LAST TIME

BLOCH EQN. FOR RECEPTION

$$\begin{pmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{pmatrix} = \begin{pmatrix} -\frac{1}{T_2} & \gamma \vec{G}\cdot\vec{r} & 0 \\ -\gamma \vec{G}\cdot\vec{r} & -\frac{1}{T_2} & 0 \\ 0 & 0 & -\frac{1}{T_1} \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{T_1} \end{pmatrix} M_0$$

TRANSVERSE MAGNETIZATION DECOUPLES $M_{xy} = M_x + iM_y$

$$\dot{M}_{xy} = \left(-\frac{1}{T_2} - i\gamma \vec{G}(t)\cdot\vec{r}\right) M_{xy}$$

$$M_{xy}(\vec{r},t) = M_{xy}(\vec{r},0)\, e^{-\frac{t}{T_2}}\, e^{-i2\pi \vec{k}(t)\cdot\vec{r}}$$

WHERE $k(t) = \frac{\gamma}{2\pi}\int_0^t \vec{G}(s)\,ds$

K-SPACE TRANSFORM

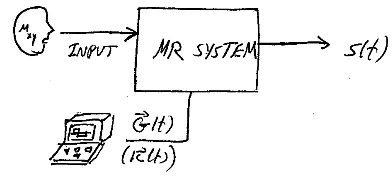
RECIEVED SIGNAL

$$s(t) = \int_{\vec{R}} M_{xy}(\vec{r},0)\, e^{-\frac{t}{T_2}}\, e^{-i2\pi \vec{k}(t)\cdot\vec{r}}\, d\vec{r}$$

$$\simeq \int_{\vec{R}} M_x(\vec{r},0)\, e^{-i2\pi \vec{k}(t)\cdot\vec{r}}\, d\vec{r}$$

(AS LONG AS $T_2 \gg t$)

$$s(t) = \mathcal{F}\left\{ M(\vec{r},0) \right\}\Big|_{k(t)}$$



WE CONTROL $\vec{G}(t)$ $(\vec{k}(t))$, CHOOSE WISELY SO,

WE CAN GET $M_{xy}(\vec{R})$ from $s(t)$

$$|G| < G_{max} \quad (\sim 4 \tfrac{G}{cm}) \Rightarrow \left|\frac{\partial k(t)}{\partial t}\right| < \frac{\gamma}{2\pi} G_{max}$$

LIMITED K-SPACE VELOCITY

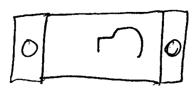
$$\left|\frac{\partial G}{\partial t}\right| < S_{max} \quad \left(\sim 15 \tfrac{G}{cm/ms}\right) \left(150\ T/m/s\right) (SR\ 150) \approx 60\ ms\ RT$$

$\Downarrow$

$$\left|\frac{\partial^2 k(t)}{\partial t^2}\right| < \frac{\gamma}{2\pi} S_{max}$$ LIMITED K-SPACE ACCELERATION

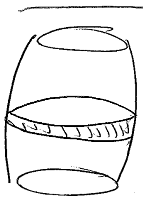
$\vec{k}(t)$ IS SMOOTH, CONTINUOUSE TRAJECTORY

LIKE ETCH-SKETCH



WHAT SHOULD WE CHOOSE?

SLICE SELECTIVE 2D EXAMPLE



SLICE SELECT IN Z

RESOLVE IN X, Y

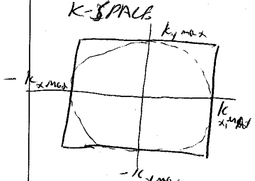
NEED $G_x(t)$, $G_y(t)$

AQUIRE $k_x(t)$, $k_y(t)$

$$s(t) = \iiint_{x\,y\,z} M_{xy}(\vec{r},0)\, e^{-\frac{t}{T_2}}\, e^{-i2\pi \vec{k}(t)\cdot\vec{r}}\, d\vec{r}$$

$$k_z(t) = 0 \Rightarrow \iint_{x\,y}\left[\int_z M_{xy}(\vec{r},0)\,dz\right] e^{-\frac{t}{T_2}}\, e^{-i2\pi \vec{k}(t)\cdot r}\, d\vec{r}$$

K-SPACE TRAJECTORY SHOULD COVER REGION IN K-SPACE



EXTENT $\Rightarrow$ RESOLUTION

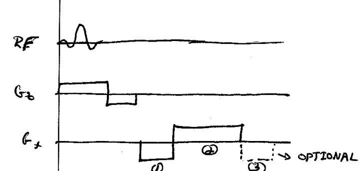
DENSITY $\Rightarrow$ FOV (NEXT TIME)

OFTEN SQURE

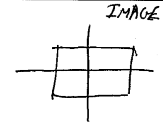
SOMETIMES CIRCLE

WHAT ARE SOME OPTIONS?

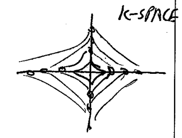
PROJECTION RECONSTRUCTION



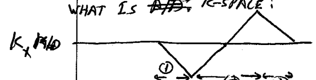
IMAGE

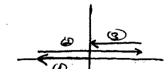


K-SPACE

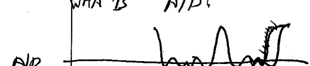


WHAT IS ~~A/D~~ K-SPACE?



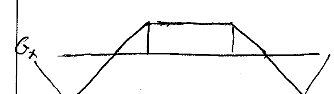


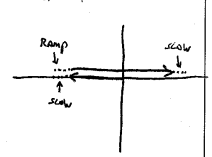
WHAT IS A/D?

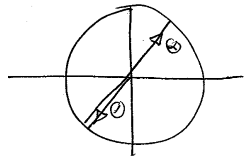


(-) BOX GRADIENTS VIOLATE $\left|\frac{\partial G(t)}{dt}\right| < S_{max}$

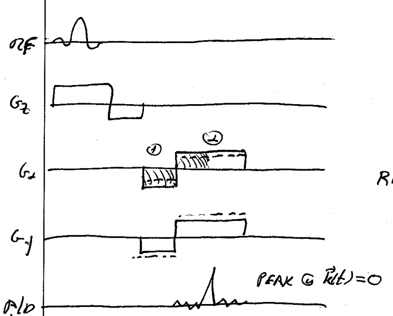
REALISTIC GRADIENT:





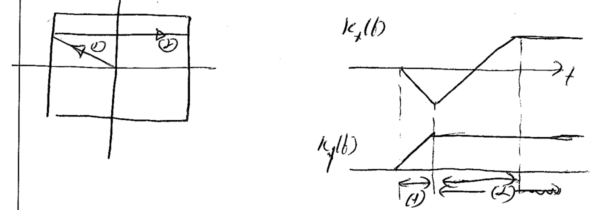


PULSE SEQUENCE

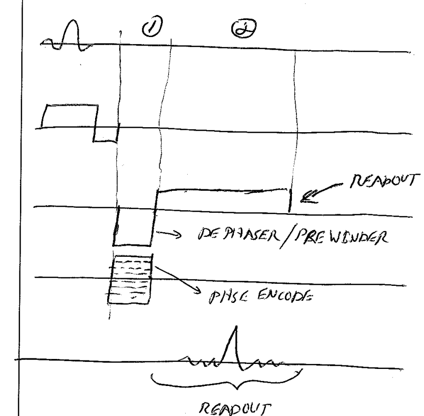


REPEAT FOR EACH DIAMETER

2DFT - (SPIN WARP)



PULSE SEQUENCE



REPEAT FOR EACH LINE

(-) BY FAR THE MOST COMMON!
(-) CALLED 2DFT, SPIN-WARP

G_y Gradient Is PHASE ENCODE

ESTABLISHES FIXED LINEAR PHASE BEFORE READOUT

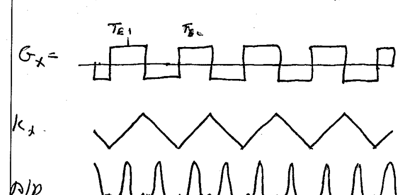
$$M_{xy}(\vec{r},0)e^{-i2\pi k_y(t_1)y}$$

G_x GRADIENT IS FREQUENCY ENCODE OR, READOUT GRADIENT

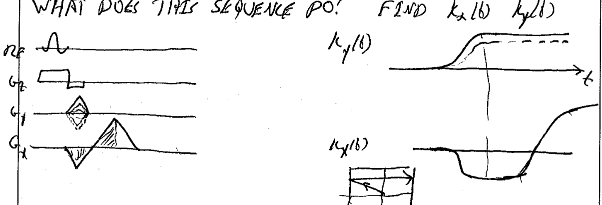
$$M_{xy}(\vec{r},0)\underbrace{e^{-i2\pi k_y(t_1)y}}_{\text{Fixed}}\,e^{-i2\pi k_x(t)x}$$

$$k_x(t)=\frac{\gamma}{2\pi}\int G_x(\tau - t_e)\,d\tau$$

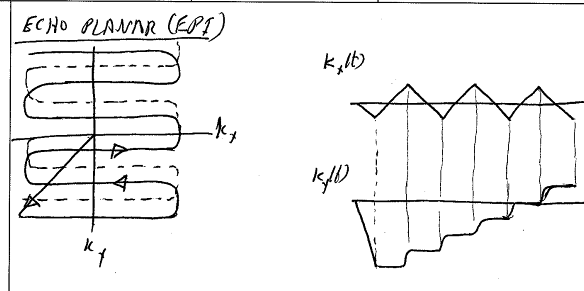
T_e IS THE TIME FOR WHICH $k_x=0$



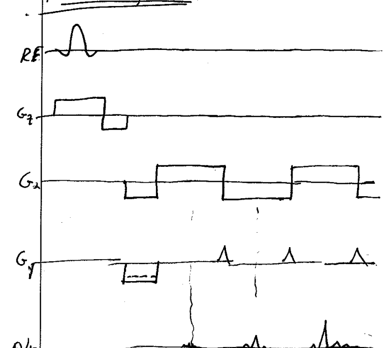
WHAT DOES THIS SEQUENCE DO? FIND $k_x(t)$ $k_y(t)$



ECHO PLANAR (EPI)

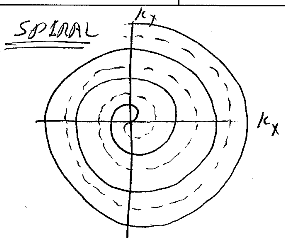


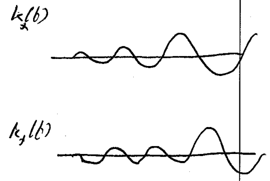
PULSE SEQUENCE



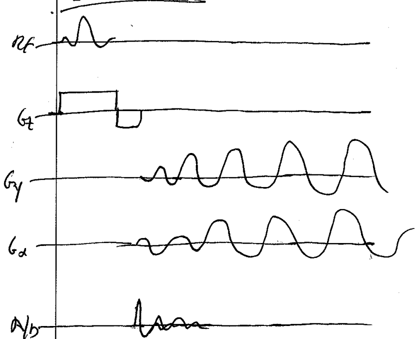
(-) MOST COMMON HIGH-SPEED ACQ. fMRI, DIFFUSION, REALTIME
(-) CAN USE MULTIPLE SHOTS

SPIRAL





PULSE SEQUENCE



VERY H/W EFFICIENT

USEFUL FOR FLOW / HEARTS (USED FOR fMRI as well)

OFTEN MULTIPLE SHOTS. (INTERLEAVES)

SAMPLING, RESOLUTION, AND FOV

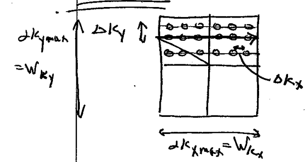
SIGNAL IS SAMPLE OF FT OF OBJECT

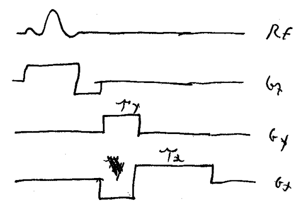
$$s(t) = M_{xy}(\vec{k}(t)) = \mathcal{F}\{M_{xy}(\vec{r},0)\}\big|_{k(t)}$$

SAMPLED AT DISCRETE TIMES!

$$s(n\Delta t) = M_{xy}(\vec{k}(n\Delta t))$$

2DFT CASE





$$k_{y,max} = \frac{\gamma}{2\pi} G_y \tau_y$$

$$2k_{x,max} = \frac{\gamma}{2\pi} G_x \tau_x \qquad W_{ky} = 2k_{y,max}$$

$$k_{x,max} = \frac{\gamma}{2\pi} G_x \left(\frac{\tau_x}{2}\right) \qquad W_{kx} = 2k_{x,max}$$

N_r – READOUT SAMPLES

N_p – PHASE ENCODES

$$\Delta k_x = \frac{2k_{x,max}}{N_r} = \frac{W_{kx}}{N_r}$$

$$\Delta k_y = \frac{2k_{y,max}}{N_p} = \frac{W_{ky}}{N_p}$$

SAMPLED FOURIER TRANSFORM

$$\hat{M}_{xy}(k_x,k_y) = \underbrace{M(k_x,k_y)}_{\text{FT of object}} \underbrace{\left[\frac{1}{\Delta k_x \Delta k_y}\, {}^2\text{III}\left(\frac{k_x}{\Delta k_x},\frac{k_y}{\Delta k_y}\right)\right]}_{\text{SAMPLING}} \cdot \underbrace{{}^2\Pi\left(\frac{k_x}{W_{kx}},\frac{k_y}{W_{ky}}\right)}_{\text{Extent}}$$

SAMPLED IMAGE

$$\hat{M}_{x,y}(x,y) = \underbrace{M_{xy}(x,y)}_{\text{OBJECT}} ** \underbrace{{}^2\text{III}(\Delta k_x x, \Delta k_y y)}_{\text{REPLICATION/ALIASING}} ** \underbrace{W_{kx}W_{ky}\,\text{sinc}(W_{kx}x)\,\text{sinc}(W_{ky}y)}_{\text{RESOLUTION}}$$

FIELD OF VIEW AND ALIASING

ASSUME INFINITE RESOLUTION (For Now...)

$$W_{kx}W_{ky}\,\text{sinc}(\,)\,(\text{sinc}(\,) = {}^2\delta(x,y)$$

THEN

$$\hat{M}_{xy}(x,y) = M_{xy}(x,y) ** {}^2\text{III}(\Delta k_x x, \Delta k_y y)$$

${}^2\text{III}(\Delta k_x x, \Delta k_y y)$ HAS AN IMPULSE WHEN

$$\Delta k_x x = n \qquad x = \frac{n}{\Delta k_x}$$

AND

$$\Delta k_y y = m \qquad y = \frac{m}{\Delta k_y}$$

when n,m are integers.

TO PREVENT ALIASING $M_{xy}(x,y)$ SHOULD BE ZERO FOR

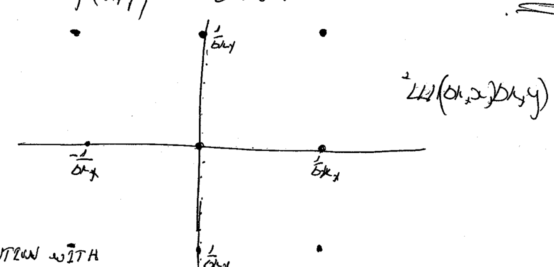
$$|x| > \frac{1}{2\Delta k_y} \qquad |y| > \frac{1}{2\Delta k_y}$$

FIELD OF VIEW IS EXTENT IN x,y

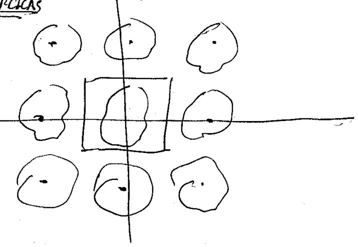
$$FOV_x = \frac{1}{\Delta k_x}$$

$$FOV_y = \frac{1}{\Delta k_y}$$

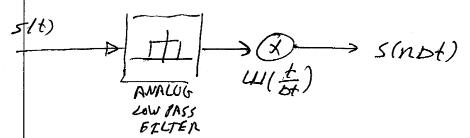
IF $M_{xy}(x,y)$ IS LARGER $\rightarrow$ ALIASING CAN OCCUR

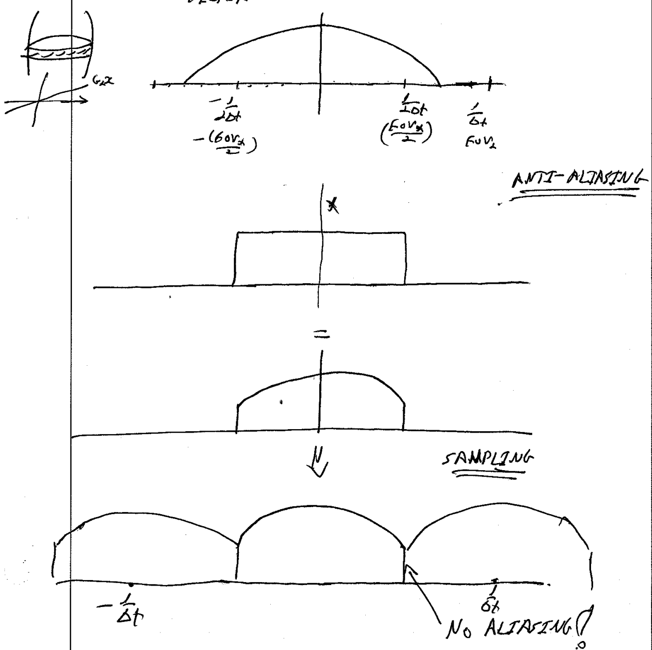


CONVOLUTION WITH M_{xy} PRODUCES REPLICAS



IN PRACTICE, ALIASING IN x DOESN'T OCCUR



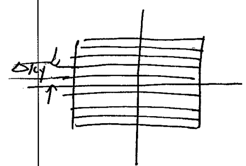


In the Readout, INCREASE ~~F~~OV BY INCREASING SAMPLING RATE

No CHANGE IN ACQUISITION GRADIENTS.
JUST MORE DATA

FOV_x IS FREE (Almost) NO ALIASING!

ALIASING DOES OCCUR IN y



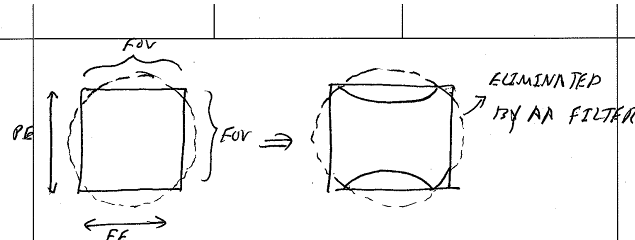
PHASE ENCODES ARE FUNDAMENTALLY DISCRETE.

FOR FIXED RESOLUTION (W_{ky})

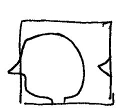
$$FOV \uparrow \Rightarrow \Delta k_y = \frac{W_{ky}}{N_p} \Rightarrow N_p \uparrow$$

MORE ACQUISITIONS $\Rightarrow$ LONGER SCAN TIME

FOV_y COSTS SCAN TIME



IF YOU SEE ALIASING YOU KNOW IT IS PHASE ENCODE DIRECTION (OTHER CLUES TOO)



$$ш(\Delta k_x x, \Delta k_y y) = ш\left(\frac{x}{FOV_x}, \frac{y}{FOV_y}\right)$$