precession about $\tilde{B}$ (all fields)
  - Transverse decay
  - Longitudinal recovery
Mathematical Description of MRI

• Plan:

  1) Derive Math for each element
  2) Put together: e.g., the BLOCH equation
  3) Solve the Bloch eqn. for special cases
     a) Excitation CH. 6 (later)
     b) Reception CH. 5 (first)
        i) Derive k-space (AGAIN!!)
        ii) Pulse sequence
        iii) Sampling
Precession

- We apply fields: $B_0$, $B_1$, $G$
Precession

Magnetization is:

\[ \vec{M} = [M_x, M_y, M_z]^T \]

- \( \vec{M} \) precesses around \( \vec{B} \)
- Frequency of rotation is
  \[ \omega = \gamma |\vec{B}| \]
- Axis of rotation is
  \[ \vec{n} = \frac{\vec{B}}{|\vec{B}|} \]

M. Lustig, EECS UC Berkeley
Precession

LAB FRAME

\[ \mathbf{B}_0 + \mathbf{G} \times \mathbf{L} \]

\[ \mathbf{B}_L(t) \quad "\text{small}" \]

\[ \text{total} \]

\[ \text{magn.} \]
Precession

- Described by cross product
  \[
  \frac{d\vec{M}}{dt} = -\gamma \vec{B} \times \vec{M}
  \]

- "-" Due to negative gyromagnetic ratio of protons

  or:
  \[
  \frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}
  \]

- \(B_0\) Dominates! Hard to see other terms
Rotating Frame

- Change coordinates:

\[
[\hat{i}_r, \hat{j}_r, \hat{k}_r]^T = [\hat{i} \cos \omega_0 t, \hat{j} \sin \omega_0 t, \hat{k}]^T
\]

- In the rotating frame at \( w_0 \):
Rotating Frame

• For \( \omega = \gamma B_0 \), MAIN FIELD GOES AWAY!

\[ \vec{B}_{\text{rot}} = \vec{B} - \vec{x} \hat{r} + B_{1,x} \hat{i} + B_{1,y} \hat{j} \]

Much simpler!

• \( \vec{M}_{\text{rot}} \) precesses about \underline{applied fields} \( \vec{B} \cdot \vec{x} \) and \( B_{1,x}, B_{1,y} \)
Examples

Excitation

\[ \omega = \gamma |B_1| \]

Precession

\[ \omega = \gamma |\vec{G} \cdot \vec{x}| \]
Examples

\( \omega = \sqrt{B_1 + \vec{\omega}^2} \)
Relaxation

• **T2 Decay**
  - Transverse magnetization decays
  - Due to loss of coherence between spins
  - Also called spin-spin relaxation
  - Not a strong function of $B_0$
  - Dipole effect stronger in solids
Transverse Relaxation (T2)

• Let

\[ M_{xy} = M_x + iM_y \]

• Then

\[ \frac{dM_{xy}}{dt} = -\frac{1}{T_2} M_{xy} \]
Transverse Relaxation (T2)

- Solution:

\[ M_{xy}(t) = e^{-\frac{t}{T_2}} M_{xy}(0) \]

Major source of contrast
Transverse Relaxation (T2)

- Example: Brain @ 1.5T
  - white matter T2=92ms, Density=0.65
  - gray matter T2=100ms, Density = 0.75

Excite, wait 100ms, collect data
T2 Example

Gray matter lighter
white matter darker
CSF Bright!
Magic Angle \(~\text{55 degrees}\)

- Longer T2 due to dipole decoupling
Relaxation

- **T1 Recovery**
  - Longitudinal relaxation
  - Due to Spin-Lattice interaction
  - Thermal bouncing of molecules - lose cone of precession - align with field
  - Strong dependency on $B_0$, since energy level depends on $B_0$
  - $B_0$ strong - hard to transition - T1 long
T1 Recovery

- Cone of Precession
- Tumbling
- Precession & wondering

- Bias towards up - stable anisotropic dist.
T1 Recovery

- Magnetization recovers to equilibrium $M_0$

\[
\frac{dM_z}{dt} = -\frac{M_z - M_0}{T_1}
\]

- Solution:

\[
M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1}
\]

M. Lustig, EECS UC Berkeley
T1 Recovery

After 90° pulse, $M_z = 0$

$$M_z(t) = M_0 - M_0 e^{-\frac{t}{T_1}} = M_0 (1 - e^{-\frac{t}{T_1}})$$

Major source of contrast as well.
T1 Contrast

• Brain at 1.5T
  - Gray Matter T1 = 900ms
  - White Matter T1 = 800ms

Excite 90, wait, excite again, image....
T1 Contrast Example

Gray matter darker

white matter lighter

CSF Dark!

Fat Bright!
Relaxation
The Bloch Equation

- Combine Precession and relaxation

\[ \frac{d \vec{M}}{dt} = -\gamma \vec{B} \times \vec{M} - \frac{M_x \hat{i} + M_y \hat{j}}{T_2} - \frac{M_z - M_0 \hat{k}}{T_1} \]

- Phenomenological: Fits observations
  - Describes most of MRI
  - Sometimes Fails.... J-coupling, Magn, transfer