**LAST TIME**

**SUCH EQU. FOR RECEPTION**

\[
\begin{bmatrix}
M_y \\
M_x \\
M_z
\end{bmatrix} =
\begin{bmatrix}
-\gamma_0 \vec{B}_0 & 0 & 0 \\
0 & -\gamma_0 \vec{B}_0 & 0 \\
0 & 0 & \gamma_0 \vec{B}_0
\end{bmatrix}
\begin{bmatrix}
M_y \\
M_x \\
M_z
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
T
\end{bmatrix}
\]

**TRANVERSE MAGNIFICATION DECOUPLES**

\[
M_y = M_z = M_x
\]

\[
M_y = (-\frac{1}{2} - i \gamma_0 \vec{B}_0 \tau)M_y
\]

\[
M_y(p, t) = M_y(p, 0) e^{\frac{1}{2} \gamma_0 \vec{B}_0 p \tau} e^{-i \omega_0 \vec{B}_0 p \tau}
\]

**WHERE**

\[
k(t) = \frac{-\gamma_0 \vec{B}_0}{2} e^{i \omega_0 \vec{B}_0 p \tau}
\]

**K-SPACE TRANSFORM**

**RECEIVED SIGNAL**

\[
S(t) = \sum_{k} M_y(p, 0) e^{i \gamma_0 \vec{B}_0 k \tau} e^{-i \omega_0 \vec{B}_0 p \tau}
\]

**SHEET SELECT AND EXAMPLE**

**SLICE SELECT IN Z**

**RESOLVE IN X, Y**

**NEED G_x(t), G_y(t)**

**ACQUIRE k_x(\tau), k_y(\tau)**

\[
S(t) = \int \int \int M_y(p, 0) e^{i \gamma_0 \vec{B}_0 k \tau} e^{-i \omega_0 \vec{B}_0 p \tau} d^2 p
\]

\[
k_x(t) = 0 \Rightarrow \int \int M_y(p, 0) e^{i \gamma_0 \vec{B}_0 k \tau} e^{-i \omega_0 \vec{B}_0 p \tau} d^2 p
\]

**K-SPACE TRAJECTORY SHOULD COVER REGION IN K-SPACE**

**EXTENT OF RESOLUTION DEPENED ON MANY TIMES OF WIDTH**

**OTHER EXAMPLES**

**WHAT ARE SOME OPTIONS?**

---

**PROJECTION RECONSTRUCTION**

**IMAGE**

**K-SPACE**

**WHAT IS THE K-SPACE?**

**WHAT IS A/P?**

**A/P BOX GRADIENT VIOLATE**

\[
|\frac{dG}{dt}| < \frac{1}{2} \omega_{max}
\]

**REALISTIC GRADIENT**

\[
G
\]
**Pulse Sequence**

**$G_y$ Gradient**

**Phase Encode**

*Encoding fixed linear pie before readout*

$M_{xy}(t,0) = e^{-i\omega t/\Delta t}$

**$G_z$ Gradient**

**Frequency Encode or**

*Readout Gradient*

$M_{xy}(t,0) = e^{-i\omega t/\Delta t}$

Find $k_{xy,y} = \frac{1}{\Delta t} \left( g_0 + \tau_0 \right)$

$\tau_0$ is the time for which $k_{xy} = 0$

**$G_z$**

**$k_2$**

**$k_1$**

**Readout**

*What does this sequence do? Find $k_{xy}$ and $k_{y1}$*

**$k_{xy,y}$**

*Most common high-speed read. Phase diffusion, resetting*

*(Can use multiple shots*)
Sampled Fourier Transform

$$\hat{M}_y(k_x, k_y) = M_y(x,y) \hat{W}(k_x, k_y)$$

where

$$\hat{W}(k_x, k_y) = \frac{1}{\sqrt{N_x N_y}} \sum_{x=0}^{N_x-1} \sum_{y=0}^{N_y-1} w(x,y) e^{-j 2\pi (k_x x + k_y y)}$$

Field of View and Aliasing

Assume infinite resolution (for now...)

$$W_0 W_0 \text{sinc}(\cdot) \text{sinc}(\cdot) \to \text{Dirac}\delta$$

Then

$$\hat{M}_y(x,y) = M_y(x,y) \ast \hat{W}(k_x, k_y)$$

$$\hat{W}(k_x, k_y)$$ has an impulse when

$$k_x = \frac{n}{N_x}$$

and

$$k_y = \frac{m}{N_y}$$

where $$n, m$$ are integers.

Signal is sample of FT of object

$$s(n) = M_y(R(n)) = \hat{F}\{M_y(R,\rho, \phi)\}(n)$$

Sampled at discrete times

$$s(n \delta t) = M_y(R(n \delta t))$$

SPECT CASE

$$k_{y, max} = \frac{N_x}{2}$$

$$k_{x, max} = \frac{N_y}{2}$$

To prevent aliasing, $$M_y(x,y)$$ should be zero for

$$|x| > \frac{N_x}{2}$$

$$|y| > \frac{N_y}{2}$$

Field of View is extent in $$x,y$$

$$\text{FOV}_x = \frac{A}{k_x}$$

$$\text{FOV}_y = \frac{A}{k_y}$$

If $$M_y(x,y)$$ is larger $$\Rightarrow$$ aliasing can occur

$$\hat{W}(k_x, k_y)$$

Compensation with $$M_y$$ produces $$\text{ALIAS}$$
In practice, aliasing in $x$ does not occur.

In the readout, increase $f_{OV}$ by increasing sample rate.

No change in acquisition gradients. Just move data.

If $f_{OV}$ is free (as it is) no aliasing.

Aliasing does occur in $y$.

Phase encodes are fundamentally discrete.

For fixed resolution ($W_{y1}$),

$FOV \uparrow \implies \Delta x = \frac{W_{y1}}{N_{y}} \implies N_{y} \uparrow$

More acquisitions $\implies$ longer scan time.

More acquisitions $\implies$ longer scan time.

If you see aliasing, you know it is phase encode direction (other clues too).

$\bar{L_{xy}}(\Delta k_{x}, \Delta k_{y}) = L_{xy}(\frac{\Delta k_{x}}{N_{x}}, \frac{\Delta k_{y}}{N_{y}})$