Principles of MRI

EE225E / BIO265

Lecture 22

Instructor: Miki Lustig
UC Berkeley, EECS

Signal to Noise

- Definitions

$$\mathrm{SNR} \triangleq \frac{\text{Signal Amplitude}}{\text{Std of Noise}}$$

$$\mathrm{CNR} \triangleq \frac{\text{Signal Difference}}{\text{Std of Noise}}$$

SNR Considerations

- Physical & Instrumental
  - B0
  - Coil Geometry
  - Conductivity of Coil and Sample
  - Noise figure of pre-amplifier
- Imaging Sequence Parameters
  - A/D time
  - Resolution
  - Timing in sequence (relaxation...)
  - Tip angle
  - k-space coverage

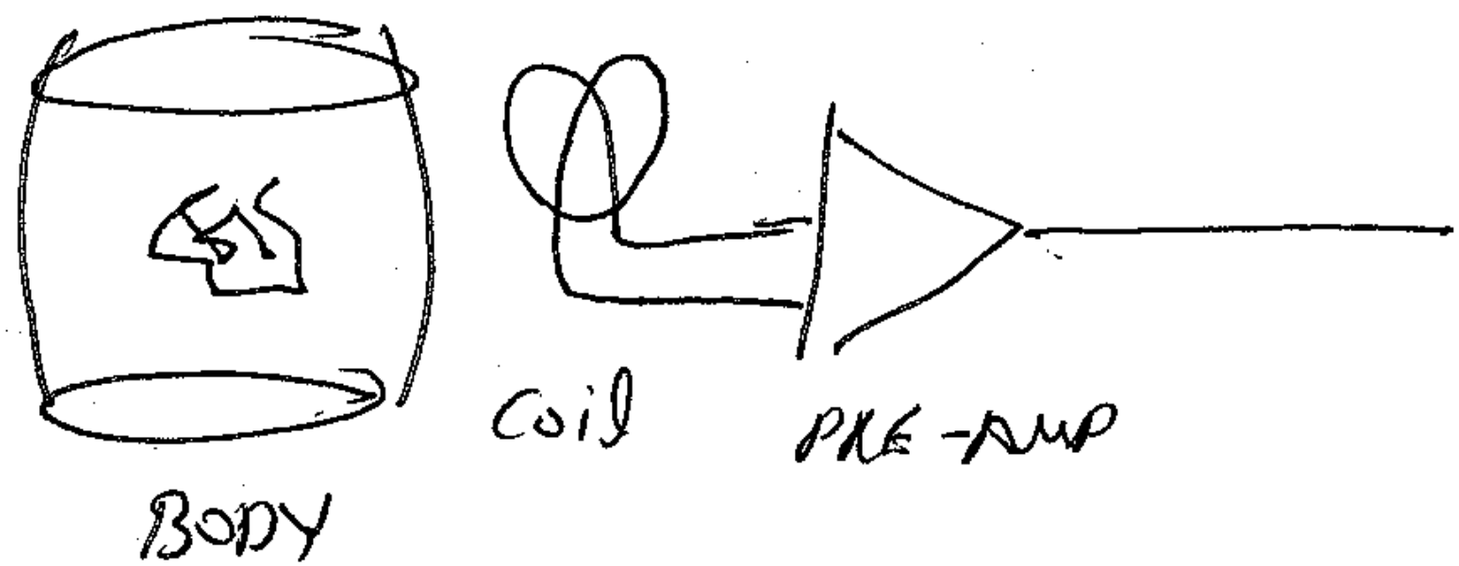
NOISE SOURCES

MR: Gaussian additive, RESISTIVE NOISE

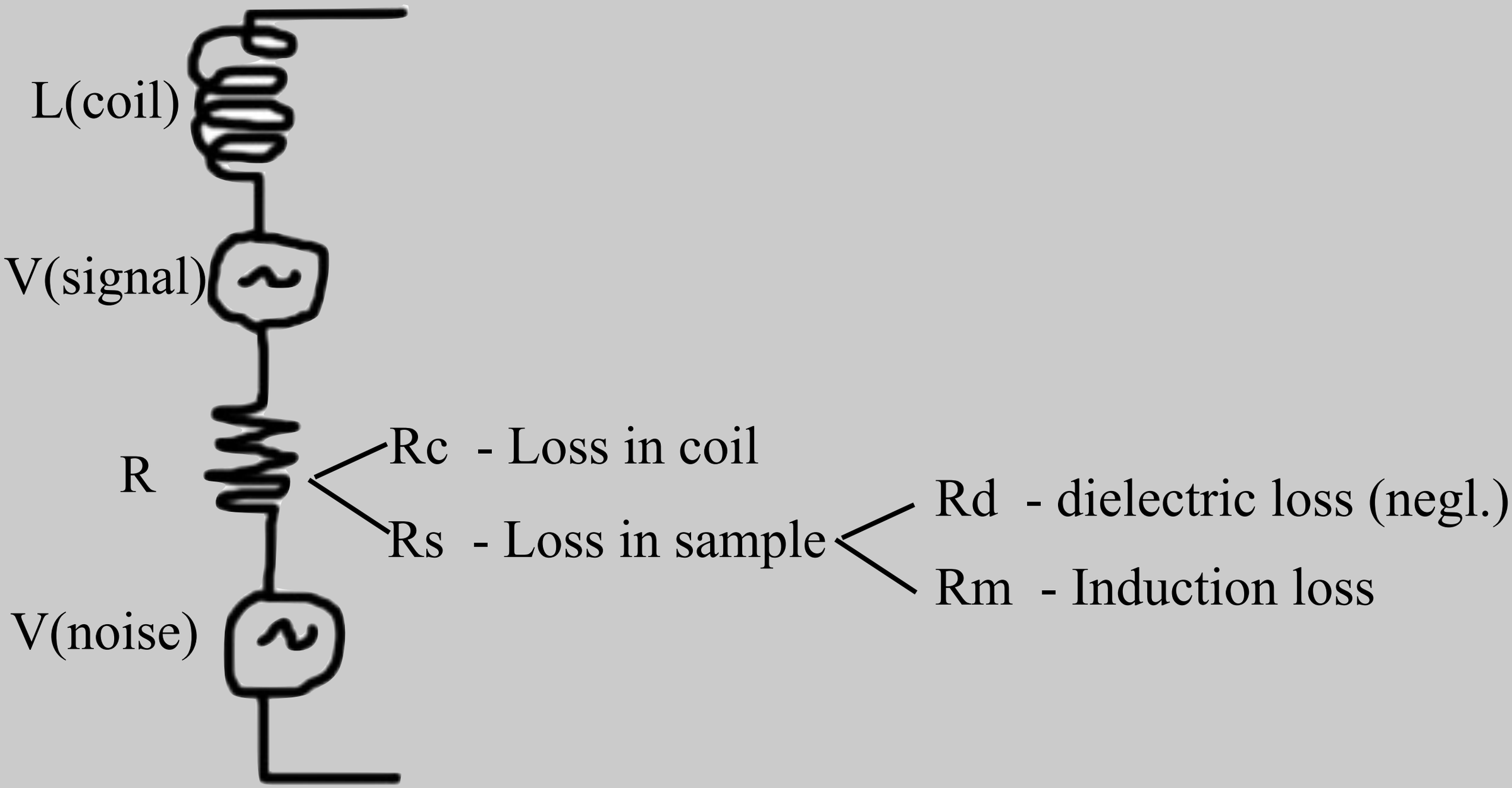
DUE TO BROWNIAN MOTION → ELECTRONS IN COIL
→ ELECTROLYTES IN BODY

$$N(f) = 4kTR \Rightarrow \sigma = \sqrt{4kTR(BW)}$$

R → RESISTANCE



Model


L(coil)
V(signal)
R
Rc - Loss in coil
Rs - Loss in sample
Rd - dielectric loss (negl.)
Rm - Induction loss
V(noise)

Noise sources

- System $R_c \propto \omega_0^{\frac{1}{2}}$
  - Freq. dependence due to skin-depth
- Body : $R_m \propto \omega_0^2 r^5$ *
  - Due to electrical eddy currents in body
  - Not an MRI process
  - Not localized
  - Body can not be cooled - Must live with it!
  - dependence on w_0 due to differentiation in Faraday's law

* See noise in MRI by Macovski

$$\sigma_m \propto \sqrt{B_0^2 r^5} \qquad \sigma_c \propto \sqrt{B_0^{\frac{1}{2}}}$$

- Body noise dominance (desired) for:
  - High Field (>0.5T)
  - Large Coils (> 1in)

SNR

$$s(t) \propto M_0 \cdot w_0 \propto B_0^2$$

$$\mathrm{SNR} \propto \frac{B_0^2}{\sqrt{\alpha B_0^{\frac{1}{2}} + \beta B_0^2}}$$

SNR

- For a well designed system (e.g. body noise dominance):
  - SNR proportional to B0

- How to (instrumentally) improve SNR?
  - small coils (up to coil-noise dominance)
  - Higher field

EMBEND: THE MR SIGNAL IS (BASEBAND)

$$\xi(t) = s(t) + n(t)$$

QUADRATURE DETECTION

$$n(t) = n_I(t) + j\, n_Q(t)$$

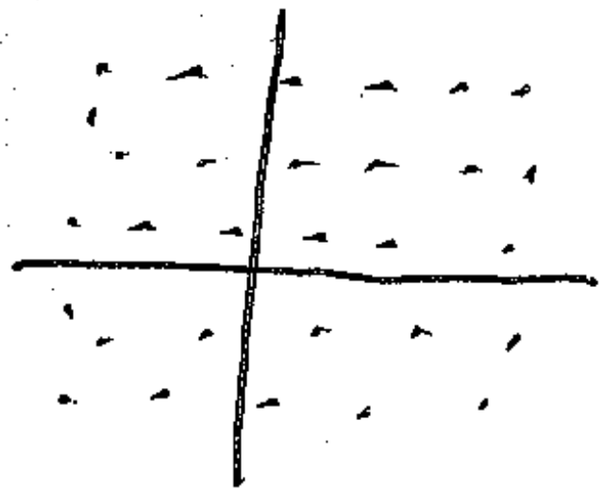
BIVARIATE GAUSSIAN

INDEPENDENT

POWER SPECTRAL DENSITY

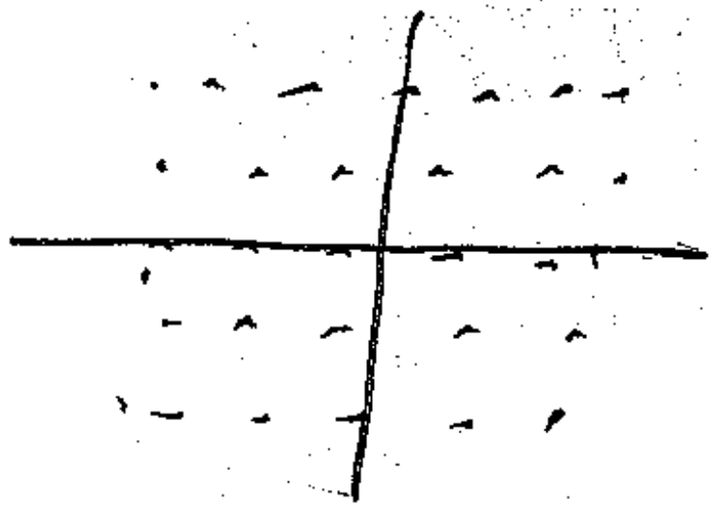
$$N(f) = N_Q(f) = \sigma^2 \, |\underbrace{H(f)}_{LPF}|^2$$

FOR 2D FT



$\xrightarrow{FFT^{-1}}$

UNITARY



BIVARIATE GAUSSIAN

BIVARIATE GAUSSIAN

RADIAL, SPIRAL NOT TRUE ANYMORE.

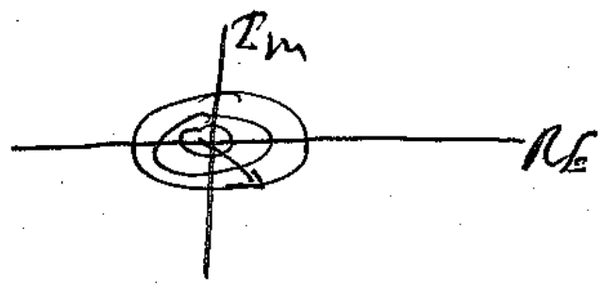
NOTE

DISPLAYED IMAGE IS:

$$|I(a,b)| = |m(a,b) + n(a,b)|$$

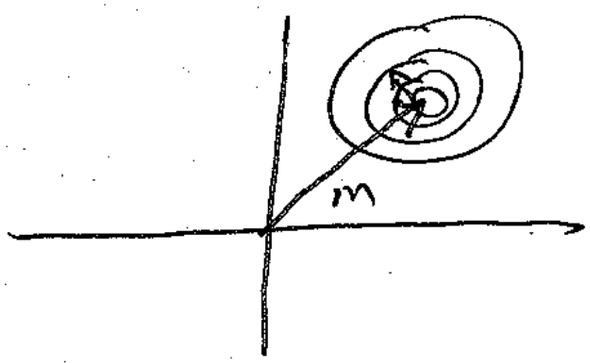
2 CASES

(a) Background pixel mean = 0



$|n| \sim$ Rayleigh

(b) $m \neq 0$



$|m+n| \sim$ Rician

FOR m LARGE $\sim$ APPROX GAUSSIAN

SNR dependence on sequence parameters

Acq. TIME

ASSUME:

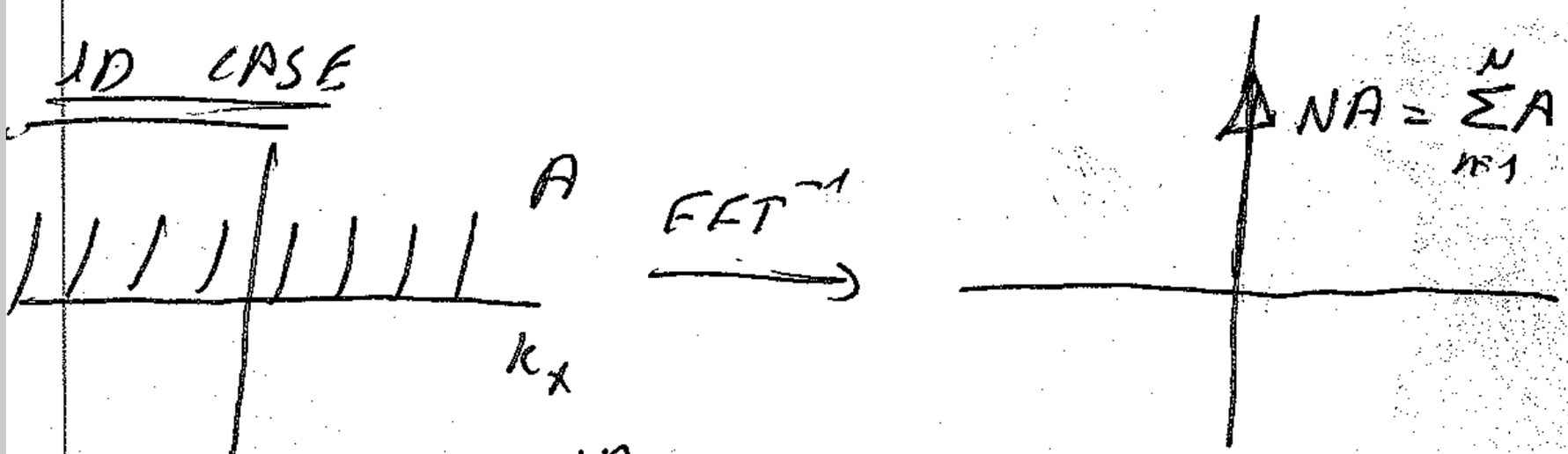
(1) CONST. RESOLUTION

(2) IMPULSE OBJECT AT (0,0) (SIGNAL A)

(3) N READOUT SAMPLES

(4) NOISE VAR/SAMP $= \sigma_n^2$

1D CASE



$$SNR = \frac{NA}{\sqrt{N\sigma_n^2}} = \frac{\sqrt{N}A}{\sigma} = SNR_{REF}$$

Case 1: Average two signals

$$\left.\begin{aligned} SIG &= 2NA \\ VAR &= 2N\sigma_n^2 \end{aligned}\right\} \quad SNR = \frac{2NA}{\sqrt{2N\sigma_n^2}} = \sqrt{2}\sqrt{N}\frac{A}{\sigma_n}$$

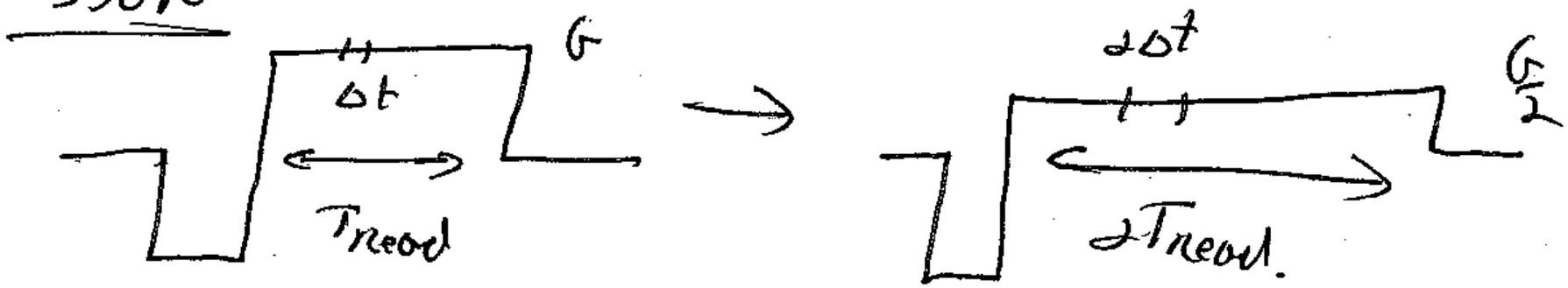
improve

$$SNR = \sqrt{N_{avg}}\; SNR_{REF}$$

Case 2: Increase readout time

ex $T_{read} \rightarrow 2T_{read}$.

SNR??



summary

$2T_{read}$
$\frac{1}{2}G_r$
$2\Delta t$
$\frac{1}{2}f_{samp}$
$\frac{1}{2}BW$ (LPF)

} res & N THE SAME!

SIGNAL = NA

VARIANCE = $\frac{N\sigma_n^2}{2}$ ← LPF BW HALVED

SO, BW/PIXEL HALVED

$$\therefore \; SNR = \frac{NA}{\sqrt{\frac{N}{2}\sigma_n^2}} = \sqrt{2}\sqrt{N}\frac{A}{\sigma_n}$$

↑ IMPROVEMENT

$$SNR \propto \sqrt{T_{READ}}$$

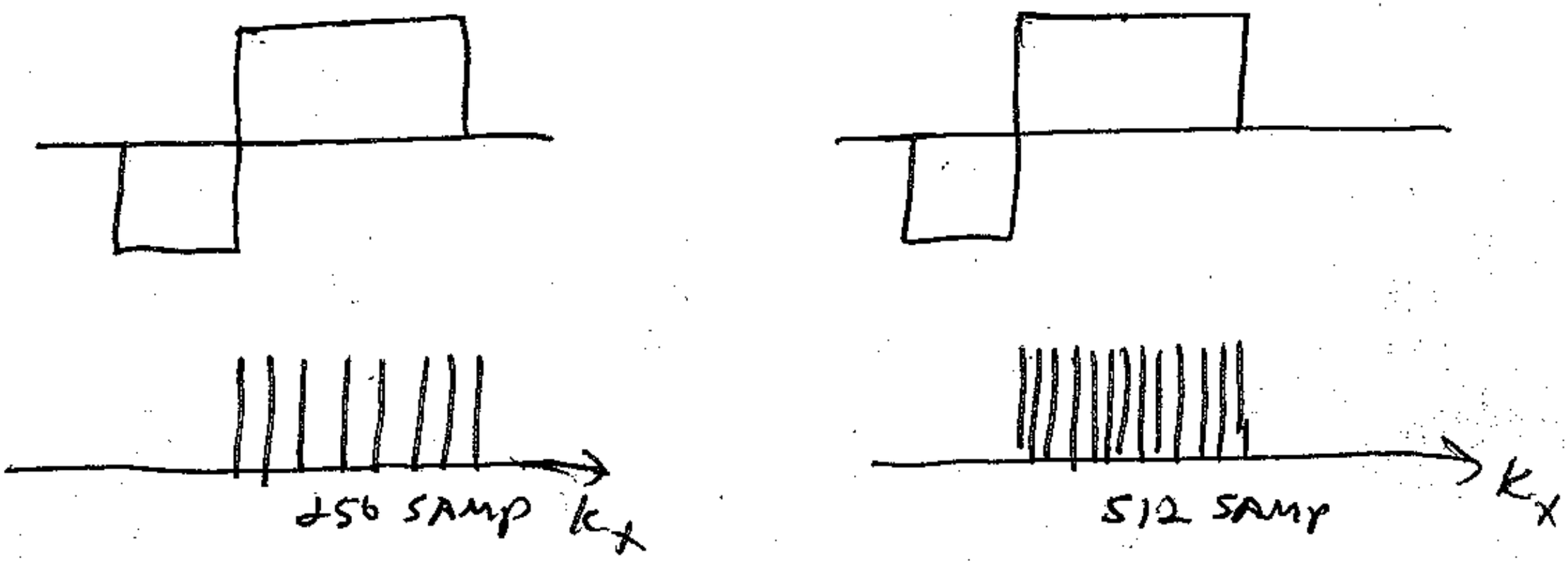
$$SNR \propto \sqrt{\text{TOTAL READ TIME}}$$

IDEAL:

$$SNR \propto \sqrt{N_{avg} \cdot N_{pe} \cdot T_{READ}}$$

EXAMPLE:

CONSIDER READOUTS:



Q WHAT IS RELATIVE SNR?

A. = SAME!

$$SNR_1 = \frac{256A}{\sqrt{256\sigma^2}} = \frac{\sqrt{256}A}{\sigma_\lambda}$$

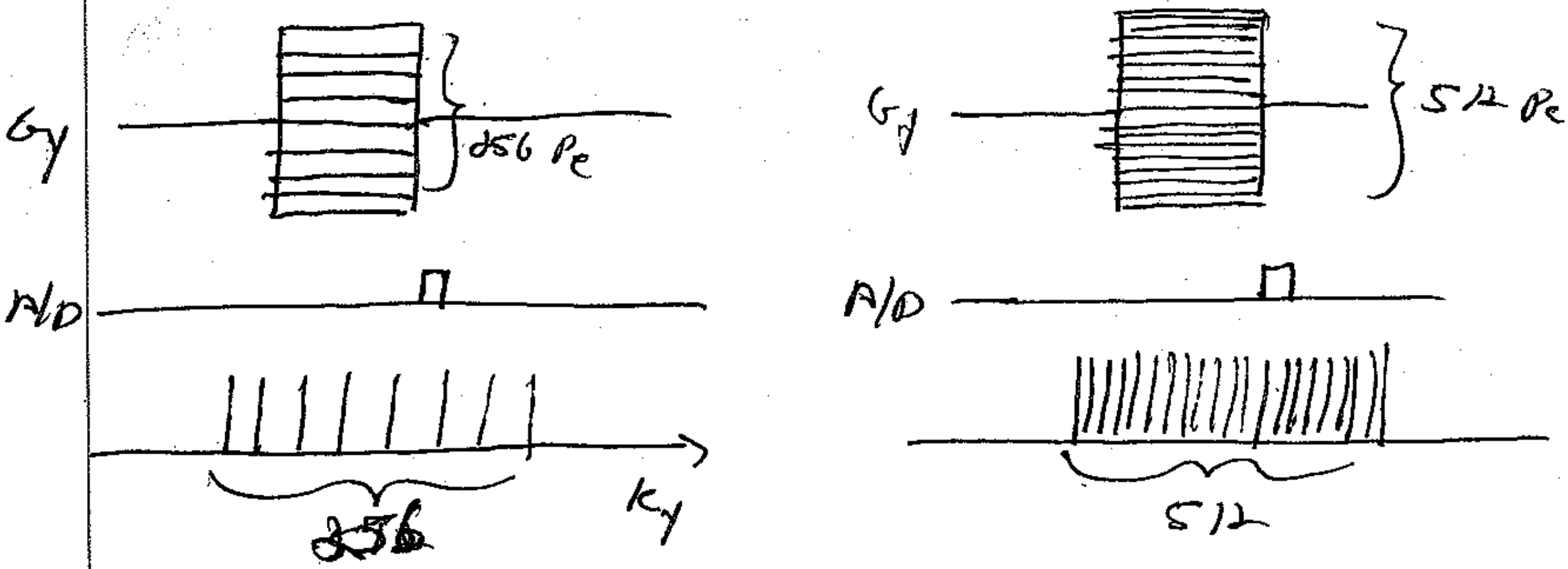
$$SNR_2 = \frac{512A}{\sqrt{(512)(2\sigma_n^2)}} = \frac{\sqrt{512}A}{\sqrt{2}\sigma_n} < \frac{\sqrt{256}\ A}{\sigma_\lambda} = SNR_1$$

MORE SAMPLES.

BW DOUBLED

ALSO... SAME READOUT TIME!

EXAMPLE: PURE PHASE ENCODE



Q WHAT IS THE RELATIVE SNR?

A. $SNR_2 = \sqrt{2}\, SNR_1$

$$SNR_1 = \frac{256A}{\sqrt{256\,\sigma_n^2}} = \frac{\sqrt{256}\,A}{\sigma_n^2}$$

$$SNR_2 = \frac{512A}{\sqrt{512\,\sigma_n^2}} = \frac{\sqrt{512}\,A}{\sigma^2} = \sqrt{2}\, SNR_1$$

SCAN TIME IS DOUBLED!

SPATIAL RESOLUTION

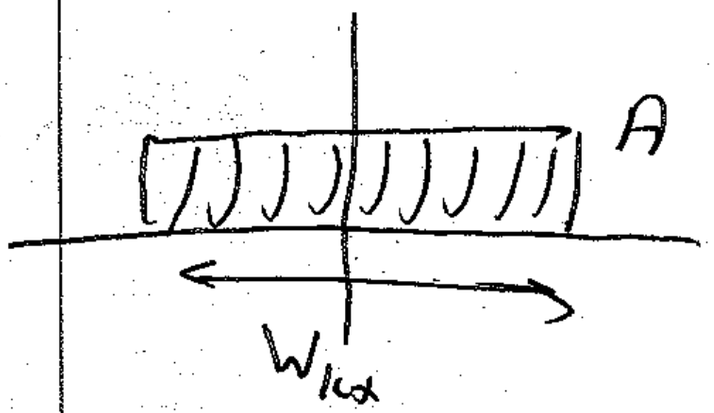
ASSUME:

(1) Readout & SCAN-TIME FIXED

(2) N readout samps

(3) $\sigma_n^2 = Var/samp$

(4)* 1D CASE FT OF OBJ $= A\,M\left(\frac{k_x}{W_{kx}}\right)$

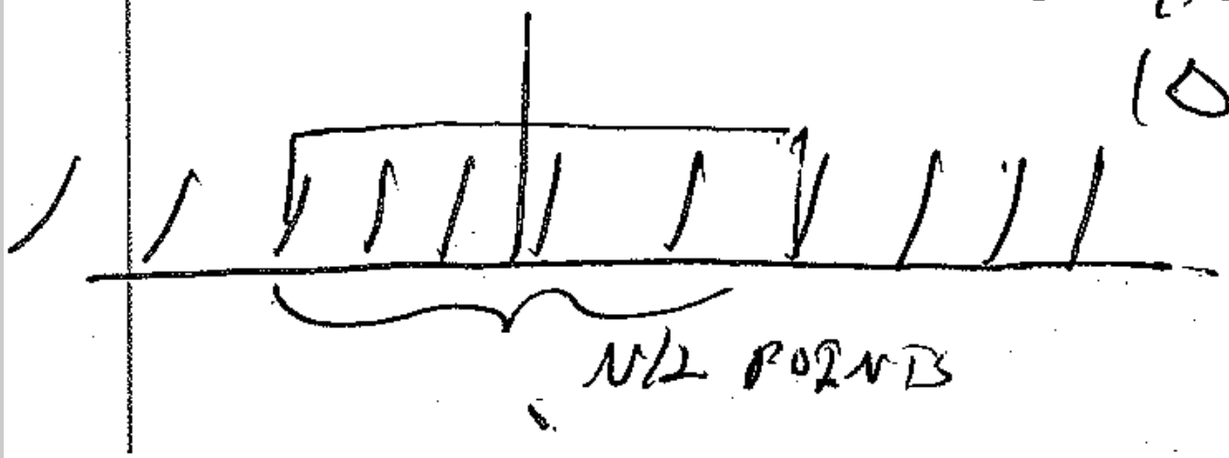


RESOLUTION $= \delta_x \simeq \frac{1}{W_{kx}}$

RECONSTRUCT AT ORIGIN

$SNR_{REF} = \sqrt{N}\,\frac{A}{\sigma_n}$

DOUBLE GRADIENT



→ EOV HALVED

(ΔΦ) HALVED

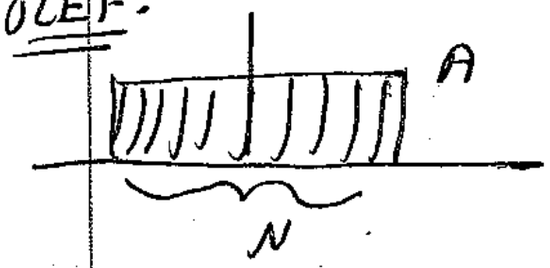
∴ SIG $\frac{N}{2}A$

VAR = $N\sigma_n^2$

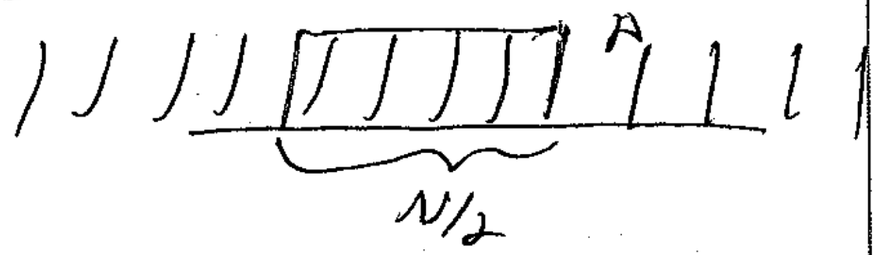
$$SNR = \frac{\frac{N}{2}A}{\sqrt{N\sigma_n^2}} = \frac{1}{2}SNR_{REF}$$

FILTER IN POST PROCESS TO IMPROVE SNR:

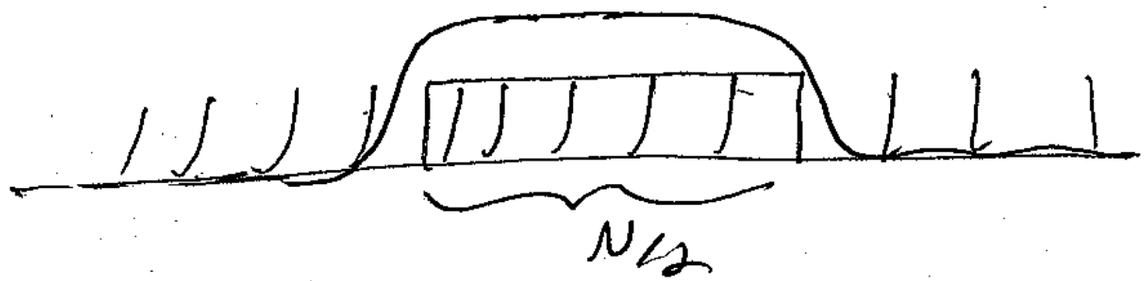
REF:



ACQ:



FILTER:



SO!

$$SIG = \frac{N}{2}A$$

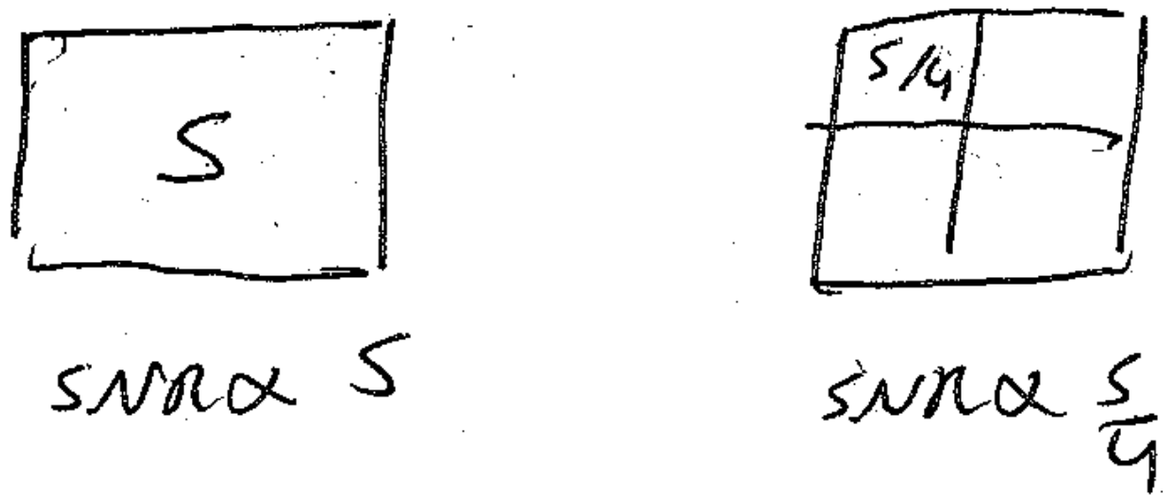
$$VAR = \frac{N}{2}\sigma_\lambda^2 \Rightarrow SNR = \frac{\frac{N}{2}A}{\sqrt{\frac{N}{2}\sigma^2}} = \frac{\sqrt{\frac{N}{2}}A}{\sigma^2} = \frac{1}{\sqrt{2}}SNR!$$

MORAL: SELECT RESOLUTION CAREFULLY

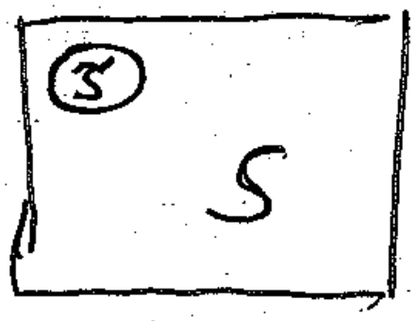
OBJ DOMAIN PERSPECTIVE!

$\frac{\text{SIGNAL}}{\text{VOXEL}}$ REDUCE LINEARLY, BUT NOISE THE SAME

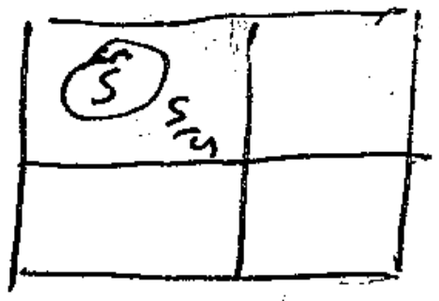
SNR $\propto$ VOXEL SIZE GOOD "RULE OF THUMB"



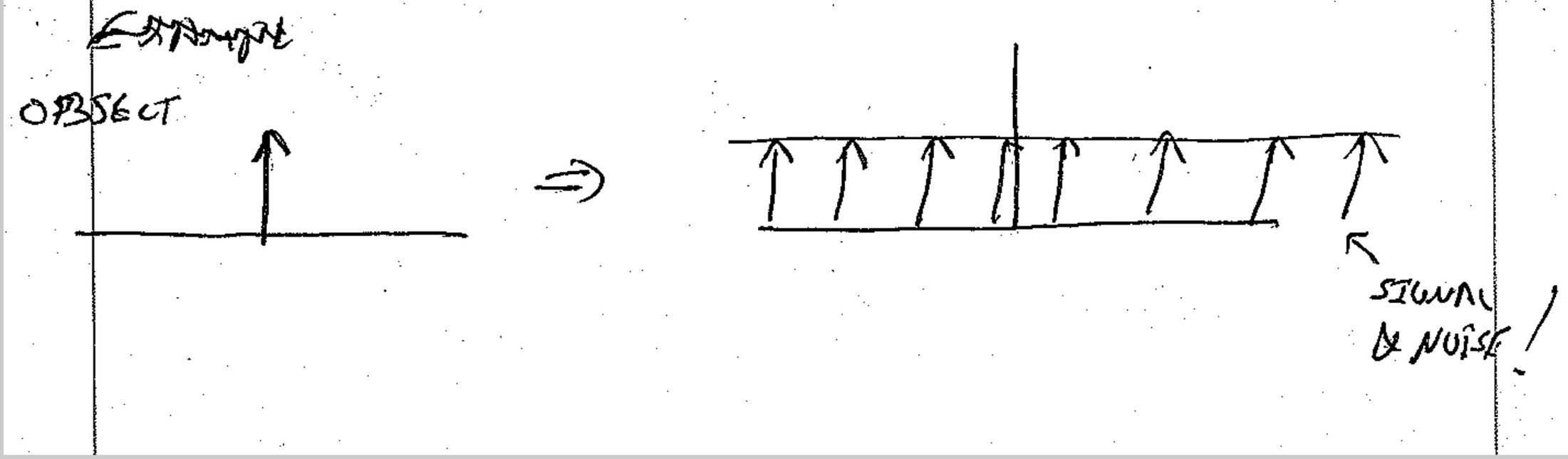
BUT, NOT TRUE IF OBJECT IS SMALLER THAN RESOLUTION.



SNR ∝ $\tilde{S}$!



SNR ∝ $\tilde{S}$!



IN GENERAL:

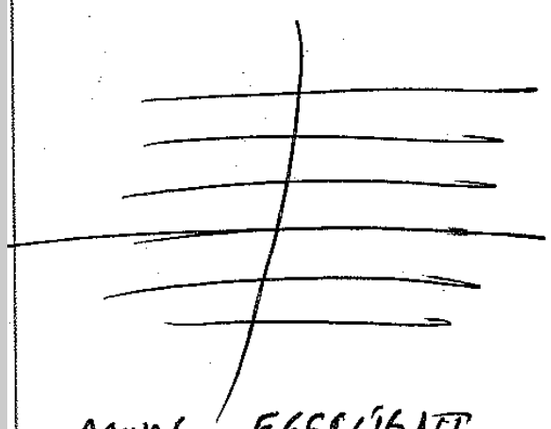
$$SNR \sim f(\rho, T_1, T_2, \ldots)\sqrt{T_{A/D}}\,(\delta_x \cdot \delta_y \cdot \delta_z)$$

$f(\rho, T_1, T_2, \ldots)$ – HOW MUCH MAGNETIZATION YOU HAVE

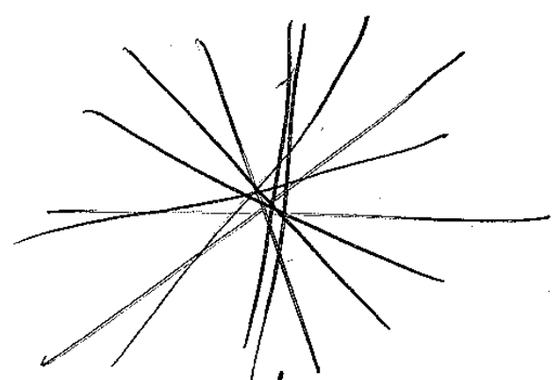
$\sqrt{T_{A/D}}$ HOW LONG YOU ACQUIRE DATA TOTAL A/D TIME

$\delta_x \delta_y \delta_z$ HOW MANY SPINS

OTHER FACTORS: UNIFORMITY OF K-SPACE



MORE EFFICIENT



COLORED NOISE