What is this?

- The first NMR spectrum of ethanol 1951.

Spatial Frequency

- Vinyl Record
  - Transforms a temporal signal to a spatial signal

Today

- Last time:
  - Linear systems, Fourier Transforms, Sampling

- Today:
  - Ch 3. Overview
    - Classical description of MRI
    - Basic Imaging

- Homework Due tonight!
What is the frequency?

\[ \cos(2\pi (f_x x + f_y y)) \]

(-1,1) (1,1)

(-1,-1) (1,-1)

a) \( f_x = 1, f_y = 2 \)
b) \( f_x = 2, f_y = 1 \)
c) \( f_x = 4, f_y = 2 \)
d) \( f_x = 2, f_y = 4 \)

What is the Temporal Frequency?

Vinyl rotates at 1 Hz

(-1,1) (1,1)

(-1,-1) (1,-1)

a) \( \cos(2\pi 8t) \)
b) \( \cos(2\pi 8t^2) \)
c) \( \cos(2\pi 4t) \)
d) \( \cos(2\pi 4t^2) \)

What is the Temporal Frequency?

Vinyl rotates at 1 Hz

(-1,1) (1,1)

(-1,-1) (1,-1)

a) \( \cos(2\pi 100t) \)
b) \( \cos(2\pi 100t^2) \)
c) \( \cos(2\pi 40t) \)
d) none of the answers

Aliasing!
Classical Description of MR

• Atoms with odd # of protons/neutrons have nuclear spin angular momentum
  – Intrinsic QM property (ch. 4)
  – Also intrinsic magnetic moment
• Like Spinning magnetic dipoles
• In biological tissue:
  – Mostly $^1$H in H$_2$O
  – Sometimes $^{31}$P, $^{13}$C, $^{23}$Na - Exotic

Interaction of magnetization with 3 fields
  – B0 - Main field ⇒ Polarization and resonance
  – B1 - RF field ⇒ Signal production + reception
  – G - Gradient fields ⇒ Spatial encoding

B0 - Main Field

• Produces polarization of sample $M_0$
• Resonance at Larmor frequency
  \[ \omega = -\gamma B_0 \]
  \[ \gamma = \frac{\gamma}{2\pi} = 4.257 \text{ KHz/G} \]
• For Protons:
  \[ \gamma = 2\pi 4.257 \text{ Krad/G} \]
• Others:
  \[ \frac{\gamma}{2\pi} = 1.127 \text{ KHz/G} \]
  \[ \frac{\gamma}{2\pi} = 1.071 \text{ KHz/G} \]
  \[ \frac{\gamma}{2\pi} = -0.43 \text{ KHz/G} \]

Typical $B_0$

<table>
<thead>
<tr>
<th>$B_0$</th>
<th>Frequency</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1T</td>
<td>4.2MHz</td>
<td>Very Low!</td>
</tr>
<tr>
<td>0.5T</td>
<td>21MHz</td>
<td>Low (permanent/ resistive)</td>
</tr>
<tr>
<td>1.5T</td>
<td>63MHz</td>
<td>&quot;High&quot; Diagnostic (superconducting)</td>
</tr>
<tr>
<td>3T</td>
<td>127MHz</td>
<td>&quot;High&quot; Diagnostic (superconducting)</td>
</tr>
<tr>
<td>4T</td>
<td>170MHz</td>
<td>Rare</td>
</tr>
<tr>
<td>7/9.4T</td>
<td>300/400 MHz</td>
<td>Very High - research only</td>
</tr>
</tbody>
</table>
B0 Field

• For Spatial/Spectral Localization we require homogeneity
  \[ \Delta B_0 \sim 1 \text{ppm} \text{ over } 40\text{cm}^3 \text{ FOV} \]

• This is: 64Hz @ 1.5T
  – Pretty remarkable!

Why Resonance

• For a bar magnet
  – Torque, but no resonance
  – Missing angular momentum

• Resonance is like a spinning-top

B1 - RF Field

• Can’t Directly Detect \( M_0 \)
  – \( M_{\text{induced}} = \mu_0^{-1} V \chi B \quad \chi \approx 4 \cdot 10^{-9} \)
  Q: Why?
  A: Huge field!

• Resonance is the key!
  – \( B_0 \) is DC while spins resonate \( \Rightarrow \) Detection!
  – Sample resonates at \( \omega_0 = -\gamma B_0 \)

• Excite magnetization off the z direction
  – Apply RF field at \( \omega_0 = -\gamma B_0 \) in the x-verse plane
  – Has to be resonant to do something

RF Excitation

• In the lab frame
  \[ B_1(t) = A\hat{y} \]
RF excitation

• In the lab frame

\[ B_1(t) = A e^{-i\omega_0 t} \]

RF excitation

• In the rotating frame @\( \omega_0 \)

\[ B_1(t) = A \hat{y}_{\text{rot}} \]

RF Excitation

• In the rotating frame: Precession about \( B_1 \)

• Typical \( B_1 \)'s: 0.14 - 0.35G ± (10%/20%) accuracy @ (1.5T/3T)
• Duration 1-3ms which is a long time at 64MHz!
• Peak power 20KW!

\[ \omega_0 = \gamma B_1 = 4.257 \cdot 0.16 = 0.68 \text{ KHz} \]
\[ 0.367 \text{ms } 90^\circ \Rightarrow 23000 \text{ rotations} \]

Relaxation

• \( T_1 \): Longitudinal relaxation \( \sim 10\text{-}2000\text{ms} \)
• \( T_2 \): Transverse relaxation \( \sim 10\text{-}300\text{ms} \)
• Main source of contrast (Later!)
• \( T_2 < T_1 \) Always!
RF Reception

- Precession enduces EMF in coil: Faraday’s law ⇒ Free induction decay

\[ V(t) \]

\[ FID \]

After demodulation

\[ t \]

Gradient Fields

- B1 has poor localization \( \lambda @ 64 \text{MHz} \sim 0.5 \text{m} \) in tissue
- Instead encode position in frequency

\[ \vec{G} = \left[ \frac{\partial B_z}{\partial x}, \frac{\partial B_z}{\partial y}, \frac{\partial B_z}{\partial z} \right] \]

- Small concomitant fields Bx, By are also created. These do not contribute much to precession - fields are NOT oscillating at Larmor freq.

\[ \omega(x) = \omega_0 + \gamma G_x x \]

- Typical #
  - \( G = 1-10 \text{ G/cm} = 10-100 \text{ mT/m} = 4.2-42 \text{ KHz/cm} \)
  - Waveforms in audio frequency
  - Slew-Rate = 15-20 G/cm/ms
    - Safety concern is in dB/dt
    - Peripheral nerve stimulation can happen
  - Big Amplifiers: 1200 Volts, 200 Amps