Principles of MRI
EE225E / BIO265
Lecture 13
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Solution for general Eq. at time t=T

\[ M_{xy} (\mathbf{r}, T) = i M_0 \int_{-\infty}^{T} \gamma B_1 (t) e^{-i 2\pi \mathbf{k}(t) \cdot \mathbf{r}} dt \]

where

\[ \mathbf{k}(t) = \frac{\gamma}{2\pi} \int_{t}^{T} \mathbf{G}(\tau) d\tau \]

k(t) is area of the remaining gradient

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Small Tip-Angle Approximation

Example: Slice Selection

\[ M_{xy} (z, T) = i M_0 \int_{0}^{T} \gamma B_1 (t) e^{-i 2\pi k_z (t) z} dt \]

- This is not exactly a Fourier transform
- Would like:

\[ M_{xy} (z, T) = i M_0 \int_{K} W(k) e^{-i 2\pi k_z z} dk_z \]

\[ W(k) = \frac{2\pi B_1 (k)}{|G(k)|} \]

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Example: Slice Selection

\[ M_{xy} (z, T) = i M_0 \int_{0}^{T} \gamma B_1 (t) e^{-i 2\pi k_z (t) z} dt \]

- First find k_z(t)
- Map B1(t) to k_z(t) -- account for speed!
- Compute the integral (Fourier transform)
Example: Slice Selection

- Plot $B_1(t)$ vs $k_z(t)$:

- $B_1$ is not centered in k-space
- We get the right magnitude $M_{xy}$

- But we get phase across the slice... signal cancels!
**k-Space Weighting Example**

- $B_1$
- $G_z$
- $k_z$

$B_1(t) \propto \text{sinc}(t)$

$\text{rect}(f)$

$B_1(t) \propto \text{sinc}(t)$

$\text{rect}(f)$

Not practical, since sinc is continuous indefinitely

**RF Pulse Design**

- Choose, $B_1(t)$ with a nice transform

$B_1(t) \propto \text{sinc}(t)$

$\text{rect}(f)$

**Truncated sinc**

$B_1(t) \propto \text{sinc}(t) \text{rect}(t/2N)$

$\text{rect}(f)*2N \text{sinc}(2Nf)$

Too much ripple!

**Windowed sinc**

$B_1(t) \propto \text{sinc}(t)w(t/2N)$

$\text{rect}(f)*2N W(2Nf)$

Hanning Window
Characterization of Pulse Shape

Time-Bandwidth Product

\[ T(BW) = (2N)1 = 2N \Rightarrow \text{Total # of zero crossings} \]

- TBW=2
- TBW=4
- TBW=8
- TBW=12

rapid imaging 180 90 slab and saturation

Slice Profile

TBW=2

TBW=12

IF WE FIX BANDWIDTH AND MAKE T LONGER

TBW=4

TBW=8

MORE SELECTIVE PROFILE.

IF WE FIX DURATION INCREASE BW

TBW=4

TBW=8

WIDER EXCITATION

SAME SHAPE

SAME SHAPE

TWICE AMPLITUDE

TWICE WIDTH
Typically in MRI we fix duration, and add the gradient amp. to compensate for the increase in BW.

\[ \text{SNW} \]

\[ \text{G}_2 \]

\[ \text{K}_2 \]

\[ \Rightarrow \text{same width} \]

**Example**

We want a T_{BW} = 8 ms pulse with 1 cm duration. What is the gradient amp. if BW = 4 kHz?

\[ T_{BW} = 8 \text{ms} \]

\[ 1 \text{cm} \approx 0.1 \text{m} \Rightarrow 0.1 \text{m} \times 4 \text{kHz} = 4 \text{kHz} \]

\[ \Delta z = 1 \text{cm} \]

\[ \frac{G \cdot \Delta z}{\text{BW}} = 4 \text{kHz} \]

\[ \left( \frac{4.25 \text{ kHz}}{G} \right) \cdot \left( \frac{0.01 \text{ m}}{0.1 \text{ m}} \right) = 4 \text{kHz} \Rightarrow G = 0.425 \text{ m}^{-1} \]

What if \( \Delta z = 0.05 \text{ cm} \)?

\[ G = 2.125 \text{ m}^{-1} \]

What if \( G_{max} = 0.94 \? \)

\[ T_{BW} = 4 \text{ms} \Rightarrow \text{less selective.} \]

\[ T = 4 \text{ms} \Rightarrow \text{low SNR duration.} \]
TBW=4, flip = 10, slice = 10mm, duration = 2ms

TBW=8, flip = 10, slice = 5mm, duration = 2ms
TBW = 8, flip = 45, slice = 5mm, duration = 2ms

Increased sidebands

TBW = 8, flip = 90, slice = 5mm, duration = 2ms

Profile still OK!

TBW = 8, flip = 180, slice = 5mm, duration = 2ms

180 small-tip Pulse
180
SLR Pulse

not a sinc!

higher bandwidth

Spectral-Spatial Pulse

“small-tip” 180

SLR 180

not a sinc!

higher bandwidth