Principles of MRI
EE225E / BIO265

Lecture 06

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Last Time

• Relaxation
  – Longitudinal: $T_1$
  – Transverse: $T_2$

• B1 - Reception RF field
  – Precession causes EMF in coil

Last Time

• B0
  – Polarization $\rightarrow$ $M_0$
  – Resonance $\omega = -\gamma B_0$

• B1 - Transmit RF field
  – Magnetization precesses around rotating field and is tipped away
  – Very Small Field! (usually < 0.35 G)
  – Resonance is essential - easy to describe in rotating frame
    \[
    \Delta B_1 \sim \pm 10\% \at 1.5T
    \]
    • Leads to different flips in space

Last Time

• Gradient Fields
  – Encode position onto frequency
    \[
    \mathbf{G} = \left[ \frac{\partial B_z}{\partial x}, \frac{\partial B_z}{\partial y}, \frac{\partial B_z}{\partial z} \right]
    \]
  – Small concomitant fields $B_x$, $B_y$ are also created. These do not contribute much to precession - fields are NOT oscillating at Larmor freq.
Imaging

1. Place sample in $B_0$
   
   $m_z$ develops $\sim 5T_1$

2. Excite using $B_1(t)$
   
   Creates transverse magnet.

3. Instantaneous precession of $M_{xy}$
   
   Induces EMF in coil

4. Encode position in freq. using gradients

   1-D "Projection"

Example: Square RECT

- What is the signal after excitation?
  
  - Signal proportional to $m_{xy}$
  
  - Comes from the entire volume

Some Limitations

Gradient strength $\Rightarrow$ Resolution

\[ \downarrow \]

Signal decay ($T_2$)

Field inhomogeneity ($T_1^*$)

Diffusion

Typical resolution: $\leq 1$ mm

($\sim 20$ pm in small animal scanners)

Example: Square RECT

- Magnetization at position $x=x_0$

  \[ m_{xy}(x) = m(x)e^{-i(\omega_0 + \gamma G_x x)t} \]

- Signal from everything

  \[ s(t) = \int_x m(x)e^{-i(\omega_0 + \gamma G_x x)t} dx \]
Example: Square RECT

\[ s(t) = \int_x m(x)e^{-i(\omega_0 + \gamma G_xt)x}dx \]
\[ = e^{-i\omega_0t} \int_x m(x)e^{-i\gamma G_xt}dx \]
\[ = e^{-i\omega_0t} \int_x m(x)e^{-i2\pi(\frac{\gamma}{2\pi} G_xt)x}dx \]
\[ = e^{-i\omega_0t} \int_x m(x)e^{-i2\pi k_x x}dx \]
\[ = e^{-i\omega_0t} \mathcal{F}\{m(x)\}|_{k_x = \frac{\gamma}{2\pi} G_xt} \]

Inverse FT of baseband signal yields 1D projection (Almost..)

Example: Square RECT

\[ s(t) = e^{-i\omega_0t} \mathcal{F}\{m(x)\}|_{k_x = \frac{\gamma}{2\pi} G_xt} \]
\[ m(x) = \text{rect}(\cdot) \Rightarrow \mathcal{F}\{m(x)\} \sim \text{sinc}(\cdot) \]

Selective Excitation

want to excite a slice

Gradient (G_t) maps position to bandwidth

Bandlimited B(t) \Rightarrow Excites a slab in space
Excitation Field

Choose $B_1(t)$

$\tilde{B}_1(t) \approx \text{rect}(t/T) \times \text{sinc}(5000t)$

finite in time

ripple in slice

Quiz

$B_1(t) = e^{-i\omega_0 t} \text{sinc}(5000t)$

$G_z = 1 \text{ G/cm}$

What is the excited slice thickness?
The Central Section Theorem

(Projection slice)

\[ \mathcal{F}_y \mathcal{F}_x \{ f(x, y) \} \]

\[ k_y \]

\[ f(x, y) \]

\[ k_x \]

Transform (2D) of projection of an object

\[ \Rightarrow \text{Diameter through transform (2D) of object} \]

Very cool! ... and useful...

Projection slice = basis for CT

Proof (for \( \theta = 0 \))

\[ p(x) = \int_{-\infty}^{\infty} m(x, y) dy \]

\[ M(k_x, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x, y) e^{-i\pi(k_x x + k_y y)} dx dy = \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x, y) e^{-i\pi k_x x} dx dy = \]

\[ = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} m(x, y) dy \right] e^{-i\pi k_x x} dx = \]

\[ = \int_{-\infty}^{\infty} p(x) e^{-i\pi k_x x} dx = \frac{1}{2\pi} \mathcal{F}^{-1} p_0^2 \]

Let's see if it makes sense...

How to reconstruct?

1. Filtered back-projection
2. Interpolation (in k-space)

Good class project
Better Idea:

- RF
- $G_z$
- $G_y$
- $G_x$

(Still not perfect)
- Make "horizontal projections"
- Encode phase in y prior to readout in x

Other Approaches to Imaging:

1. $\mathbf{B}$ localization
2. Saddle point
3. Local magnet
4. RF
5. Local coils
6. Local excitation
7. Non-linear gradients

Projection Reformation (Lauterbur)

$P_{-1} \cdot \{\text{Diameter}\} \Rightarrow \text{Projection}$

However... not quite perfect yet...
Complex signal & Demodulation

Physical Signal:
\[ s_p(t) = A(t) \cos(\omega_c t + \phi(t)) = \]
\[ \text{real} \rightarrow A(t) \cos(\phi(t)) \cos(\omega_c t) - \sin(\phi(t)) \sin(\omega_c t) \]
\[ I(t) \text{ in phase} \]
\[ Q(t) \text{ quadrature} \]

Analysis Signal (convenient):
\[ s_r = A(t) e^{-i \omega_c t + i \phi(t)} \]
\[ s_p = \text{Re} \{ s_r \} \]

Baseband Signal
\[ s(t) = s_r(t) e^{i \omega_c t} = A(t) e^{-i \phi(t)} = I(t) + i Q(t) \]

Quadrature Phase Sensitive Detection:
\[ s(t) \rightarrow \]
\[ \text{LPF} \rightarrow I(t) \]
\[ \sin(\omega_c t) \]
\[ \text{LPF} \rightarrow Q(t) \]
\[ S(t) = I(t) + i Q(t) \]

Often used in communication

\[ I(t) \rightarrow M_x \text{ in rotating frame} \]
\[ Q(t) \rightarrow M_y \text{ in rotating frame} \]