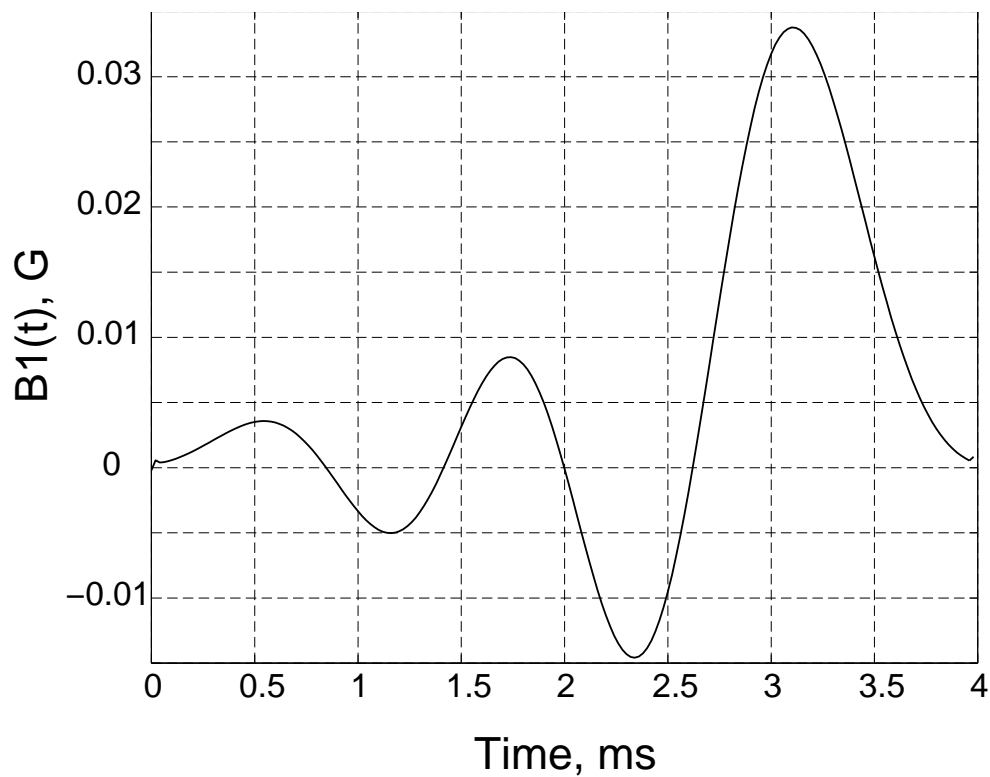


Assignment 6

Due Friday March 4th, 2016, Self Grading Due Monday March 7th, 2016

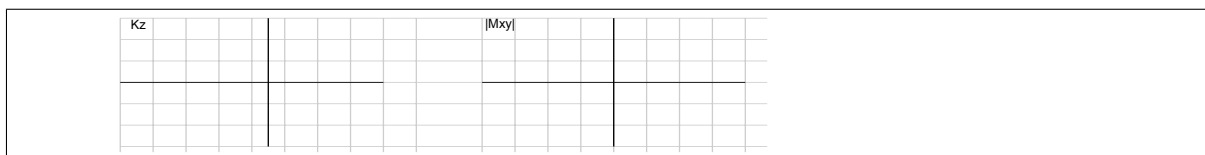
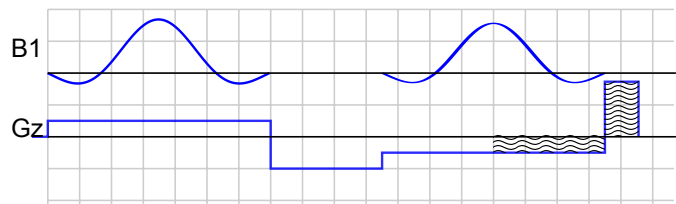
1. Read Nishimura Ch. 6
2. Nishimura assignment 6.5
3. Minimum-Phase RF Excitation
 - a) We would like to use the waveform plotted below as an RF excitation pulse. Assume that the slice select gradient is applied during the pulse, and then inverted to refocus the slice. What is the duration of the refocusing gradient that produces the maximum signal?
 - b) If we refocus for 2 ms, which is 1/2 the slice select gradient length, how does the signal compare to that of part (a)?



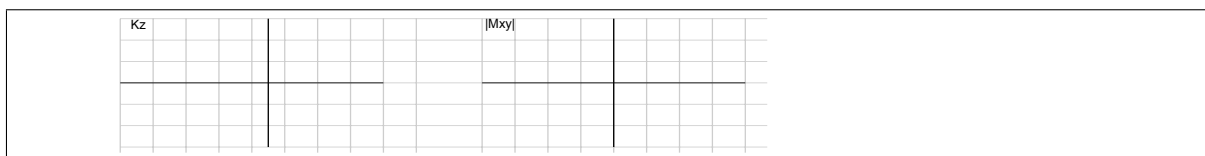
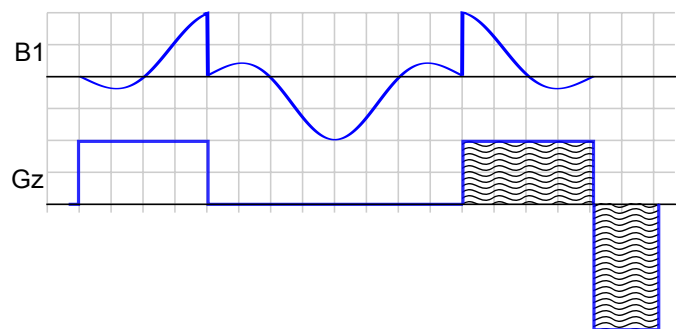
4. From Midterm I 2011: RF Excitation and Excitation k -space

For each of the following selective excitations, find (qualitatively) the associated excitation k -space and the magnitude of the slice profile at the end of the pulse, e.g., $|M_{xy}(z, T)|$. Draw them qualitatively, pointing out the “interesting” parts. Assume small tip-angle approximation.

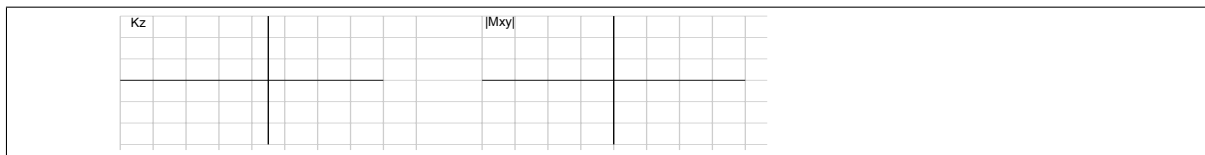
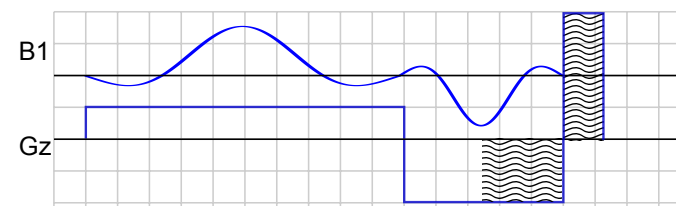
a) What do these pulses do?



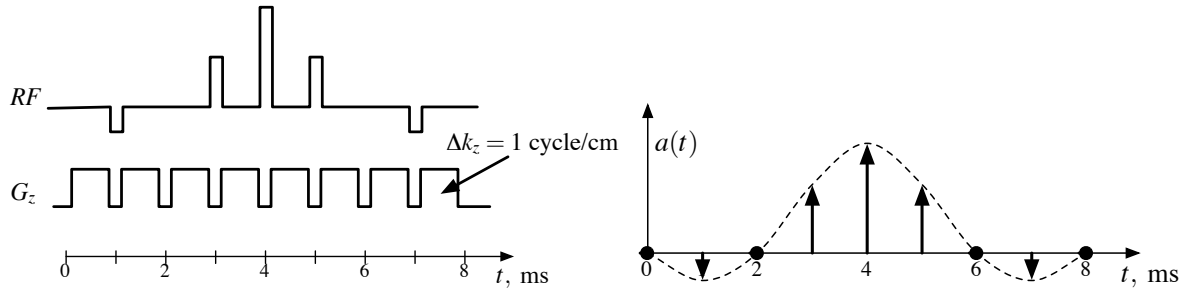
b) What do these pulses do?



c) What do these pulses do?



5. In this problem we will determine the excitation profile of the following pulse, below on the left,

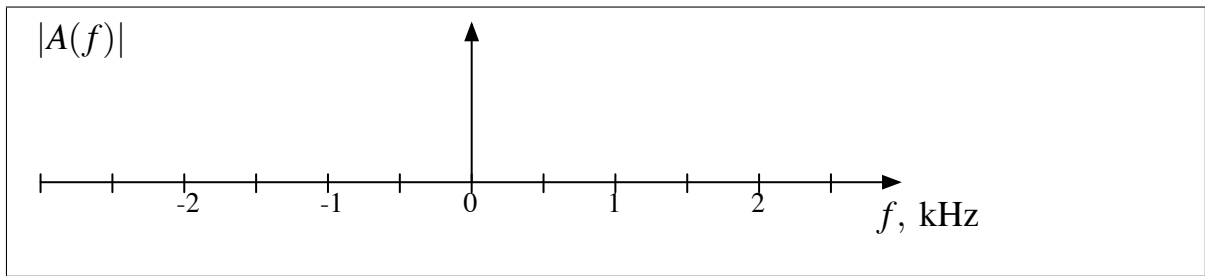


This is a small-tip-angle pulse. The RF and gradient are applied alternately. One lobe of the gradient produces a change in k_z of

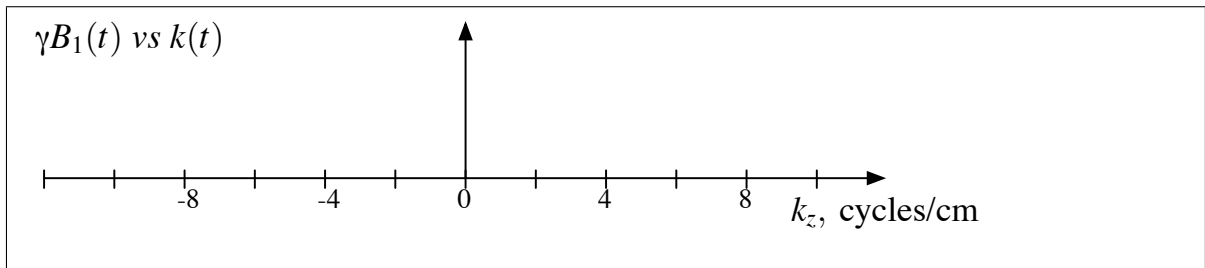
$$\Delta k_z = \frac{\gamma}{2\pi} \int_0^1 G(\tau) d\tau = 1 \text{ cycle/cm}$$

The area of each RF subpulse corresponds to a sample of a $TBW = 4$ windowed sinc shown above on the right.

- a) Plot the magnitude of the spectrum $A(f)$ of the sampled windowed sinc. You do not have to compute an expression. Label the axes of the plot.

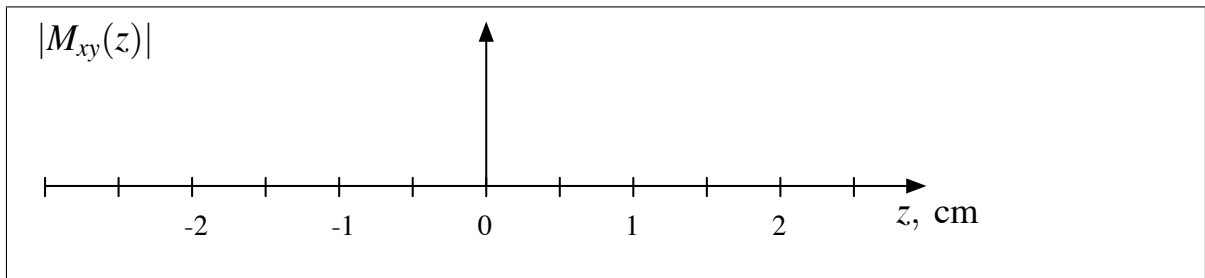


- b) Plot the k-space weighting that the RF pulse produces.



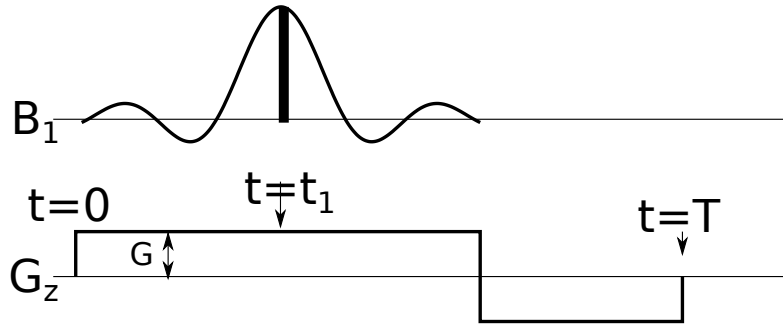
- c) Plot the magnitude of the magnetization $|M_{xy}(z)|$ produced by the RF pulse.

Hint: Approximate the RF pulse as an ideal sampled pulse as in (a) with a constant gradient.



6. Relaxation During RF Pulses (From Midterm II 2012)

When deriving the small-tip-angle approximation for slice selective RF pulses we assumed that the RF pulse duration is much smaller than T_2 , and so it can be neglected. This is a good approximation in general, but fails when the T_2 relaxation is short. In this question we will explore the case when T_2 relaxation can not be neglected anymore. We will still assume the small-tip-angle approximation in which $M_z = M_0$ for the entire pulse duration. Under this approximation, at each time point new transverse magnetization is created and evolves independently. For example, consider the following pulse



The B_1 field at time $t = t_1$ will produce new transverse magnetization $\Delta M_{xy}(z, t_1) = iM_0 \sin(\gamma B_1(t_1)\Delta t) \approx iM_0 \gamma B_1(t_1)\Delta t$

- a) The new x-verse magnetization $\Delta M_{xy}(z, t_1)$ will evolve over time. Find an expression for it at the end of the pulse (at time T). Assume transverse relaxation T_2 .

$\Delta M_{xy}(z, t)$ at time $T =$

- b) Show that the slice profile $M_{xy}(z, T)$ has the form of

$$M_{xy}(z, T) = i\gamma M_0 \int_0^T \tilde{B}_1(t) e^{-i2\pi k_z(t)z} dt$$

Find the expression for the "effective RF field" $\tilde{B}_1(t)$ as a function of the original $B_1(t)$. What is $k_z(t)$?

$\tilde{B}_1(t) =$	$k_z(t) =$
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- c) The slice profile can be expressed as a Fourier transform in excitation k-space

$$M_{xy}(z, T) = i\gamma M_0 \int_{k_z} \tilde{B}_1(k_z) e^{-i2\pi k_z z} dk_z.$$

For the above pulse, find the expression for t as a function of k_z , then find the "effective" RF field in k -space $\tilde{B}_1(k_z)$.

$\tilde{B}_1(k_z)$ has the form: $\tilde{B}_1(k_z) = C \cdot W(k_z) B_1(k_z)$ where C is a constant and $W(k_z)$ is a function of k_z

$\tilde{B}_1(k_z) =$	$t(k_z) =$
----------------------	------------

- d) What will be the (two) effects of T_2 relaxation on the actual slice profile? Explain.

Effect I:

Effect II:

- e) The T_2 of white matter is two orders of magnitude longer than the T_2 of myelin. Design (qualitatively) a slice selective RF pulse that will mostly excite myelin.

B_1 _____

G_z _____

7. Design the following slice-selective excitation pulses. In each case, choose a time-bandwidth product that is a multiple of four. The maximum gradient strength is 4 G/cm. Assume the excitation pulse is a Hamming windowed sinc, as we described in class, and that we want the sharpest profile within the constraints.
- Design a pulse to excite a 3 mm slice. The pulse should be 1 ms in duration. Find the time-bandwidth of the pulse, and the amplitude of the slice-select gradient.
 - Design a pulse to excite a 8 cm slab. Assume we want to use a time-bandwidth 16 pulse, and that the shortest time we can play this waveform is 2 ms, due to limits on peak RF amplitude. What is the gradient amplitude?
 - A 2 ms, time-bandwidth 8 pulse is to be used to excite a 1 cm slice centered at +12 cm from gradient isocenter (the zero point of the gradients). Find the gradient amplitude for this pulse, and the frequency for the RF pulse compared to the slice at isocenter.
8. Introduction: This assignment concerns typical Fourier transform designs of excitation pulses. This includes designing windowed sinc pulses, calculating the RF amplitude required, simulating slice profiles, designing a pulse for a specific application, and computing the relative SAR of a pulse sequence. For simulating the slice profile, you will need the bloch simulator.

a. Design of Windowed Sinc RF Pulses Write an m-file that computes a Hamming windowed sinc pulse, given a time-bandwidth product, and number of samples.

```
>> rf = wsinc(timebandwidth, samples)
```

Write the mfile so that it scales the waveform to sum to one, $\text{sum}(\text{rf}) = 1$. Plot windowed sines with TBW of 4, 8, and 12. The TBW=4 pulse is common for 180 degree pulses, the TBW of 8 is typical for excitation pulses, and a TBW of 12 or 16 is typical for slab select pulses.

b. Plot RF Amplitude For convenience, we will assume that the RF waveforms are normalized so that the sum of the RF waveform is the flip angle in radians. The sampled RF waveform can then be thought of as a sequence of small flips. This eliminates the need to explicitly consider the pulse duration in the design and simulation. However, it is sometimes important to compute the RF pulse amplitude in Gauss. In this problem you will write an m-file that takes a normalized RF pulse, and then, given a overall pulse length, scales the waveform to Gauss.

First, generate a 3.2 ms, $TBW = 8$ windowed sinc RF pulse, and scale it to a $\pi/2$ flip angle

```
>> rf = (pi/2)* wsinc(8,256);
```

Then, write an m-file called `rfscaleg` that takes a normalized RF pulse and a pulse duration, and returns a waveform that is scaled to Gauss,

```
>> rfs = rfscaleg(rf, pulseduration);
```

Plot the pulse you generated, scaled to Gauss. Label the axes. What is the peak amplitude?

c. Simulated Slice Profiles We will use the bloch simulator for simulating the RF pulse.

Calculate the slice thickness of the $TBW = 8$ pulse from problem (b), based on the relations presented in class. Assume the slice select gradient is 0.6 G/cm. Simulate the RF pulse using

```
> dp = linspace(-2,2,512).'; % simulate from -2cm to 2cm
> mx0 = zeros(512,1); my0=zeros(512,1);, mz0 = ones(512,1);
> dt = 3.2e-3/256;
> [mx,my,mz] = bloch(rfs,g,dt,100,100,0,dp,0,mx0,my0,mz0);
> mxy = mx+i*my;
> figure, plot(dp,abs(mxy)), xlabel('cm'), ylabel('amplitude');
```

Is the slice the expected width?

d. Design a Slab Select Pulse You are designing a 3D pulse sequence, and you need a slab select pulse in the z dimension. You have 6 ms for the pulse, and want it to be as sharp as possible. You also have a peak RF amplitude constraint of 0.17 G.

- (a) What is the highest time-bandwidth you can allow, given that the maximum flip angle will be 90 degrees?

- (b) Assume we want the minimum slab thickness to be 8 cm. What is the gradient amplitude that this requires?
- (c) Simulate the slice profile. How wide is the transition band compared to the passband? Assume that the passband edge is 95% of the middle of the passband, and the stopband edge is 5% of the passband.

e. Compute the Relative SAR of a Pulse Sequence SAR stands for specific absorption rate and is the amount of RF energy deposition in the body. Usually, this is calculated using software model that gives the SAR limit measured in terms of 1 ms rectangular 180 degree pulses (“hard” 180’s). This depends on the patient weight, body part, and RF coil. The limit might be 100 hard 180s per second, for example. The power of a particular RF pulse is

$$P = \int B_1^2(t) dt.$$

The relative SAR of a particular RF pulse is it’s power divided by the power in a 1 ms hard 180. The relative SAR of a pulse sequence is computed as the sum of the relative SAR’s of each of the pulses, measured in equivalent hard 180’s.

Write an mfile `rsar.m` which takes a normalized RF pulse and pulse duration, and returns the relative SAR, measured in equivalent hard 180’s

```
>> eq180 = rsar(rf,t)
```

Assume that you are developing a very fast sequence. The excitation pulse is a 1 ms TBW=2 windowed sinc. Assume you need a TR of 2.25 ms. What is the maximum flip angle you can allow, given that the SAR limit is 100 equivalent 180’s per second?