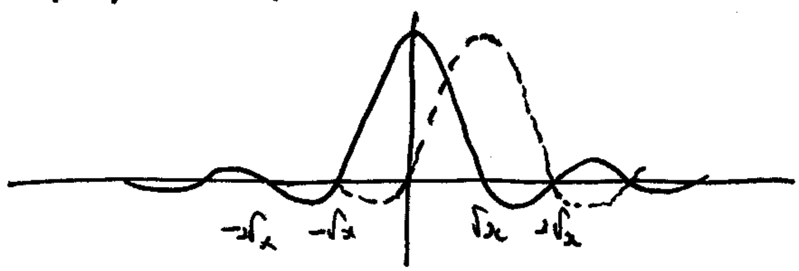
SPATIAL RESOLUTION

$$\hat{M}_{xy}(x,y) = M_{xy}(x,y) ** \underbrace{W_{ky} W_{kx} \,\mathrm{sinc}(W_{kx})\, \mathrm{sinc}(W_{ky})}_{\text{RESOLUTION}}$$

$$** \underbrace{Ш\left(\frac{x}{FOV_x}, \frac{y}{FOV_y}\right)}_{\text{REPLICATION}}$$

IMPULSE RESPONSE:



$$W_{kx} x = n$$

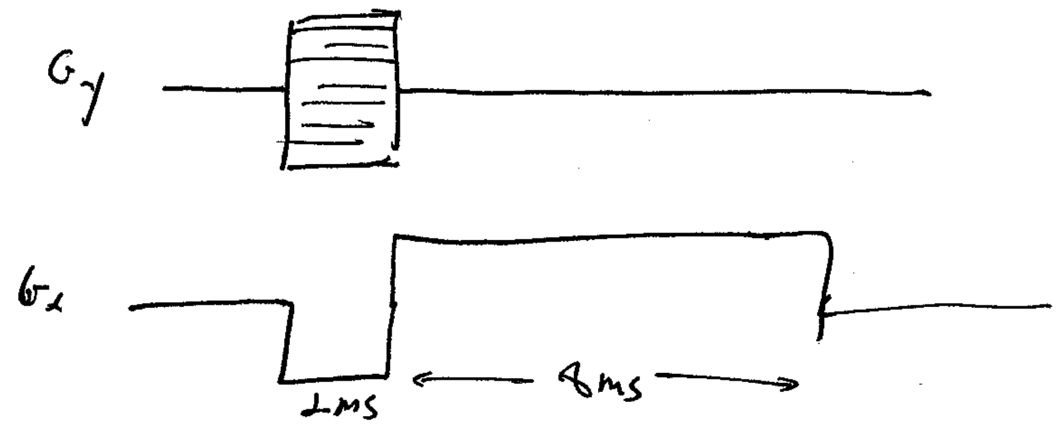
$$x = \frac{n}{W_{kx}}$$

$$\delta_x = \frac{1}{W_{kx}}$$

$$\delta_y = \frac{1}{W_{ky}}$$

RESOLUTION IS δ_x, δ_y

Example:



WHAT ARE THE AMPLITUDES FOR

$$FOV_x = FOV_y = 25.6 \text{ [cm]}$$

$$\delta_x = \delta_y = 0.1 \text{ [cm]}$$

FIRST $\quad N_p = N_r = \dfrac{25.6}{0.1} = 256$ SAMPLES

NEXT $\quad \delta_x = \dfrac{1}{W_{kx}}$

SO $\quad W_{kx} = \dfrac{1}{\delta_x} = \dfrac{1}{0.1 \text{ cm}} = 10 \text{ cycles/cm}$

$\left(\pm 5 \dfrac{\text{cycles}}{\text{cm}}\right)$

THEN $\quad \dfrac{\gamma}{2\pi} G_x \tau_x = 10 \dfrac{\text{cyc}}{\text{cm}}$

$$G_x = \frac{10}{\left(4.257 \frac{\text{kHz}}{\text{G}}\right)(8 \text{ ms})} \cong 0.3 \frac{\text{G}}{\text{cm}}$$

THE DEPHASER HAS SAME AREA AS HALF THE READOUT

$$(G_{x,d})\cdot(2ms) = (G_x)\left(\frac{8ms}{2}\right)$$

$$G_{x,d} = \left(0.3\ \frac{G}{cm}\right)\frac{4ms}{2ms} \cong 0.6\ \frac{G}{cm}$$

IN THE Y DIMENSION

$$\delta_y = 0.1\ cm$$

SO, $W_{ky} = \frac{1}{\delta_y} = 10\left[\frac{\text{cycles}}{cm}\right]$

PHASE ENCODE GRADIENT INTEGRATES TO $\pm W_{ky}/2$

$$\frac{\gamma}{2\pi} G_y \tau_y = \frac{W_{ky}}{2}$$

$$G_y = \frac{\left(10\ \frac{\text{cycles}}{cm}\right)/2}{\left(4.257\ \frac{kHz}{G}\right)(2ms)} \simeq 0.6\left[G/cm\right]$$

BOOK HAS Expressions FOR MANY SPECIAL CASES.

SIMPLER TO REMEMBER

EXTENT IN ONE DOMAIN (FOV, W_k)

= INVERSE ~~OF~~ IN OTHER DOMAIN (δ, Δk)

RESOLUTION $\quad$ $1/\text{Extent}$

$$\Delta k = \frac{1}{FOV}$$

$$\delta = \frac{1}{W_k}$$

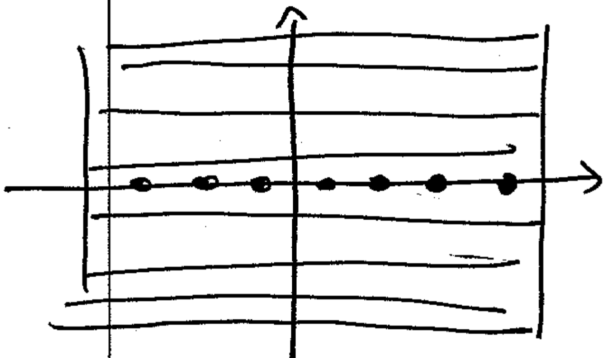
EXAMPLE

10 CM FOV $\Rightarrow \Delta k = 0.1 \ \frac{\text{cycles}}{\text{CM}}$

0.1 CM RES. $\Rightarrow W_k = 10 \ \frac{\text{cycles}}{\text{CM}}$.

RECONSTRUCTING 2DFT MR IMAGES

DATA IS ON A 2D GRID IN k-SPACE



$$u \in \left\{ -\frac{N_r}{2}+1 : \frac{N_r}{2} \right\}$$

$$v \in \left\{ -\frac{N_p}{2}+1 : \frac{N_p}{2} \right\}$$

$$N_r = 8 \quad u \in \{-3:4\}$$

$$N_p = 8 \quad v \in [-3:4]$$

SAMPLED DATA

$$\hat{m}_{xy}(k_x,k_y) = m_{xy}(k_x,k_y) \cdot \left(\frac{1}{\Delta k_x \Delta k_y} {}^{2}\text{Ш}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y} \right) \right) {}^{2}\Pi\left(\frac{k_x}{W_{kx}}, \frac{k_y}{W_{ky}} \right)$$

IMAGE

$$\hat{M}_{xy}(x,y) = \mathcal{F}^{-1}\left\{ \hat{m}(k_x,k_y) \right\}$$

$$\underbrace{\hat{M}_{xy}(a\delta_x, b\delta_y)}_{\text{SAMPLED IMAGE}} = \sum_{u=-\frac{N_r}{2}+1}^{N_r/2} \sum_{v=-\frac{N_p}{2}+1}^{N_p/2} \underbrace{\hat{m}(u\Delta k_x, v\Delta k_y)}_{\text{SAMPLED DATA}} e^{i2\pi(u\Delta k_x a\delta_x + v\Delta k_y b\delta_y)}$$

WHAT IS:

$u \Delta k_x a \delta_x$

$\Delta k_x = \frac{W_{kx}}{N_r} \qquad \delta_x = \frac{1}{W_{kx}}$

SO $\quad u \Delta k_x a \delta_x = u \frac{W_{kx}}{N_r} a \frac{1}{W_{kx}} = \frac{ua}{N_r}$

AND $\quad v \Delta k_y b \delta_y = v \frac{W_{ky}}{N_p} b \frac{1}{W_{ky}} = \frac{vb}{N_p}$

THE RECON IS:

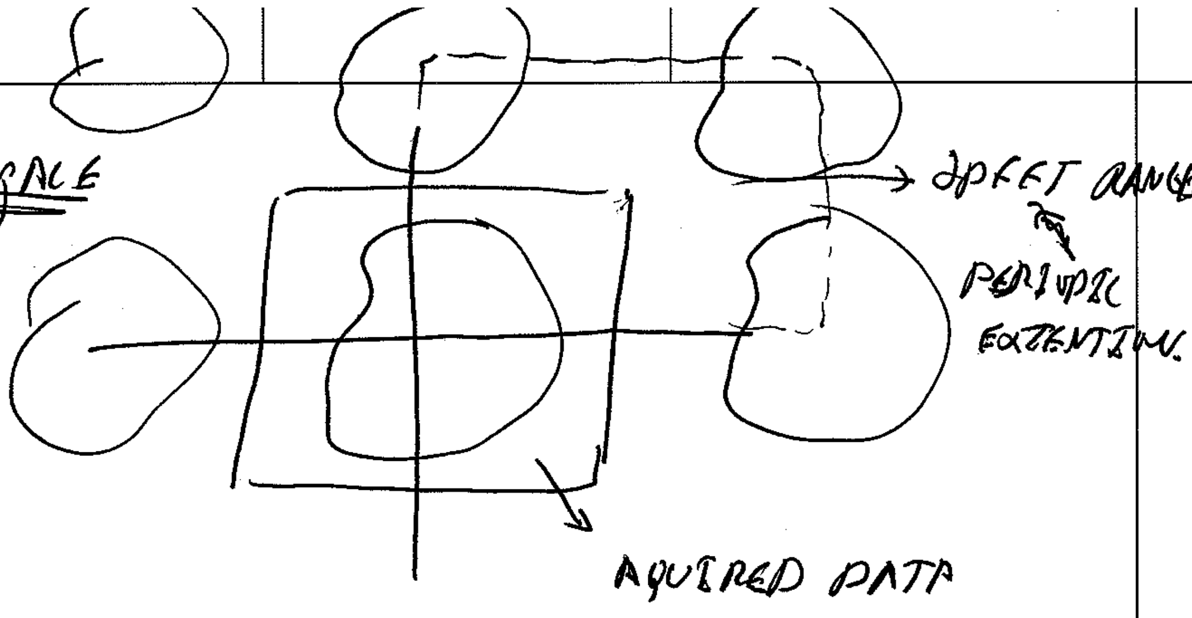
$$\hat{m}(a\delta_x, b\delta_y) = \sum_{u=-\frac{N_r}{2}+1}^{\frac{N_r}{2}} \sum_{v=-\frac{N_p}{2}+1}^{\frac{N_p}{2}} \tilde{m}(u\Delta k_x, v\Delta k_y) e^{+i2\pi\left(\frac{ua}{N_r}+\frac{vb}{N_p}\right)}$$

THIS IS A 2DFT, CENTERED AT $\left(\frac{N_r}{2}+1, \frac{N_p}{2}+1\right)$

WE WOULD LIKE TO USE A 2DFFT
WHICH HAS ZERO FREQ AT $(0,0)$

$$\hat{m}(a\delta_x, b\delta_y) = \sum_{u=0}^{N_r-1} \sum_{v=0}^{N_p-1} \tilde{m}(u\Delta k_x, v\Delta k_y) e^{i2\pi\left(\frac{ua}{N_r}+\frac{vb}{N_p}\right)}$$

K-SPACE



SWAP QUADRANTS FOR 2DFFT

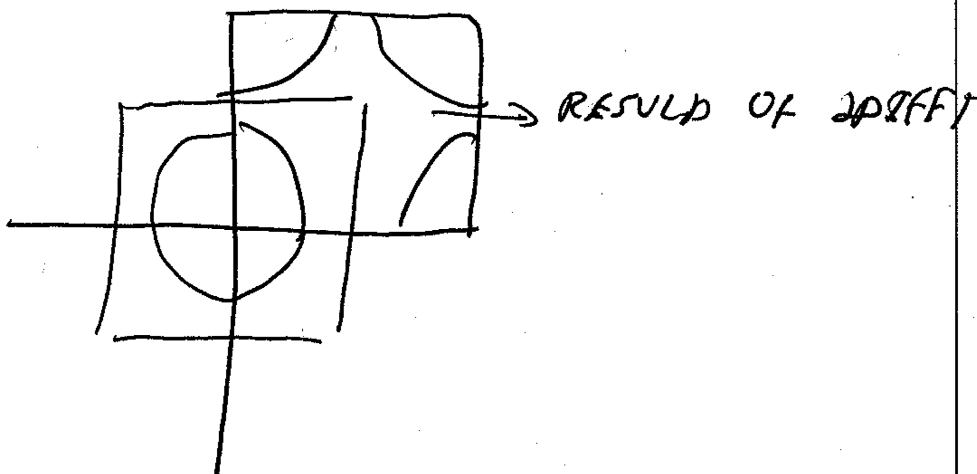


IMAGE SPACE

SWAP QUADRANTS TO PIECE TOGETHER

IN MATLAB:

FFTSHIFT(IFFT2(IFFTSHIFT($\hat{m}$)))

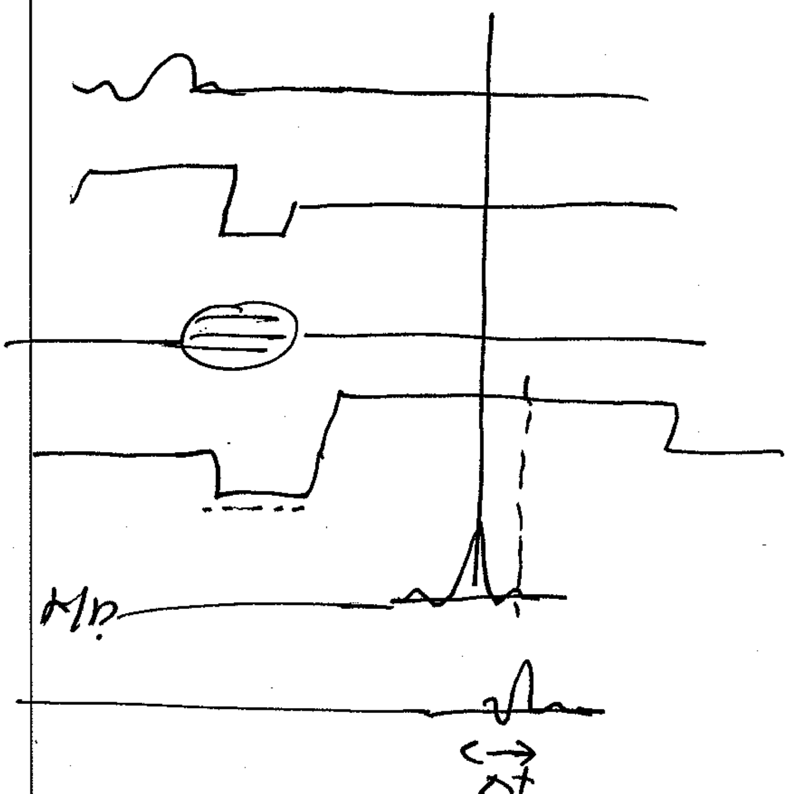
PRACTICAL ISSUES

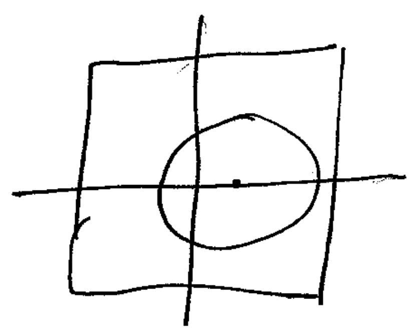
MAGNITUDE RECONSTRUCTION

(-) MOST MR IMAGES ARE MAGNITUDE

(-) MANY CASES IMAGES SHOULD BE REAL, BUT:

1) TIMING ERRORS.
2) FREQUENCY VARIATIONS
3) COIL SENSITIVITIES





SHIFT IN k-SPACE