Followup on Homework

- Gradient non-Linearities

![Graph showing actual gradient and linear model](image1)

Followup on Last time

- Magnetization:

\[ M_0 = \frac{N \gamma^2 \hbar^2 I_z (I_z + 1) B_0}{3kT} \]

where:
- \( M_0 \) is the magnetization
- \( N \) is the number of spins
- \( \gamma \) is the gyromagnetic ratio
- \( \hbar \) is the reduced Planck constant
- \( I_z \) is the spin quantum number
- \( B_0 \) is the magnetic field strength
- \( k \) is the Boltzmann constant
- \( T \) is the temperature

The graph shows the relationship between position and frequency, with linear and quadratic components.

Increase in signal-to-noise ratio of >10,000 times in liquid-state NMR

![Graph showing signal-to-noise ratio](image2)

The polarization of nuclei and electrons is shown, with a peak at 3.35T (~1.2K) for polarization transfer from electrons to nuclei.

Slide: Simon Hu, UCSF
Precession

• Recall from Last time:

\[ \vec{S} + \vec{\mu} + \vec{B} \Rightarrow \text{Precession} \]

Solution: \( \omega = \gamma |\vec{B}| \)

\[ \frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \]

Mathematical Description of MRI

• Three Elements:
  - Precession about \( \vec{B} \) (all fields)
  - Transverse decay
  - Longitudinal recovery

Principles of MRI

EE225E / BIO265

Lecture 09

Instructor: Miki Lustig
UC Berkeley, EECS

Mathematical Description of MRI

• Plan:
  1) Derive Math for each element
  2) Put together: e.g., the BLOCH equation
  3) Solve the Bloch eqn. for special cases
     a) Excitation CH. 6 (later)
     b) Reception CH. 5 (first)
        i) Derive k-space (AGAIN!!!)
        ii) Pulse sequence
        iii) Sampling
Precession

• We apply fields: $B_0, B_1, G$

Precession

Magnetization is:

$$\vec{M} = [M_x, M_y, M_z]^T$$

• $\vec{M}$ precesses around $\vec{B}$
• Frequency of rotation is
  $$\omega = \gamma |\vec{B}|$$
• Axis of rotation is $\vec{\Omega} \times \vec{B}$

Precession

• Described by cross product
  $$\frac{d\vec{M}}{dt} = -\gamma \vec{B} \times \vec{M}$$
• “-” Due to negative gyromagnetic ratio of protons
  or:
  $$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$
• $B_0$ Dominates! Hard to see other terms
Rotating Frame

- Change coordinates:
  \[ [\hat{i}_r, \hat{j}_r, \hat{k}_r]^T = [\hat{i} \cos \omega_0 t, \hat{j} \sin \omega_0 t, \hat{k}]^T \]

- In the rotating frame at \( \omega_0 \):
  \[ \hat{\mathbf{r}} = \begin{bmatrix} \hat{i} \cos \omega_0 t, \hat{j} \sin \omega_0 t, \hat{k} \end{bmatrix} \]

Examples

- Excitation
  \[ \omega = \gamma |B_1| \]

- Precession
  \[ \omega = \gamma |\vec{G} \cdot \hat{z}| \]

Examples

- For \( \omega_0 = \gamma B_0 \) MAIN FIELD GOES AWAY
  \[ \vec{B}_{\text{rest}} = \vec{G} \cdot \hat{z} \hat{k}_r + B_{1z} \hat{i}_r + B_{1y} \hat{j}_r \]
  Much simpler!

- \( \vec{M}_{\text{rot}} \) precesses about \( \text{applied fields} \)
  \( \vec{G} \cdot \hat{z} \) and \( B_{1z}, B_{1y} \)
Relaxation

- **T2 Decay**
  - Transverse magnetization decays
  - Due to loss of coherence between spins
  - Also called spin-spin relaxation
  - Not a strong function of $B_0$
  - Dipole effect stronger in solids

Transverse Relaxation (T2)

- Let
  \[ M_{xy} = M_x + iM_y \]

- Then
  \[ \frac{dM_{xy}}{dt} = -\frac{1}{T_2} M_{xy} \]

Transverse Relaxation (T2)

- Solution:
  \[ M_{xy}(t) = e^{-\frac{t}{T_2}} M_{xy}(0) \]

- Example: Brain @ 1.5T
  - white matter $T_2=92\text{ms}$, Density=0.65
  - gray matter $T_2=100\text{ms}$, Density = 0.75

  Excite, wait 100ms, collect data
T2 Example

Gray matter lighter

white matter darker

CSF Bright!

Magic Angle ~55 degrees

- Longer T2 due to dipole decoupling

Relaxation

- **T1 Recovery**
  - Longitudinal relaxation
  - Due to Spin-Lattice interaction
  - Thermal bouncing of molecules - lose cone of precession - align with field
  - Strong dependency on $B_0$, since energy level depends on $B_0$
  - $B_0$ strong - hard to transition - T1 long

T1 Recovery

- Bias towards up - stable anisotropic dist.
T1 Recovery

- Magnetization recovers to equilibrium $M_0$

$$\frac{dM_z}{dt} = -\frac{M_z - M_0}{T_1}$$

- Solution:

$$M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1}$$

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T1 Recovery

After 90° pulse, $M_2 = 0$

$$M_2(t) = M_0 - M_0 e^{-\frac{t}{T_1}} = M_0 (1 - e^{-\frac{t}{T_1}})$$

Major source of contrast as well.

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T1 Contrast

- Brain at 1.5T
  - Gray Matter T1 = 900ms
  - White Matter T1 = 800ms

Excite 90, wait, excite again, image....

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T1 Contrast Example

Gray matter darker

white matter lighter

CSF Dark!

Fat Bright!

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Relaxation

The Bloch Equation

- Combine Precession and relaxation

\[
\frac{d\vec{M}}{dt} = -\gamma \vec{B} \times \vec{M} - \frac{M_x \hat{i} + M_y \hat{j}}{T_2} - \frac{M_z - M_0 \hat{k}}{T_1}
\]

- Phenomenological: Fits observations
  - Describes most of MRI
  - Sometimes Fails.... J-coupling, Magn, transfer