Principles of MRI
EE225E / BIO265

Lecture 20

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Spin Echo

\[ e^{-\frac{t}{T_E}} \]

\[ e^{-\frac{t}{T_2}} \]
Any signal due to depinning $W_k(r) t$ is refocused.

If $W_k(r, t) \Rightarrow$ refocusing NOT perfect

Field fluctuating

Shows as $T_2$ decay!
Spin Echo Imaging

\[ \hat{\mathbf{k}}(T) = (k_x(T), k_y(T)) \]

\[ -\mathbf{k}(T) \]

\( k\)-space goes to conjugate position! \underline{conjugate position!}
REGULAR HDFT GRADIENT ECNU!
NEGATE GRADIENT LEFT OR 180

TRIVIAL ADAPTATION.

MINIMUM MOTION ARTIFACTS.

WANT GRADIENT ECHO TO HAPPEN WITH SPIN-ECOHO.
imperfect 180 causes parasitic which is not phase encoded (HW)
MULTI-SLICE

SIMPLE T2-WEIGHTED SPIN-ECNU.

\[ \text{max}(\text{no}) e^{-\frac{t}{T_2}} \]

FENS 100 MILLISECONDS

SECONDS
INEFFICIENT! ADD MULTISlice MAKE NO SELECTIVE

WHEN IMAGING ONE SLICE, OTHER RECOVER!
VERY EFFICIENT ALWAYS ACQUIRING DATA.
Other non-idealities

- RF Inhomogeneity
  - Affects Excitation
    - Spatially varying flip-angle
    - Spatially varying phase
    - Can cause problem for fat saturation
  - Worse @ high field (wave effects)

- Affects receive
RF excitation inhomogeneity
AFFECTS RECEIVE:

(2) VARYING COMPLEX SENSITIVITY

\[ s(b) = \iint c(x, y) \tilde{m}(x, y) e^{-i2\pi \langle k, x \rangle} \, dx \, dy \]
CAn BE A GOOD THING: SENSITIVITY ENCODING

\[ S_1(b) = \int m(r, \rho) c_1(r) e^{-i2\pi (\rho_1 \cdot \rho)} \, d\rho \]

\[ S_2(b) = \int m(r, 0) c_2(r) e^{-i2\pi (\rho_1 \cdot r)} \, dr \]

To reconstruct \( MC_1 \) & \( MC_2 \) need \( n \) encodings
but to reconstruct \( m \) need (maybe) \( \frac{n}{2} \) encodings.

More later!
Gradient non-idealities

GRADIENTS

(-) Non Linearity - gradient rolls off

\[ z \uparrow \Rightarrow \square \Rightarrow \text{for } \uparrow \text{ res } \]

Easy to correct by interpolation
Concomitant Gradient (Maxwell Terms)

- Concomitant Gradients

Ideal Gradients Violate Maxwell Eqn.

For cylindrical coils used in MRI

\[
\begin{align*}
B &= B_0 + 6_x x + 6_y y + G_z Z + \frac{1}{2B_0} \left[ \frac{G_z^2}{4} (x^2 + y^2) + (G_x + G_y)^2 Z^2 - 6_x G_x x - 6_y G_y y \right] \\
\text{main gradients}
\end{align*}
\]

Example: At \( B = 20 \text{cm} \) \( 6_x = 40 \text{ mT/m} \) \( B_0 = 1.5 \text{T} \)
Concomitant Gradient (Maxwell Terms)

CONCOMITANT GRADIENTS

IDEAL GRADIENTS VIOLATE MAXWELL Eqs.

For cylindrical coils used in MRI

\[ \mathbf{B} = \mathbf{B}_0 + \mathbf{G}_x \mathbf{x} + \mathbf{G}_y \mathbf{y} + \mathbf{G}_z \mathbf{z} + \frac{1}{4\mathbf{B}_0} \left[ \frac{G_x^2}{4} (x^2 + y^2) + (G_y + G_z)^2 z^2 - G_x z^2 - G_y z^2 \right] \]

\[ G_x \xrightarrow{\text{main gradient}} \]

EXAMPLE: AT \( B = 20 \text{cm} \) \( G_x = 40 \text{ mT/m} \) \( B_0 = 1.5 \text{T} \)

\[ \frac{G_x^2 z^2}{2B_0} = \frac{(40 \cdot 10^{-3} - 0.2)^2}{2 \cdot 1.5} = 2.13 \cdot 10^{-5} T = 14.2 \text{ ppm} \]

\( G = 10 \text{mT/m} \Rightarrow 0.889 \text{ ppm} \)

\( B_0 = 0.7 \text{T} \Rightarrow 650 \text{ ppm} \)
Concomitant Gradients

\[ \frac{G_x z^2}{2B_0} \]

FIG. 3. Sagittal spiral scans acquired at isocenter \((x/1.1005, 0)\) with (a) \(g_m/1.1005 = 2.18\) G/cm, and (b) \(g_m/1.1005 = 1.11\) G/cm, using field strength \(B_0/1.1005 = 1.5\) T. Scan prescriptions are (a) 8 interleaves, 4096 readout points, 125 kHz bandwidth, 4 NEX; and (b) 16 interleaves, 2048 readout points, 64 kHz bandwidth, 2 NEX. Both (a) and (b) use spin echo, 27-cm field of view, TE/1.1005 = 15, TR/1.1005 = 2000, 1-cm thick slice. Spatial resolution, SNR, and off-resonance blurring are approximately the same for both cases. Arrows show areas of blurring differences.

FIG. 2. Axial spiral scans acquired at slice locations \(z/1.1005 = 0\) and \(z/1.1005 = 10\) cm with (a) \(g_m/1.1005 = 2.1\) G/cm, and (b) \(g_m/1.1005 = 0.524\) G/cm, using field strength \(B_0/1.1005 = 1.5\) T. Scan prescriptions are (a) 8 interleaves, 4096 readout points, 62.5 kHz bandwidth, 4 NEX; and (b) 32 interleaves, 1024 readout points, 16 kHz bandwidth, 1 NEX. Both (a) and (b) use spin echo, 14-cm field of view, TE/1.1005 = 15, TR/1.1005 = 2000, 1-cm slice. Spatial resolution, SNR, and off-resonance blurring are approximately the same for all cases.
@z=10cm

king et. al, MRM 41:103-112(1999)

\[ g_m = 2.1 \text{ G/cm} \quad \text{and} \quad g_m = 0.524 \text{ G/cm} \]

1.0 T

1.5 T

(a)  

(b)
Gradient Finite Rise-Time

Even finite-rise-time rate! (often ignored)
Eddy Currents

Generated by electric field from changing magnetic flux, build up in time varying gradients and decay in constant or slow rate.
\[ B_e(\vec{\gamma},t) = b_0(\beta) + \int \varphi(\beta) \, \tau \, \cdot \]
Eddy Currents

\[ q(t) = -\frac{d\phi}{dt} \otimes e(t) \]

\[ e(t) = H(t) \geq \alpha_n e^{-\frac{t}{\tau}} \]

↑

step function

↑

intensity

↑

time constant

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Pre-emphasis

\[ B_0 \rightarrow \text{chung raister phase} \]

gradient

\[ \Rightarrow \]

For short time constants

\[ \text{Gnet}(t) = \text{Gapplied}(t - \alpha t) \]

Benzstein (Ch. 10.3)
IMAGE CONTRAST

SO FAR ASSUMED

\[ \hat{M}(0) = \{0, 0, M_0 \}^T \]

SAMPLE IS FULLY RELAXED \( TR > 3 T_1 \)

IN PRACTICE THIS IS Seldom TRUE

\[ I(x,y) = f(p, T_1, T_2, \Theta, TR, TE) \]

IMAGE

PHYSICAL PARAMETERS

INSTRUMENTAL PARAMETERS

Tissue has inherent variability in \( T_1, T_2, \Theta \)

Want to emphasize it
$m(x,y,t)$ is a function of time. A good approximation is $m(x,y)$. for all $(x,y)$

We will consider: $T_4 > T > T_{2.3}$

- May decays between acquisitions
- $M_2$ not fully recovered.
SATURATION RECOVERY

RF

\[ M_0 \]

\[ M_z \]

\[ M_0 \left( 1 - e^{-\frac{TR}{T_1}} \right) \]

\[ ||M_{xy}||_0 \]

\[ M_0 \left( 1 - e^{-\frac{TR}{T_1}} \right) \]

\[ ||M_{xy}|| \]

1st TR usually not used
STEADY-STATE SIGNAL: $M_0 (1 - e^{-\frac{TR}{T_1}})$

GAIN CONSTANT

$TE = 0$

$I(x,y) = k \rho(x,y) \left[ 1 - e^{-\frac{TR}{T_1(x,y)}} \right]$  

SPIN DENSITY  

$T_1$ WEIGHTING

$TR < T_1$

$I(x,y) \equiv k \rho(x,y) \left( \frac{TR}{T_1(x,y)} \right)$  

SHORT $T_1$ BRIGHT

$TR > T_1$

$I(x,y) = k \rho(x,y) \left[ 1 - e^{-\frac{TR}{T_1(x,y)}} \right] e^{-\frac{TE}{T_2}}$  

LONG $T_1$ DARK
\[ I(x, y) = k p(a, y)[1 - e^{-\frac{TR}{T_2(a, y)}}] e^{-\frac{TE}{T_1(a, y)}} \]

**Proton Density:**

\[ \begin{align*}
TE \text{ short } & \ll T_2 \\
TR \text{ long } & \gg T_1
\end{align*} \]

\[ I \approx k p(a, y) \]

**T_1 Weighing:**

\[ \begin{align*}
TE \text{ short } & \ll T_1 \\
TR & \approx T_1
\end{align*} \]

\[ I \approx k p c \cdot e^{-\frac{TR}{T_1}} \]

(TR = 400 ms, TE = 15 ms \( \odot \) fast brain)

**T_2 Weighing:**

\[ \begin{align*}
TR \text{ long } & \gg T_2 \\
TE & \approx T_2
\end{align*} \]

\[ I \approx k p e^{-\frac{TE}{T_2}} \]

(TR = 2500 ms, TE = 90 ms \( \odot \) fast)

\[ T_1 - \text{Anatomy} \]

\[ T_2 - \text{Lesion sensitive} \]
GENERAL EXCITATION RECOVERY
GENERAL EXCITATION RECOVERY

- RF
- TI
- TR

M₀ (1)
Mₓ

(1) 180° pulse
(2) M₀
(3) 90° pulse
(4) M₀
(5) 180° pulse
(6) M₀
(7) 90° pulse
(8) M₀