Principles of MRI
EE225E / BIO265

Lecture 22

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Signal to Noise

• Definitions

\[
\text{SNR} \triangleq \frac{\text{Signal Amplitude}}{\text{Std of Noise}}
\]

\[
\text{CNR} \triangleq \frac{\text{Signal Difference}}{\text{Std of Noise}}
\]
SNR Considerations

- **Physical & Instrumental**
  - B0
  - Coil Geometry
  - Conductivity of Coil and Sample
  - Noise figure of pre-amplifier

- **Imaging Sequence Parameters**
  - A/D time
  - Resolution
  - Timing in sequence (relaxation...)
  - Tip angle
  - k-space coverage
Noise Sources

MR: Gaussian additive, Resistive Noise

Due to Brownian Motion

\[ N(f) = 4kTR \]

\[ \Rightarrow \sqrt{V} = \sqrt{4kTR(BW)} \]

Body

Coil

Pre-Amp
Model

L(coil)

V(signal)

R

Rc - Loss in coil

Rs - Loss in sample

Rd - dielectric loss (negl.)

Rm - Induction loss

V(noise)
Noise sources

- **System** \( R_C \propto \omega_0^2 \)
  - Freq. dependence due to skin-depth

- **Body** :
  - \( R_m \propto \omega_0^2 r^5 \)
  - Due to electrical eddy currents in body
  - Not an MRI process
  - Not localized
  - Body can not be cooled - Must live with it!
  - dependence on \( \omega_0 \) due to differentiation in Faraday’s law

* See noise in MRI by Macovski
\[ \sigma_m \propto \sqrt{B_0^2 r^5} \quad \quad \sigma_c \propto \sqrt{B_0^{\frac{1}{2}}} \]

- Body noise dominance (desired) for:
  - High Field (\(>0.5\)T)
  - Large Coils (\(>1\)in)
\[ s(t) \propto M_0 \cdot w_0 \propto B_0^2 \]

\[ \text{SNR} \propto \frac{B_0^2}{\sqrt{\alpha B_0^{\frac{1}{2}}} + \beta B_0^2} \]
SNR

• For a well designed system (e.g. body noise dominance):
  - SNR proportional to $B_0$

• How to (instrumentally) improve SNR?
  - small coils (up to coil-noise dominance)
  - Higher field
EMVELO: THE MR SIGNAL IS (BASE BAND)

\[ \ell b = s(\ell b) + n(\ell b) \]

QUADRATURE DETECTION

\[ n(\ell b) = n_i(\ell b) + i n_\phi(\ell b) \]

132VARIATE
GAUSSIAN

INDEPENDENT

POWER SPECTRAL DENSITY

\[ N(\ell b|f) = \frac{1}{4\pi} \int \frac{|H(f)|^2}{\sigma^2} df \]
For 2DFT

BIVARIATE GAUSSIAN

RADIAL SPIRAL NOT TRUE ANYMORE

UNITARY
**NOTE**

**Displayed Image is:**

\[ |z(a, b)| = |m(a, b) + n(a, b)| \]

2 Cases

(i) Background pixel mean = 0

\[ \text{Data } |M| \sim \text{Rayleigh} \]

(ii) \( m \neq 0 \)

\[ |m+n| \sim \text{Rician} \]

For \( M \) \text{ LARGE } \sim \text{APPROX Gaussian}
SNR dependence on sequence parameters

Assume:

1) CONST. RESOLUTION
2) IMPULSE OBJECT AT (u, v) (SIGNAL A)
3) N RANDOM SAMPLES
4) NOISE \( \text{VAR/SAMP} = \sigma_n^2 \)

ID CASE

\[
\Delta NA = \sum_{m=1}^{N} A
\]

\[
\text{SNR} = \frac{NA}{\sqrt{N \sigma_n^2}} = \sqrt{\frac{NA}{\sigma_n^2}} = \text{SNR}_{\text{REF}}
\]
Case 1: Average two signals

\[
\begin{align*}
\text{SIG} &= 2\text{NA} \\
\text{VAR} &= 3\text{NA}^2 \\
\text{SNR} &= \frac{3\text{NA}}{\sqrt{2\text{NA}^2}} = \sqrt{2}\frac{\text{NA}}{\text{NA}} \\
\text{SNR} &= \sqrt{N_{\text{avg}}} \text{SNR}_{\text{REF}}
\end{align*}
\]
Case 2: Increase readout time

\[ \text{ex} \quad T_{\text{read}} \to 2T_{\text{read}}. \]

summary

\[ \begin{align*}
2T_{\text{read}} & \leq t_0 \\
2t & \leq t_{\text{amp}} \\
\frac{1}{2}f_{\text{amp}} & \leq \frac{1}{2}\text{BW}(50\text{Hz})
\end{align*} \]

yes & no the same?
\[ \text{SIGNAL} = NA \]
\[ \text{VARIANCE} = \frac{N \sum_h \nu_h}{2} \]
\[ \Rightarrow \text{LPF BW HALVED} \]
\[ \Rightarrow \text{BW/Pixel HALVED} \]

\[ \therefore \text{SNR} = \frac{NA}{\sqrt{\sum_h \nu_h}} = \sqrt{A \frac{N \sum_h A}{\sum_h}} \]

\[ \uparrow \text{IMPROVEMENT} \]

\[ \text{SNR} \propto \sqrt{T_{\text{READ}}} \]
SNR \propto \sqrt{\text{TOTAL READ TIME}}

\text{OFT:}

\frac{\text{SNR}}{\sqrt{N_{\text{OVG}} \cdot N_{\text{PE}} \cdot T_{\text{READ}}}}
Example:

Consider readouts!

What is relative SNR?
$A = \text{SAME!}$

$$\text{SNR}_1 = \frac{256A}{\sqrt{256} \sigma^2} = \frac{\sqrt{256}A}{\sigma}$$

$$\text{SNR}_2 = \frac{\sqrt{12}A}{\sqrt{(\sqrt{12})(2\sqrt{2})}} = \frac{\sqrt{512}A}{\sqrt{24} \sigma} < \frac{\sqrt{256}A}{\sigma} = \text{SNR}_1 \uparrow \text{BW DOUBLED more samples,}$$

Also... SAME READOUT TIME!
Example: Pure Phase Encode

\[ \text{by} \]

\[ \text{A/D} \]

\[ \text{512} \]

\[ \text{What is the relative SNR?} \]
\[ SNR_2 = \sqrt{2} \times SNR_1 \]

\[ SNR_1 = \frac{256A}{\sqrt{256} V_n} = \frac{\sqrt{256} A}{V_n} \]

\[ SNR_2 = \frac{512A}{\sqrt{512} V_n} = \frac{\sqrt{512} A}{V_n} = \sqrt{2} \times SNR_1 \]

SCAN TIME IS DOUBLED!
**Spatial Resolution**

**Assume:**
1. Readout & scan-time fixed
2. N readout samples
3. $\sigma_n^2 = \text{Var/samp}$
4. 1D case: FT of OBJ = AM($\frac{k_y}{W_{k_y}}$)

$$A \quad \text{Resolution} = \sigma_x = \frac{1}{W_{k_y}}$$

Reconstruct at Origin

$$\text{SNR}_{\text{REF}} = \sqrt{W} \frac{A}{\sigma_n}$$
\[
\text{SNR} = \frac{\frac{N/2}{\sigma_0^2}}{\sqrt{N \gamma_{\text{REF}}}} = \frac{1}{\frac{1}{2} \text{SNR}_{\text{REF}}}
\]
FILTER IN POST PROCES TO IMPROVE SNR:

REF:

\[ \begin{align*}
\text{ACQ:} & & \text{FILTER:} \\
& & \text{FILTER:} \\
& & \text{FILTER:}
\end{align*} \]

\[
\text{SNR} = \frac{N \frac{A}{2}}{\sqrt{\frac{N}{2} B^2}} = \frac{\sqrt{N} A}{\sqrt{N} B^2} = \frac{A}{B^2} \text{SNR}!
\]

MORAL: SELECT RESOLUTION CAREFULLY
Because the SNR is inversely proportional to the square root of the voxel size, reducing the size of the voxels linearly will not reduce the noise proportionally. A good rule of thumb is to reduce the voxel size by a factor of 2 in order to reduce the noise by a factor of \(\sqrt{2}\).
But, not true if object is smaller than resolution.

\[ \text{SNR} \propto \frac{S}{\bar{S}} \]

\[ \text{SNR} \propto \frac{S}{\bar{S}} \]

\[ \text{OBJECT} \quad \Rightarrow \quad \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \quad \text{SNR & Noise} \]
**IN GENERAL:**

\[ s_{\text{NN}} \cdot f(\rho, T_1, T_2, ...) \sqrt{T_{\text{tot}}} \cdot (s_x \cdot s_y \cdot s_z) \]

\[ f(\rho, T_1, T_2, ...) - \text{HOW MUCH MAGNETIZATION YOU HAVE} \]

\[ \sqrt{T_{\text{tot}}} \cdot \text{HOW LONG YOU ACQUIRE DATA} \]

\[ q \cdot q \cdot \text{TOTAL Acquisition Time} \]

\[ q \cdot q \cdot \text{HOW MANY SPINS} \]

**OTHER FACTORS:**

**UNIFORMITY OF K-SPACE**

- More Efficient
- Colored Noise