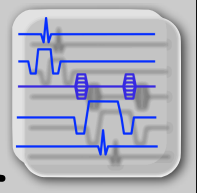


Principles of MRI

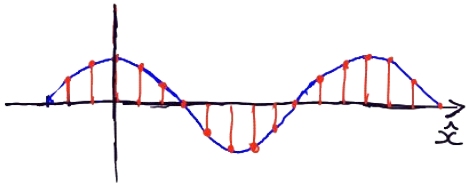
EE225E / BIO265

Lecture 04

Instructor: Miki Lustig
UC Berkeley, EECS

Sampling:

IDEA: How frequently do you need to sample?

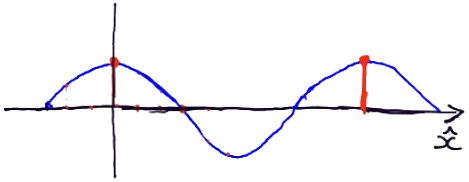


LOOKS RIGHT!

(2)

Sampling:

IDEA: How frequently do you need to sample?

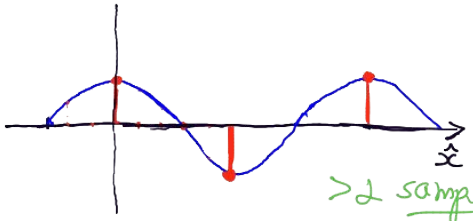


Too slow!
(Looks constant)

(3)

SAMPLING

#1 is right, but seems over excessive
#2 is too slow
What is the minimum rate?



>2 sample period

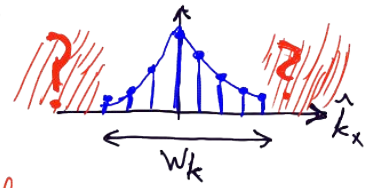
In general rate $> 2 \cdot BW$

show aliasing

sampling in k-space

(4)

In MRI $F(k)$ is sampled over a limited range



Problems:

Now → (1) DISCRETE SAMPLING

(2) FINITE SUPPORT

sampling in k-space

(4a)

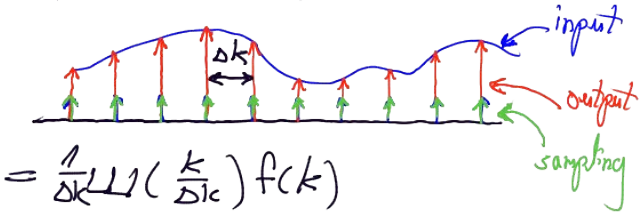
Recall:

$$f(k)\cdot\delta(k-k_0) = f(k_0)\,\delta(k-k_0)$$

1 sample @ $f(k_0)$

Model sampling using impulse train:

$$\left[\sum_{k=-\infty}^{\infty}\delta(k-n\Delta k)\right] f(k) =$$



$$= \frac{1}{\Delta k}\,Ш\!\left(\frac{k}{\Delta k}\right) f(k)$$

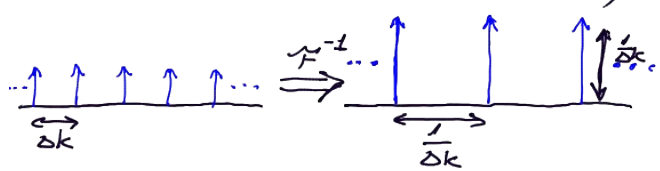
Recall:

(5)

$$F\left(\frac{k}{a}\right) \xRightarrow{\mathcal{F}^{-1}} |a|\, f(ax)$$

So,

$$\frac{1}{\Delta k}\,Ш\!\left(\frac{k}{\Delta k}\right) \xRightarrow{\mathcal{F}^{-1}} Ш(\Delta k\, x)$$



also recall

$$F(k)\cdot G(k) \xRightarrow{\mathcal{F}^{-1}} f(x) * g(x)$$

So,

(6)

if $\hat{F}(k) = F(k)\cdot\frac{1}{\Delta k}\,Ш\!\left(\frac{k}{\Delta k}\right)$ (sampled)

$$\Downarrow \mathcal{F}^{-1}$$

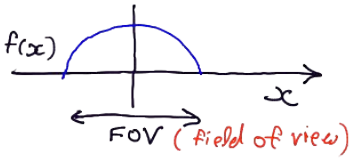
then

$$\hat{f}(x) = \mathcal{F}^{-1}\{\hat{F}(k)\} = f(x) * Ш(\Delta k\, x)$$

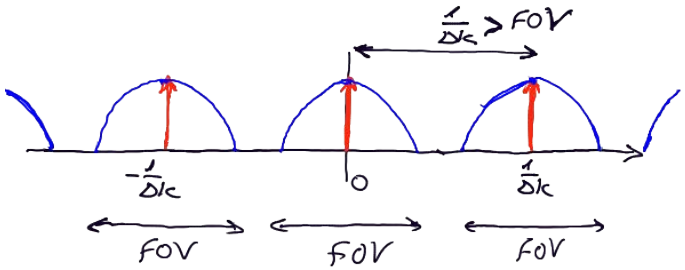
Recall:

$$f(x) * \delta(x-x_0) = f(x-x_0)$$

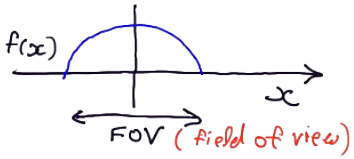
If $f(x)$ is **SPACE LIMITED**: (7)



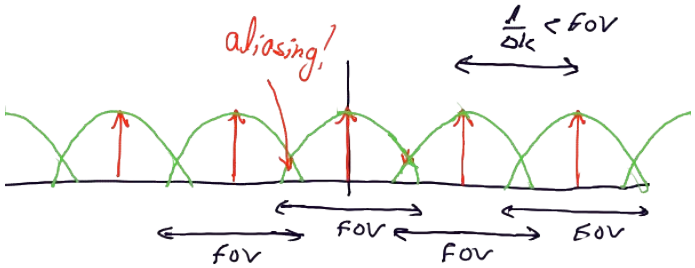
$$\hat{f}(x) = f(x) * \text{Ш}(\Delta k x)$$



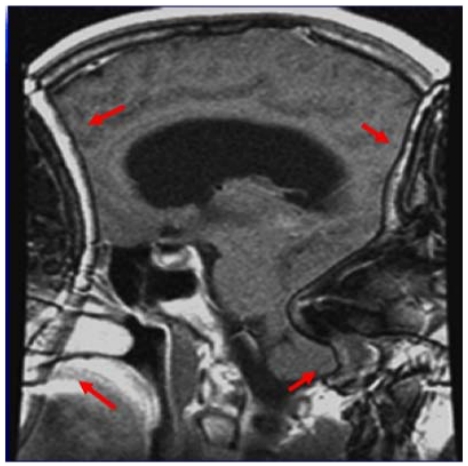
If $f(x)$ is **SPACE LIMITED**: (8)



$$\hat{f}(x) = f(x) * \text{Ш}(\Delta k x)$$

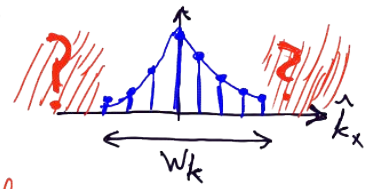


Aliasing in x due to large Δk_x (8a)



<u>sampling in k-space</u> (9)

In MRI $F(k)$ is sampled over a limited range



Problems:

1. DISCRETE SAMPLING

Now → 2. FINITE SUPPORT

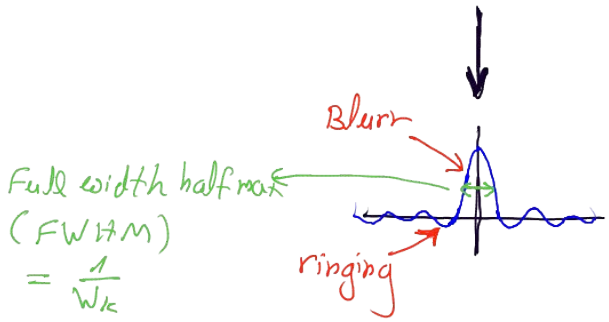
Finite support sampling

(10)

$$F_W(k) = F(k)\,\Pi\left(\frac{k}{W_k}\right)$$

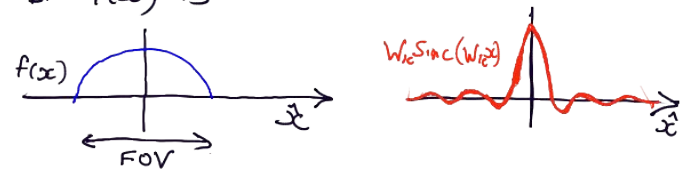
"windowed"

$$\Rightarrow f_W(x) = f(x) * W_k \,\mathrm{sinc}(W_k x)$$

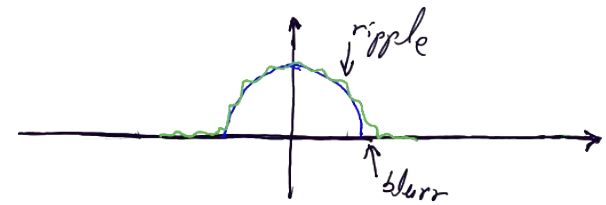


(11)

If $f(x)$ is



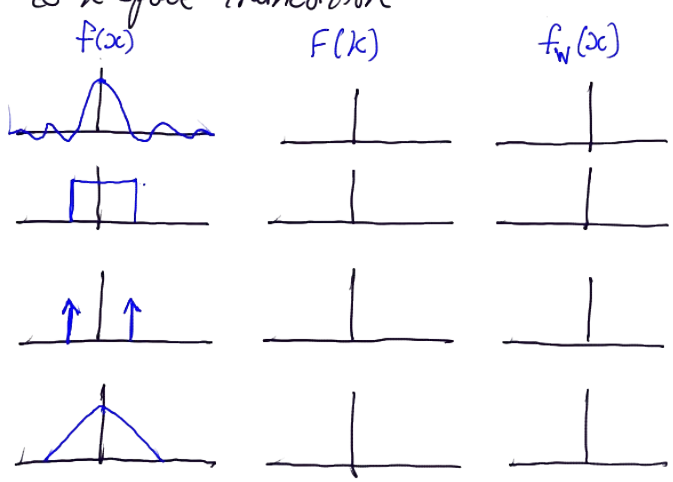
$$f_W(x) = f(x) * W_k \,\mathrm{sinc}(W_k x)$$



QUIZ

(11a)

Rank the "strength" of artifact due to k-space truncation



Discrete & finite sampling

(12)

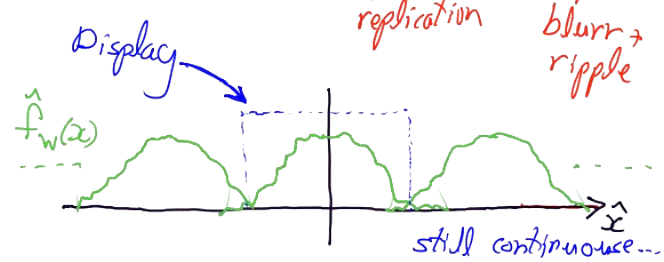
Together:

$$\hat{F}_W(k) = F(k)\,\frac{1}{\Delta k}\,\mathrm{Ш}\left(\frac{k}{\Delta k}\right)\Pi\left(\frac{k}{W_k}\right)$$

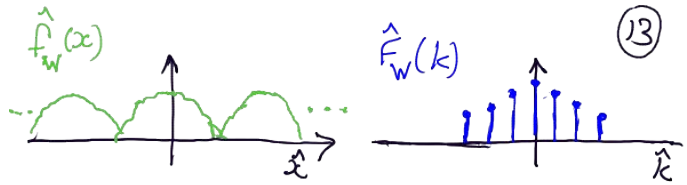
$$\overset{\mathcal{F}^{-1}}{\Longrightarrow} \quad \hat{f}_W(x) = f(x) * \mathrm{Ш}(\Delta k x) * W_k \,\mathrm{sinc}(W_k x)$$

replication

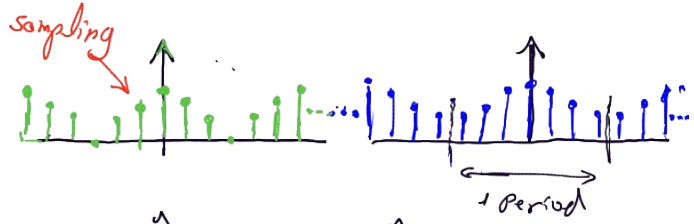
blurr + ripple



(13)



...But images are discrete pixels...
so $\hat{f}_w(x)$ is sampled



$$\hat{f}_w(x) \Longleftrightarrow \hat{F}_w(k)$$

$$f_w[n] \overset{DFT}{\Longleftrightarrow} F_w[m]$$

integer

(14)

DFT (centered)

$$F[m] = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} f[n]\, e^{-i2\pi \frac{mn}{N}}$$

$$f[n] = \frac{1}{N} \sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} F[m]\, e^{+2\pi \frac{mn}{N}}$$

$\boxed{\text{FAST computation with FFT}}$

MATLAB:

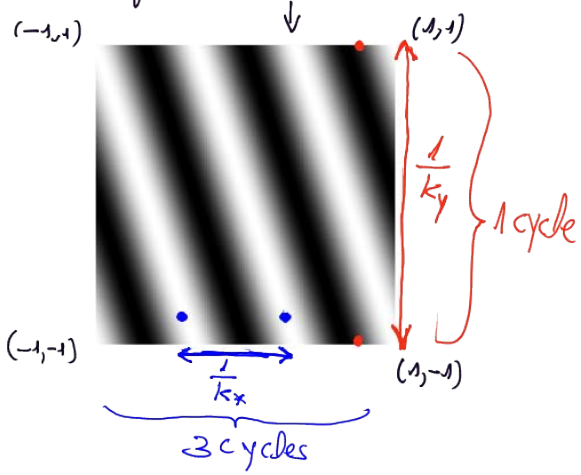
DFT = fftshift(fft(fftshift))

IDFT = fftshift(ifft(ifftshift))

(15)

SPATIAL HARMONICS & 2D FT

This is a spatial harmonic



$$f(x,y) = \cos(2\pi(3x+y))$$

(16)

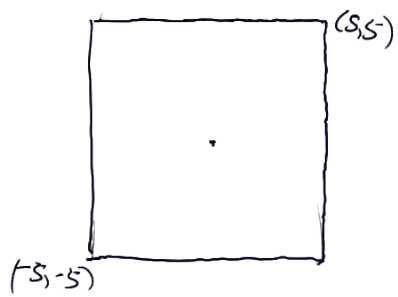
2D FOURIER TRANSFORMS

$$F(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-i2\pi(k_x x + k_y y)}\, dx\, dy$$

Q: what is the 2D-FT of

$$f(x,y) = \cos(2\pi(3x+y))$$

A:



seperable functions

(17)

$f(x,y) = f(x)f(y)$

Q. Which is the 2D-FT?

A. (1) $F(k_x) * F(k_y)$

(2) $F(k_x) \cdot F(k_y)$

sampling & truncation

(18)

$$\hat{F}_W(k_x,k_y) = F(k_x,k_y) \cdot \frac{1}{\Delta k_x \Delta k_y} \text{Ш}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right) \cdot \Pi\left(\frac{k_x}{W_x}, \frac{k_y}{W_y}\right)$$

aliasing

$$\hat{f}_W(x,y) = f(x,y) ** \text{Ш}(\Delta k_x x, \Delta k_y y) ** W_x W_y \operatorname{sinc}(W_x x, W_y y)$$

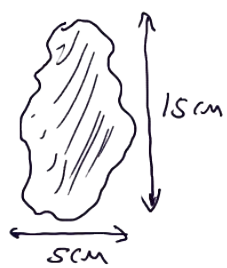
Blurr

more degrees of freedom for sampling!

2D SAMPLING EXAMPLE

(19)

GIVEN $f(x,y)$:



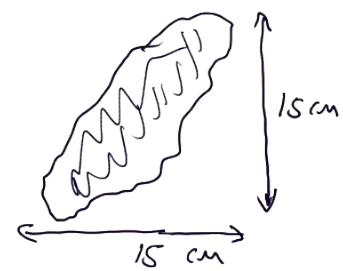
what is Δk_x, Δk_y?

. (to avoid aliasing)

2D SAMPLING EXAMPLE

(20)

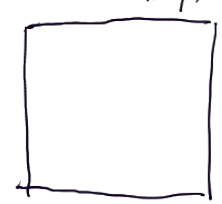
GIVEN $f(x,y)$:



what is Δk_x, Δk_y?

. (to avoid aliasing)

$F(k_x,k_y)$



CIRCULARLY SYMMETRIC

(4)

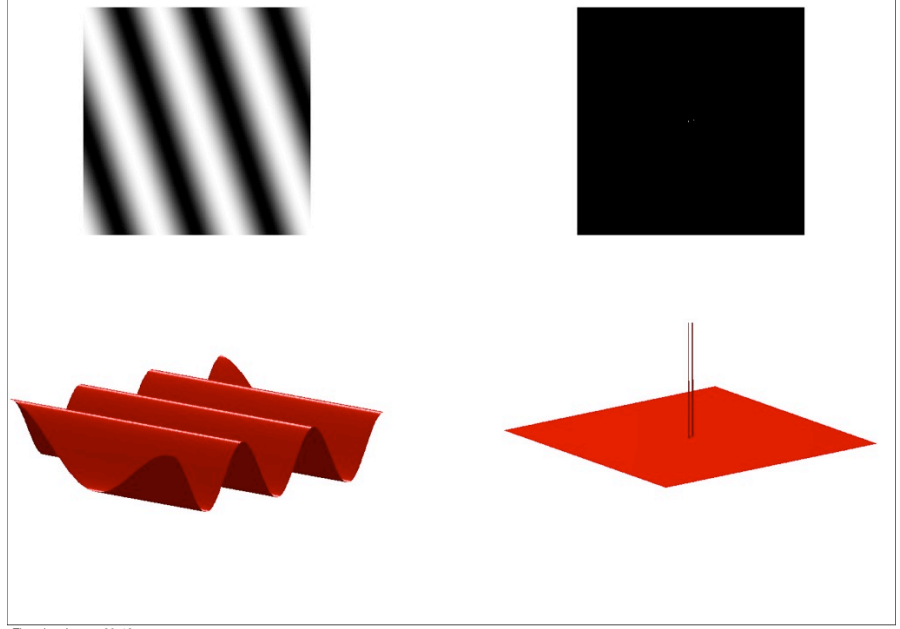
$$f(x,y) = f(r)$$

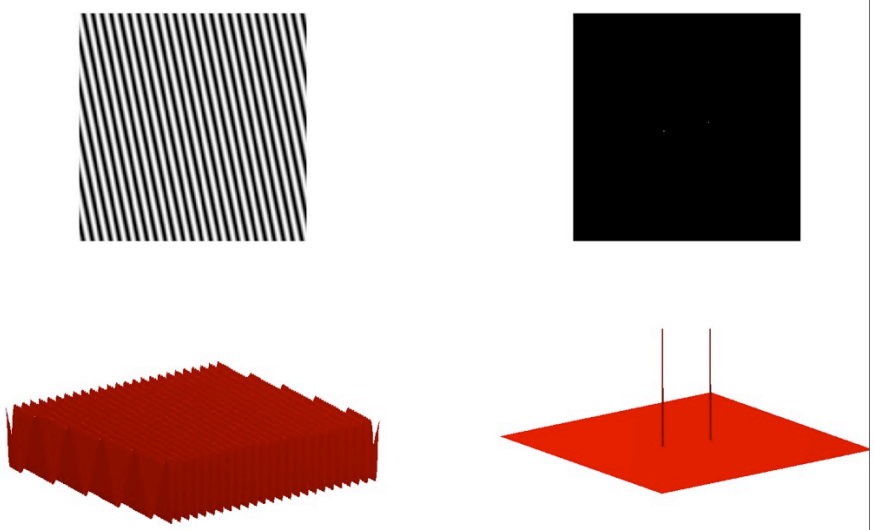
$$\mathcal{F}_{2D}\{f(r)\} = 2\pi \int_0^\infty r f(r) J_0(2\pi r \rho_k)\, dr$$

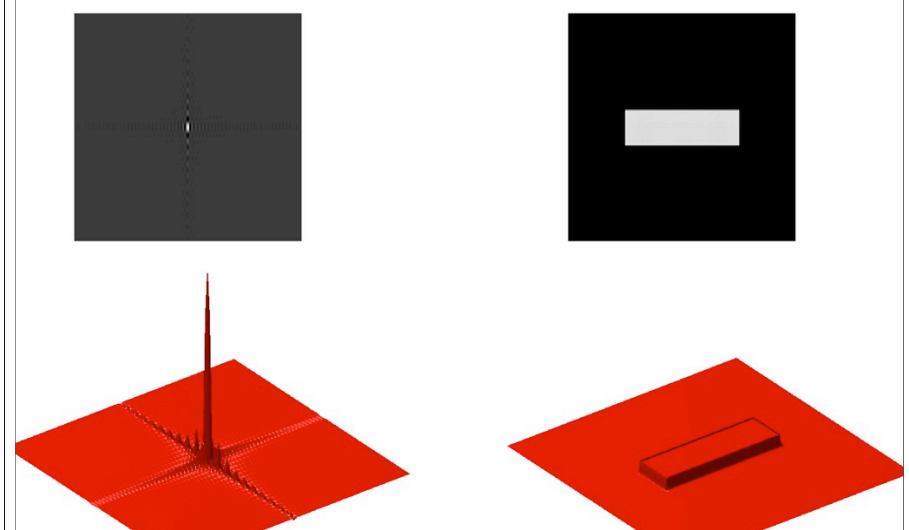
Hankel xform of zero'th order
... also circularly symmetric

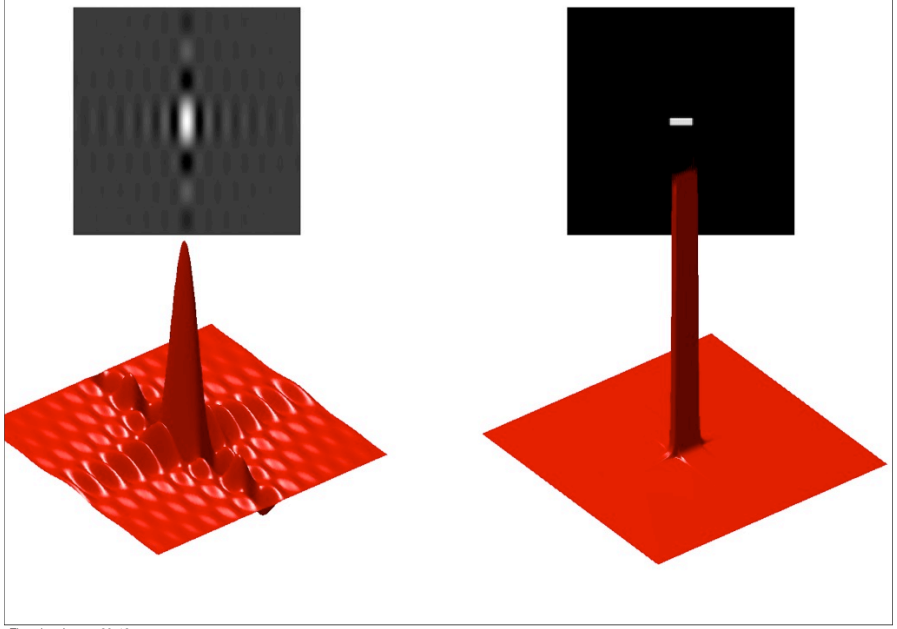
$$M(r) \Longrightarrow \frac{J_1(\pi \rho_k)}{2\rho_k} \triangleq \mathrm{Jinc}(\rho_k)$$

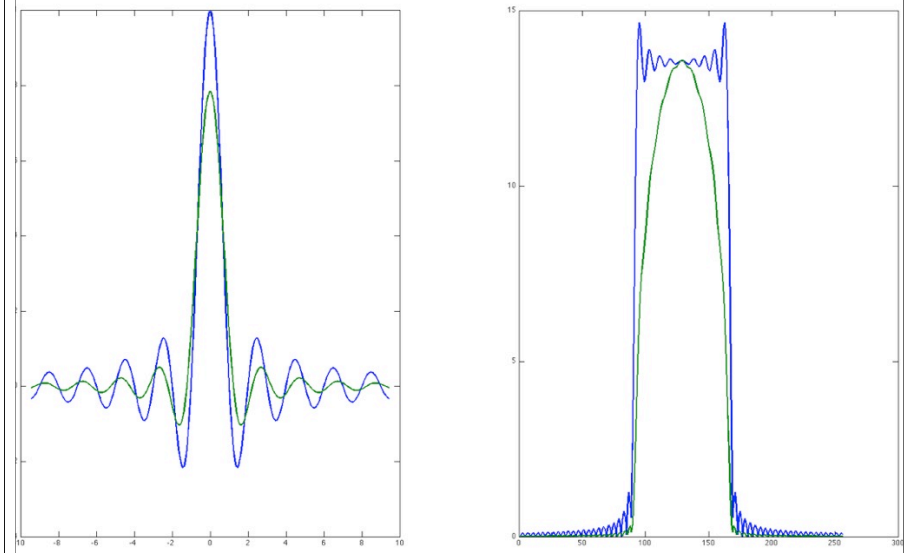
like a sinc ... but different



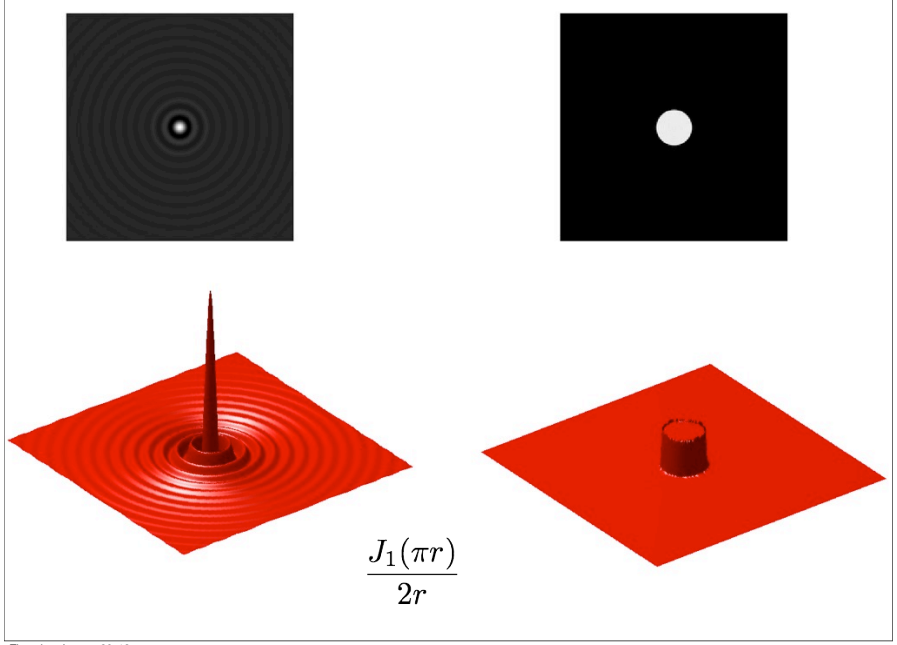








$$\frac{J_1(\pi x)}{2x} \qquad \frac{\sin(\pi x)}{\pi x}$$



$$\frac{J_1(\pi r)}{2r}$$