LAST TIME   EXCITATION

SIMPLIFIED BLOCH EQN:

$$\frac{dM}{dt} = -\gamma \vec{B} \times \vec{M}$$

DEFINES A ROTATION, WITH AXIS

$$\vec{n} = \frac{\vec{B}}{|B|}$$

AND RATE

$$\omega = |\gamma \vec{B}|$$

IN ROTATING FRAME:

$$\vec{B}_{eff} = \left[B_{1x}, B_{1y}, \vec{G}\cdot\vec{r} \right]^T$$

BLOCH EQN

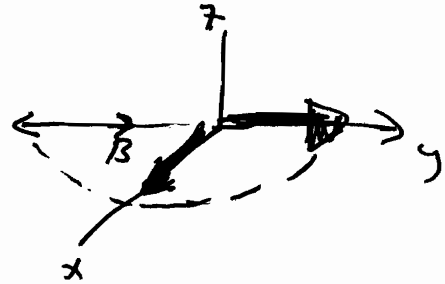
$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \begin{pmatrix} 0 & \gamma \vec{G}\vec{r} & -\gamma B_{1y} \\ -\gamma G r & 0 & \gamma B_{1x} \\ \gamma B_y & -\gamma B_{1x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

LAST TIME $\gamma \vec{G}\cdot\vec{r} = 0 \Rightarrow$ Non selective

$\vec{B} = (B_{1x}, B_{1y}, 0)$

IS IN THE TRANSVERSE PLANE, AND IS ISOTROPIC

Example:



$$\Theta = \int_0^t \gamma B_1 B_1(y)\, dt$$

OTHERS: EXCITATION, INVERSION.

TODAY:

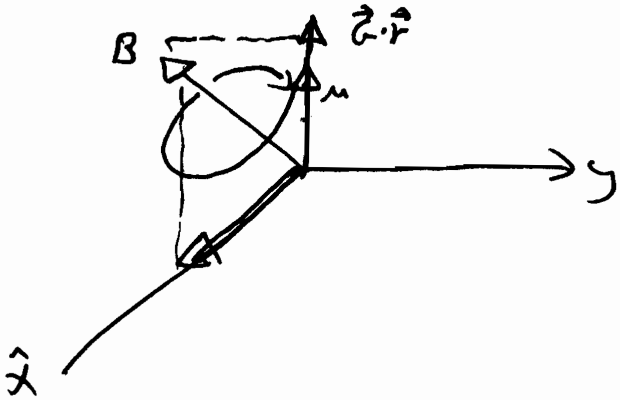
$$\gamma \vec{G}\cdot r \neq 0$$

SELECTIVE EXCITATION

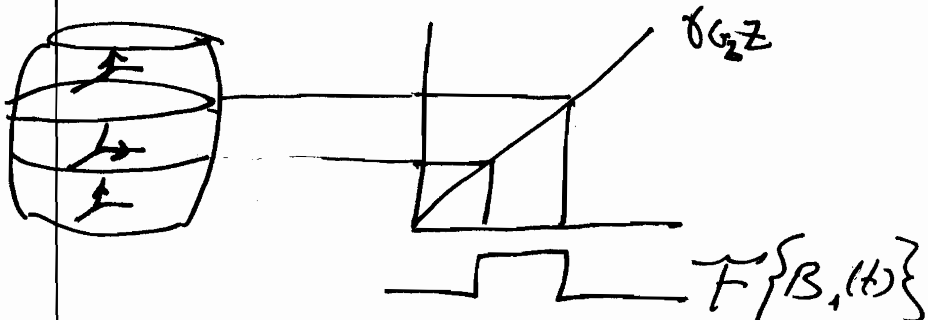
THE VECTOR

$$\vec{B} = [B_{1x}, B_{1y}, \vec{G}\cdot\vec{r}]$$

POINTS IN DIFFERENT DIRECTIONS AS A FUNCTION OF $\vec{r}$



SLICE SELECTIVE EXAMPLE $\quad \vec{G} = [0, 0, G_z]$



ONLY SPINS NEAR RESONANCE ARE EXCITED

PULSE SEQENCE

WHAT IS $M_{xy}(\vec{r}, T)$?

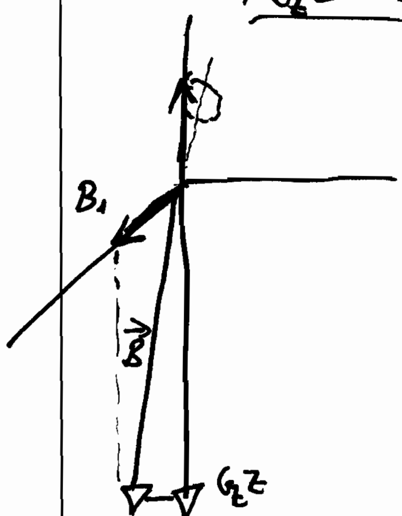
HOW DO WE DESCRIBE THIS A ROTATIONS

(1) B_{1x}, B_{1y} SAME EVERY WHERE

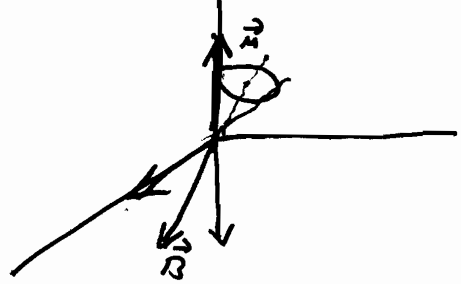
$\boxed{\text{USE THE WOLE BOARD}}$

(2) $G_z Z$ CHANGES LINEARLY IN Z

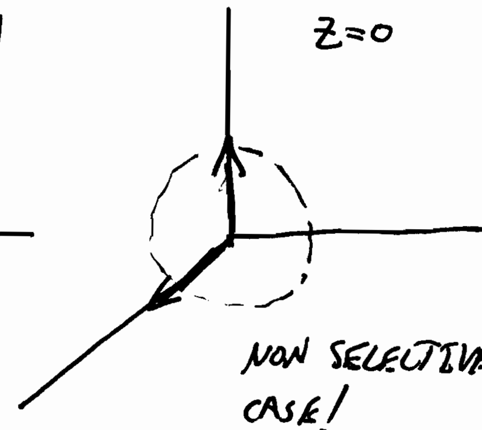
$\boxed{G_z Z << B_1}$



$\boxed{G_z Z \sim B_1}$



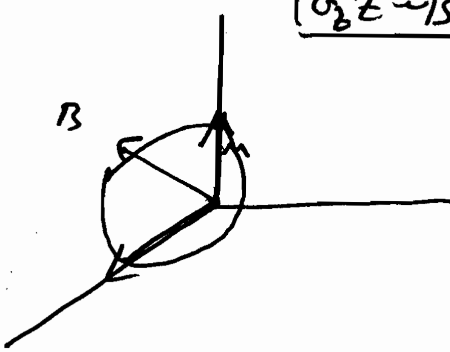
$Z=0$



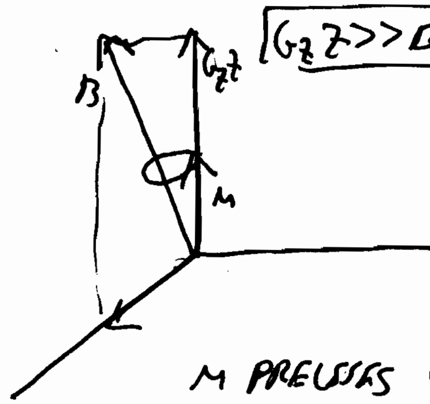
NON SELECTIVE CASE!

$\theta = \gamma \int B_1(t)dt$

$\boxed{G_z Z \sim B_1}$



$\boxed{G_z Z >> B_1}$



M PRECESSES ON A TIGHT CONE NEAR Z

LITTLE M_{xy} PRODUCED

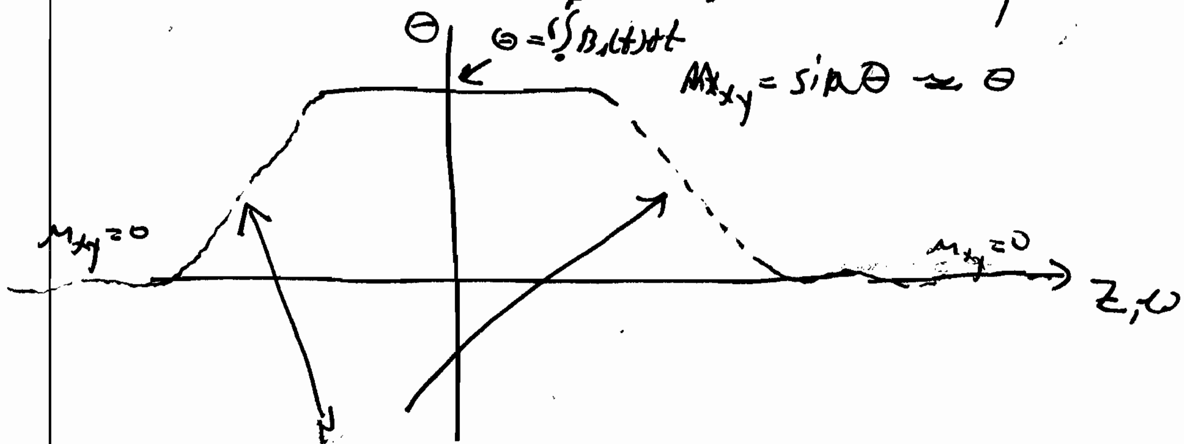
SIMPLE CASES

(1) ON-RESONANCE $\Rightarrow$ NON-SELECTIVE

$$\theta = \gamma \int_0^T B_1(t)\,dt$$

(2) FAR OFF RESONANCE

$\hat{n} \cong [0,0,1] \Rightarrow$ NO M_{xy} PRODUCED



WHAT HAPPENS HERE?

IN GENERAL A HARD PROBLEM

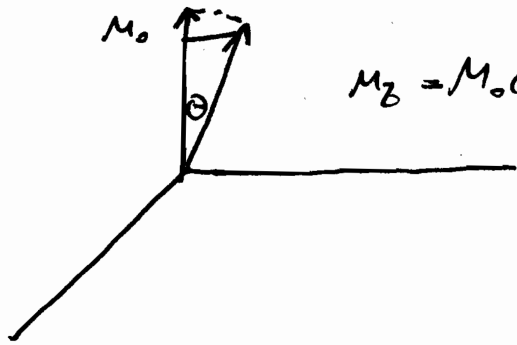
ROTATIONS $\Rightarrow$ FUNDAMENTALLY NON-LINEAR

MANY SOLUTIONS FOR SPECIAL CASES

INCLUDES MOST IMPORTANT ONES.

SMALL TIP ANGLE EXCITATION PULSES

BASIC IDEA: TIP ANGLE IS SMALL ENOUGH THAT $M_z = M_0$ THROUOUT THE PULSE



$$M_z = M_0 \cos\theta \simeq M_0$$

ACTUALLY NOT BAD FOR MUCH BIGGER TIPS (i.e. 90°)

BLOCH EQN

$$\begin{pmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{pmatrix} = \begin{pmatrix} 0 & \gamma \vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma \vec{G}\cdot\vec{r} & 0 & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

IF $M_z \simeq M_0$ THE LAST EQN DECOUPLES!

$$\begin{pmatrix} \dot{M}_x \\ \dot{M}_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 & \gamma \vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma \vec{G}\cdot\vec{r} & 0 & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_0 \end{pmatrix}$$

$$\begin{pmatrix}\dot{M}_x\\ \dot{M}_y\end{pmatrix} = \begin{pmatrix}0 & \gamma\vec{G}\cdot\vec{r} & -\gamma B_{1y}\\ -\gamma\vec{G}\cdot\vec{r} & 0 & \gamma B_{1y}\end{pmatrix}\begin{pmatrix}M_x\\ M_y\\ M_0\end{pmatrix}$$

$$\begin{pmatrix}\dot{M}_x\\ \dot{M}_y\end{pmatrix} = \underbrace{\begin{pmatrix}0 & \gamma\vec{G}\cdot\vec{r}\\ -\gamma\vec{G}\cdot\vec{r} & 0\end{pmatrix}\begin{pmatrix}M_x\\ M_y\end{pmatrix}}_{\text{PRECESSION (JUST LIKE RECEPTION!)}} + \underbrace{\begin{pmatrix}-\gamma B_{1y}\\ \gamma B_{1x}\end{pmatrix}M_0}_{\text{EXCITATION}}$$

<u>NOTE</u> $M_x, M_y, \vec{G}, B_{1x}, B_{1y}$ are function of time!

LET $M_{xy} = M_x + iM_y$

$$\dot{M}_x + i\dot{M}_y = -i^2\gamma\vec{G}\cdot\vec{r}M_y - i\gamma\vec{G}\cdot\vec{r}M_x + \left(\overset{i^2}{(-)}\gamma B_{1y} + i\gamma B_{1x}\right)M_0$$

$$= \left(-i\gamma\vec{G}\cdot\vec{r}\right)\left(M_x + iM_y\right) + i\left(\gamma B_{1x} + i\gamma B_{1y}\right)M_0$$

$$\dot{M}_{xy} = (-i\gamma \vec{G}\cdot\vec{r}) M_{xy} + i\gamma B_1 M_0$$

where $B_1 = B_{1x} + iB_{1y}$

SOLVE LIKE IN RELEPTION EQN.

$$\dot{M}_{xy} + i\gamma \vec{G}\cdot\vec{r} M_{xy} = i\gamma B_1 M_0$$

MULIPLY BY INTEGRATING FACTOR.

$$e^{i\int_{-\infty}^{t} \gamma \vec{G}(\tau)\vec{r}\, d\tau}$$

Then.

$$\frac{d}{dt}\left[M_{xy}(\vec{r},t)\, e^{i\int_{-\infty}^{t} \gamma G\cdot r \tau}\right] =$$

$$= i\gamma B_1(t) M_0 e^{i\int_{-\infty}^{t} \gamma G(\tau)\cdot\vec{r}\, d\tau}$$

~~INTEGRATE from $-\infty$ to $t=T$~~

$$\cancel{M_{xy}(\vec{r},T) = e^{i\int_0^T \gamma \vec{G}(\tau)\vec{r}\,d\tau} =}$$

$$\cancel{= \int_{-\infty}^{T} i M_0 \gamma B_1(t)\, e^{i\int_{-\infty}^{t} \gamma \vec{G}(\tau)\cdot\vec{r}\,d\tau}\, dt}$$

$$M_{xy}(\vec{r},T) = iM_0 \int_{-\infty}^{T} \gamma B_1(t)\, e^{-i\int_t^T \gamma \vec{G}(\tau)\cdot r\,d\tau}\, dt$$

DEFINE

$$\boxed{\vec{k}(t) = \frac{\gamma}{2\pi}\int_t^T \vec{G}(\tau)\,d\tau}$$

THEN

$$M_{xy}(\vec{r},T) = iM_0 \int_{-\infty}^{T} \gamma B_1(t)\, e^{-i2\pi \vec{k}(t)\cdot\vec{r}}\, dt$$

Excitation Profile is FT of $B_1(t)$!

More cases $\cancel{\gamma B_1(t)} = \omega_1(t)$

in The Book. $\cancel{\omega_1}$