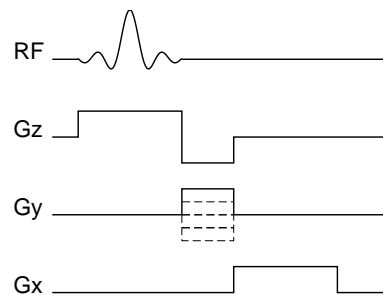


Assignment 4

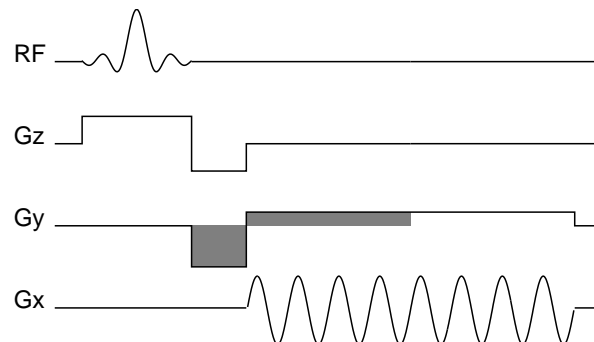
Due Friday Feb 21st, 2014, Self Grading Due Monday Feb 24th, 2014

1. Finish reading Nishimura Ch.4 and Ch. 5.
2. The following pulse sequences are defective in that they don't cover a symmetric region in k-space. For each, (i) sketch the region of k-space that each trajectory covers, and (ii) propose a modification that provides symmetric coverage.

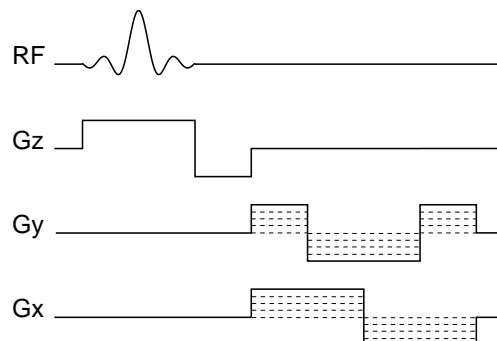
(a)



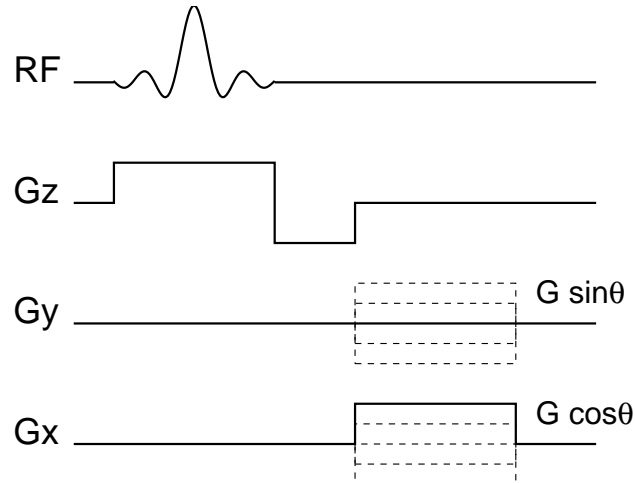
(b) The shaded areas are equal in this pulse sequence.



(c) The amplitude of the readout gradients are incremented together, so that for the i^{th} acquisition the gradient amplitudes are $G_x = G_y = G_{max} \left(\frac{i}{N} \right)$.



3. The following pulse sequence is performed repeatedly, with varying values of θ .

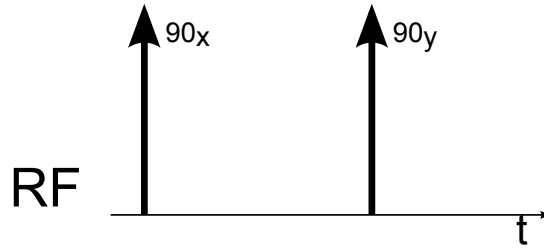


We would like the pulse sequence to cover a symmetric region of k-space.

- What range should the range of the angle θ be? Find the minimum and maximum values of θ , and don't worry about how finely θ is sampled. Sketch the resulting k-space trajectory, and k-space coverage.
- Compute the readout gradient duration if the maximum gradient is 4 G/cm, and we want 0.5 mm resolution.

4. RF Excitation (Midterm I 2012)

Consider the following sequence of two RF pulses:



The notation 90_x , 90_y means that B_1 is applied respectively in the x and y directions (in the rotating frame). Assume that the RF pulse is instantaneous. At time $t = 0$ the magnetization is: $[0, 0, M_0]^T$. You can neglect T_2 decay and T_1 recovery.

- a) What is the M_z component of the magnetization immediately after each RF pulse? explain.

$M_{z1} =$	$M_{z2} =$
------------	------------

- b) Unfortunately, your RF excitation coil produces a B_1 inhomogeneous field with $\pm 10\%$ variation in the flip angle. What is the resulting percentage variation in M_z due to the inhomogeneity (numbers!) ? Comment on the advantages or/and disadvantages of the 2-pulse scheme compared to a single RF when the target is to achieve maximum magnetization in the transverse plane.

$M'_{z1} = M_{z1} \pm$ comment :	$M'_{z2} = M_{z2} \pm$
-------------------------------------	------------------------

- c) The above analysis was done for spins at the Larmor frequency. However, due to field inhomogeneity some spins in the volume will precess at slightly different frequencies. What is the resulting M_z component of the magnetization after the second RF for spins with $\gamma \Delta B_0 = 250\text{Hz}$? Assume that the time between pulses is 1ms, so the magnetization precesses over 1/4th of a cycle in that time period. Comment on the sensitivity of this sequence to B_0 inhomogeneity.

5. This question is taken from midterm Sp '10
TRUE or FALSE

For each of the following statements, identify whether it is True or False. To get a score you need also to provide a **brief explanation** for either case.

- a) For sequences with short time-repetitions (TR), doubling the main field (B_0) always results in double the SNR.

TRUE/FALSE

- b) The FID from n Conollyum nuclei ($\frac{\gamma}{2\pi} = 4$ kHz/G) at 1.5T is identical to the FID from n Pinesume nuclei ($\frac{\gamma}{2\pi} = 1$ kHz/G) at 6T.

TRUE/FALSE

- c) (short T_2 , long T_1) species are easier to image *in vivo* than (long T_2 , short T_1) species.

TRUE/FALSE

6. **Matlab Exercise: Bloch simulation and spin visualization** In this exercise you will simulate and visualize the behavior of a spin in the presence of magnetic fields. For this purpose we will use a Matlab implementation of a Bloch simulator that was written by Prof. Brian Hargreaves of Stanford Radiology.

- **The Bloch simulator.** Download the Bloch simulator files : `bloch.m` and `bloch.c` from the class website. The file `bloch.c` is a mex file implementation that is used to accelerate the computation of the solution for the Bloch equation. You will first need to compile the executable for your particular computer architecture. In your Matlab command window type:

```
>> mex bloch.c
```

Ignore any warnings from the compilation. If you encounter problems compiling the the mex file you can try downloading the executable for your platform from the class website

Read the help for the function `bloch` by typing

```
>> help bloch
```

Here's a simple example showing T_2 decay and T_1 recovery:

```
>> dt = 4e-6; % 4 us sampling rate
```

```
>> rf_90 = 90/360/(4257*dt);
```

```
>> % impulse RF pulse
```

```
>> b1 = [rf_90;zeros(300e-3/dt,1)];
```

```
>> g = b1*0; % no gradient
```

```
% the spin is on-resonance
```

```
>> df = 0;
```

```
>> % the spin at iso-center
```

```
>> dp = 0;
```

```
>> t1 = 100e-3;
```

```
>> t2 = 50e-3;
```

```
>> % start at Mz
```

```
>> mx_0 = 0;
```

```
>> my_0 = 0;
```

```
>> mz_0 = 1;
```

```
>> % Simulate
```

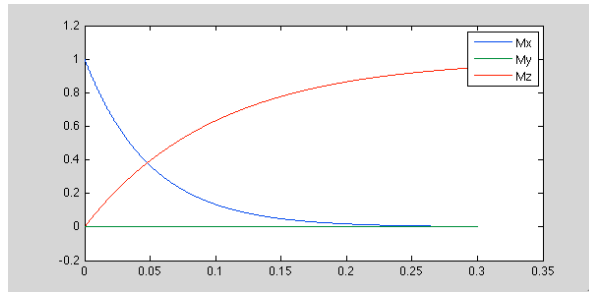
```
>> [mx,my,mz] = bloch(b1,g,dt,t1,t2,df,dp,2,mx_0,my_0, mz_0);
```

```
>> %plot
```

```
>> time = [1:length(mx)]*dt;
```

```
>> figure, plot(time,mx,time,my,time,mz); legend('Mx','My','Mz');
```

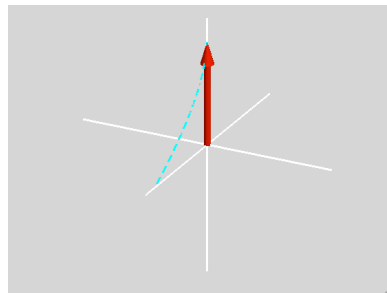
This is the plot you should get:



- Design an RF waveform, sampled at $4\mu\text{s}$ with maximum $|B_1| < 0.16\text{G}$ that produces a 90° rotation. What is the length of the pulse and what is the amplitude?
- Design a gradient waveform with $|G| < 4\text{G/cm}$ and $|\frac{dG}{dt}| < 15000\text{G/cm/s}$ that produces a rotation of 2 cycles for spins located at 0.2cm off iso-center. You can use the function you wrote in the previous homework! What is the length of the pulse?
- Perform a simulation of a period of $t = 100\text{ms}$, in which first the RF is applied, followed by the gradient and then no-field is applied. Simulate a spin at 0.2cm from iso-center with $T_1 = 30\text{e-}3$ and $T_2 = 15\text{e-}3$. Plot M_x, M_y , and M_z as a function of time.
- **Visualization:** Download the files: visualizeMagn.m, arrow3D.m, rotatePoints.m from the class website. The function visualizeMagn renders an animation of the spin and the fields. The first argument is a vector of the applied B_1 field in Gauss. The second argument is the vector of the effective B_0 field. In this case it will be the gradient field times the position of the spin. The third argument is an array of the magnetization vector vs time. It should be $[N \times 3]$ representing Mx My and Mz at each time point. The last argument is the acceleration factor of the rendering (you should use at least 10 if you are impatient).

This is the plot you should get for the previous example by running:

```
>> visualizeMagn(zeros(size(mx)), zeros(size(mx)), [mx,my,mz],100);
```



Render the simulation and submit the plot of the final result.

- Now, simulate and visualize the sequence 90RF, Gradient, -90RF for a spin at: 0cm, 0.025cm, 0.05cm, 0.075cm and 0.1cm. Use the same gradient you used previously and $T_1 = 1000$ and $T_2 = 1000$.

What can you say about the distribution of M_z in space after this sequence?

7. Bloch-Siegert B_1 Mapping ¹

Consider an MRI system with a main static magnetic field B_0 giving rise to a resonance frequency $\omega_0 = -\gamma B_0$. A transverse radio-frequency (RF) field, B_1 , is applied continuously to the system at an off-resonance frequency $\omega_{\text{RF}} = \omega_0 - \Delta\omega$. Consider a frame rotating at the RF frequency $\omega_{\text{RF}} = \omega_0 - \Delta\omega$.

- a) Write the expression for the effective applied magnetic field in the RF rotating frame.
(*Hint: no sines, cosines or exponentials are necessary*)

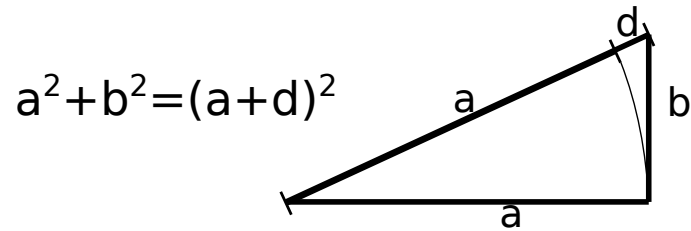
$B_{\text{eff}} = \underline{\hspace{2cm}} \hat{z} + \underline{\hspace{2cm}} \hat{i} + \underline{\hspace{2cm}} \hat{j}$

¹Based on the young investigator award paper: Sacolick L. et.al, MRM 2010;63(5):1315-22

- b) A Bloch-Siegert shift is defined as the increment in the effective magnetic field amplitude when an RF pulse is applied at an off-resonance frequency. In other words, it is how much the effective field vector lengthens when the off-resonance pulse is played. Now, consider the case where $B_1 = B_{1x}$ and the off-resonance is large, *i.e.*, $|\Delta\omega| \gg \gamma|B_1|$.

What is the increment in the magnetic field amplitude, B_{BS} , when the B_1 field is on? What is the approximate axis of precession? Can the RF field create excitation?

In your derivation, you can use the following trigonometric relationship and also assume that $d^2 \approx 0$ as long as $a \gg b$.



$B_{BS} =$	axis of precession:	excitation? Yes/No
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- c) Now, consider a spin with $M_z(0) = 0$ and transverse magnetization $M_{xy}(0) = M_0$. A constant off-resonance RF, B_1 , is applied at a frequency $\omega_{\text{RF}} = \omega_0 - \Delta\omega$ for a period of T seconds. How much phase will the spin accrue with respect to the case where no RF is applied?

$\phi_{\text{BS}} =$

- d) Note, that the relative phase accrual due to off resonance B_1 at the RF rotating will be the same relative phase accrual at the Larmor frequency rotating frame, ω_0 . So, we can go back to the usual ω_0 rotating frame. Regardless to what you got in parts (a)-(c), assume here that $M_{xy}(T) = e^{i\alpha B_1^2 T} M_0$.

The RF field B_1 has a typical amplitude variation in space of $\pm 20\%$ at 3T. In several applications, it is very useful to map the $B_1(x, y)$ amplitude variation. Based on (a)-(c), draw a slice selective imaging pulse sequence that provides information to map the magnitude of $B_1(x, y)$. Explain how would you calculate $B_1(x, y)$ from the resulting image. (You can assume that there is no off-resonance and no other spurious sources of phase)

$$B_1(x, y) =$$

Pulse sequence:

RF _____

Gz _____

Gx _____

Gy _____