**Sampling:**

**IDEA:** How frequently do you need to sample?

![Graph showing frequency and sampling](image1)

Too SLOW! (Looks constant)

**Sampling:**

#1 is right, but seems excessive. 
#2 is too slow. 
What is the minimum rate?

In general, rate $> \Delta B_w$

**Looks RIGHT!**
**Sampling in k-space**

In MRI, $F(k)$ is sampled over a limited range.

Problems:
- Now → **1. Discrete Sampling**
- 2. Finite Support

Recall:

- $F(k) \Rightarrow \text{IFFT}(\Delta k x) = \frac{1}{\Delta k} \sum_{n=-\infty}^{\infty} f(n \Delta k) e^{-i 2\pi n k}$

So,

- $\Delta k \text{IFFT}(\frac{f}{\Delta k}) = \text{IFFT}(\Delta k x)$

also recall

- $F(k) \cdot G(k) \Rightarrow \hat{f}(x) \ast \hat{g}(x)$

Recall:

- $f(x) \ast \delta(x-x_0) = f(x-x_0)$

**Model sampling using impulse train.**

- $\sum_{k=-\infty}^{\infty} \delta(k - n \Delta k)$
- $f(k) = \frac{1}{\Delta k} \sum_{n=-\infty}^{\infty} f(n \Delta k)$

**Sample at $f(k)$**

Recall:

- $f(k) \cdot \delta(k-k_0) = f(k_0) \delta(k-k_0)$
If $f(x)$ is \textit{space limited}:

$$f_{\text{alias}} = f(x) \times U_{\text{FOV}}(dx)$$

\[ \text{Aliasing due to large } \Delta x \]

\[ \text{Sampling in } k\text{-space} \]

In MRI, $F(k)$ is sampled over a limited range.

- \text{Discrete sampling}
- \text{Finite support}
Finite support sampling

\[ F_W(k) = F(k) \mathcal{H}\left(\frac{k}{W_k}\right) \]

\[ \Rightarrow f_W(x) = f(x) * W_k \text{sinc}(W_k x) \]

**Quiz**

Rank the "strength" of artifact due to k-space truncation

- \( f(x) \)
- \( F(k) \)
- \( f_W(x) \)

Discrete finite sampling

Together:

\[ \hat{F}_W(k) = F(k) \frac{\partial}{\partial k} \mathcal{H}\left(\frac{k}{W_k}\right) \mathcal{H}\left(\frac{k}{W_k}\right) \]

\[ \Rightarrow \hat{F}_W(k) = f(x) * \mathcal{H}\left(\Delta k x\right) * W_k \text{sinc}(W_k x) \]
...But images are discrete pixels... so $f_w(x)$ is sampled.

$\mathcal{F}[f_w(x)] = F_w(k)$

**Spatial Harmonics & DFT**

This is a spatial harmonic $(-1,1)$

$\hat{f}(x,y) = \cos(\frac{1}{3}(3x+y))$

**DFT (Centered)**

$$F[m] = \sum_{n=-N}^{N-1} f[n] e^{-j\frac{2\pi mn}{N}}$$

$$f[n] = \frac{1}{N} \sum_{m=-N}^{N-1} F[m] e^{j\frac{2\pi mn}{N}}$$

**MATLAB:**

```matlab
DFT = fftshift(fft(fftshift(f)))
IDFT = ifftshift(fftshift(ifft(f)))
```

**1D Fourier Transforms**

$$F(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi (k_x x + k_y y)} \, dx \, dy$$

**What is the 2D-FT of**

$$f(x,y) = \cos(\frac{1}{3}(3x+y))$$

**A:**

$$f(x,y) = \cos(\frac{1}{3}(3x+y))$$

```plaintext
(5,5)
```
 separable functions

\[ f(x,y) = f(x)f(y) \]

Q. Which is the 2D FT?

A. (i) \( F(k_x) \ast F(k_y) \)
   (ii) \( F(k_x) \cdot F(k_y) \)

2D sampling example

GIVEN \( f(x,y) \):

\[ 15 \text{ cm} \]
\[ 15 \text{ cm} \]

what is \( \Delta k_x \), \( \Delta k_y \)?

(to avoid aliasing)

2D sampling example

GIVEN \( f(x,y) \):

\[ 15 \text{ cm} \]
\[ 15 \text{ cm} \]

what is \( \Delta k_x \), \( \Delta k_y \)?

(to avoid aliasing)
CIRCULARLY SYMMETRIC

\[ f(x, y) = f(r) \]
\[ \mathcal{F}\{f(r)\} = \frac{1}{2\pi} \int_{0}^{\infty} r f(r) J_0(2\pi r) dr \]

Hankel transform of zeroth order
... also circularly symmetric

\[ M(r) \rightarrow \frac{J_0(2\pi f_k)}{2\pi f_k} \triangleq \text{sinc}(f_k) \]

like a sinc... but different
\[
\frac{J_1(\pi r)}{2r}
\]