Principles of MRI
EE225E / BIO265

Chapter 03

Instructor: Miki Lustig
UC Berkeley, EECS
What is this?

- The first NMR spectrum of ethanol 1951.
Spatial Frequency

- Vinyl Record
  - Transforms a temporal signal to a spatial signal

http://offtosognefjord.tumblr.com
What is the frequency?

\[ \cos(2\pi(f_xx + f_yy)) \]

(-1,1) \quad (1,1)

(-1,-1) \quad (1,-1)

a) \( f_x = 1, f_y = 2 \)

b) \( f_x = 2, f_y = 1 \)

c) \( f_x = 4, f_y = 2 \)

d) \( f_x = 2, f_y = 4 \)
What is the Temporal Frequency?

Vinyl rotates at 1 Hz

(-1,1)  (1,1)
(-1,-1) (1,-1)

a) $\cos(2\pi 8t)$  c) $\cos(2\pi 4t)$
b) $\cos(2\pi 8t^2)$  d) $\cos(2\pi 4t^2)$
What is the Temporal Frequency?

Vinyl rotates at 1 Hz

(-1, 1) (1, 1)

(-1, -1) (1, -1)

a) $\cos(2\pi 100t)$
b) $\cos(2\pi 100t^2)$
c) $\cos(2\pi 40t)$
d) none of the answers

Aliasing!
Classical Description of MR

- Atoms with odd # of protons/neutrons have nuclear spin angular momentum
  - Intrinsic QM property (ch. 4)
  - Also intrinsic magnetic moment
- Like Spinning magnetic dipoles
- In biological tissue:
  - Mostly $^1$H in H$_2$O
  - Sometimes $^{31}$P, $^{13}$C, $^{23}$Na - Exotic
Classical Description of MR

• Interaction of magnetization with 3 fields
  – B0 - Main field \( \Rightarrow \) Polarization and resonance
  – B1 - RF field \( \Rightarrow \) Signal production + reception
  – G - Gradient fields \( \Rightarrow \) Spatial encoding
B0 - Main Field

- Produces polarization of sample $M_0$
- Resonance at Larmor frequency
  \[ \omega = -\gamma B_0 \]
- Gyromagnetic Ratio
- For Protons:
  \[ \frac{\gamma}{2\pi} = 4.257 \text{ KHz/G} \]
- Others:
  - $^{23}\text{Na}$
    \[ \frac{\gamma}{2\pi} = 1.127 \text{ KHz/G} \]
  - $^{13}\text{C}$
    \[ \frac{\gamma}{2\pi} = 1.071 \text{ KHz/G} \]
  - $^{15}\text{N}$
    \[ \frac{\gamma}{2\pi} = -0.43 \text{ KHz/G} \]
## Typical $B_0$

<table>
<thead>
<tr>
<th>Field Strength</th>
<th>Frequency</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1T</td>
<td>4.2MHz</td>
<td>Very Low!</td>
</tr>
<tr>
<td>0.5T</td>
<td>21MHz</td>
<td>Low (permanent/resistive)</td>
</tr>
<tr>
<td>1.5T</td>
<td>63MHz</td>
<td>“High” Diagnostic (superconducting)</td>
</tr>
<tr>
<td>3T</td>
<td>127MHz</td>
<td>“High” Diagnostic (superconducting)</td>
</tr>
<tr>
<td>4T</td>
<td>170MHz</td>
<td>Rare</td>
</tr>
<tr>
<td>7/9.4T</td>
<td>300/400 MHz</td>
<td>Very High - research only</td>
</tr>
</tbody>
</table>
B0 Field

- For Spatial/Spectral Localization we require homogeneity

\[ \Delta B_0 \sim 1 \text{ppm} \]

over 40cm³ FOV

- This is: 64Hz @ 1.5T
  - Pretty remarkable!
Why Resonance

• For a bar magnet
  – Torque, but no resonance
  – Missing angular momentum

• Resonance is like a spinning-top
B₁ - RF Field

- Can’t Directly Detect $M₀$
  - $M_{\text{induced}} = μ₀⁻¹VχB$ \( χ ≈ 4 \cdot 10⁻⁹ \)

Q: Why?
A: Huge field!

- Resonance is the key!
  - $B₀$ is DC while spins resonate ⇒ Detection!
  - Sample resonates at $ω₀=-γB₀$

- Excite magnetization off the z direction
  - Apply RF field at $ω₀=-γB₀$ in the x-verse plane
  - Has to be resonant to do something
RF Excitation

- In the lab frame

\[ B_1(t) = A\hat{y} \]
RF excitation

- In the lab frame

\[ B_1(t) = A e^{-i\omega_0 t} \]
RF excitation

- In the rotating frame $\omega_0$

$$B_1(t) = A\hat{y}_{rot}$$
RF Excitation

- In the rotating frame: Precession about $B_1$

Typical $B_1$’s: 0.14 - 0.35G ± (10%/20%) accuracy @ (1.5T/3T)

Duration 1-3ms which is a long time at 64MHz!

Peak power 20KW!

\[
\omega_0 = \gamma B_1 = 4.257 \cdot 0.16 = 0.68 \text{ KHz}
\]

\[
0.367 \text{ms } 90^\circ \Rightarrow 23000 \text{ rotations}
\]
Relaxation

- $T_1$: Longitudinal relaxation $\sim 10$-$2000$ms
- $T_2$: Transverse relaxation $\sim 10$-$300$ms
- Main source of contrast (Later!)
- $T_2 < T_1$ Always!

\[ M_{xy} e^{-t/T_2} \quad \text{Time} \]

\[ M_z (1-e^{-t/T_1}) \quad \text{Time} \]
RF Reception

- Precession enduces EMF in coil: Faraday’s law ⇒ Free induction decay

\[ V(t) \]

\[ FID \]

After demodulation

\[ t \]
Gradient Fields

• B1 has poor localization $\lambda \at64\text{MHz} \sim 0.5\text{m in tissue}$
• Instead encode position in **frequency**

\[ \vec{G} = \left[ \frac{\partial B_z}{\partial x}, \frac{\partial B_z}{\partial y}, \frac{\partial B_z}{\partial z} \right] \]

• Small concomitant fields $B_x, B_y$ are also created.
These do not contribute much to precession - fields are NOT oscillating at Larmor freq.

\[ \omega(x) = \omega_0 + \gamma G_x x \]
Gradient Fields

• Typical #
  – $G = 1\text{-}10 \text{ G/cm} = 10\text{-}100 \text{ mT/m} = 4.2\text{-}42 \text{ KHz/cm}$
  – Waveforms in audio frequency
  – Slew-Rate = 15-20 G/cm/ms
    • Safety concern is in dB/dt
    • Peripheral nerve stimulation can happen
  – Big Amplifiers: 1200 Volts, 200 Amps
Followup on Homework

- Gradient non-Linearities

![Graph showing Actual Gradient and Linear Model](image)

Position, $z$, cm

Frequency, kHz

M. Lustig, EECS UC Berkeley
Followup on Homework

- Gradient non-Linearities

Before correction

Grad-warp correction
Imaging

1. Place sample in $B_0$  
   $M_z$ develops $\sim 5T_1$

2. Excite using $B_1(t)$  
   Creates transverse magnet.

3. Instantaneous precession of $M_{xy}$  
   Induces EMF in coil

4. Encode position in freq. using gradients  
   1-D "Projection"
Some Limitations

Gradient strength \( \rightarrow \) Resolution

\[ \downarrow \]

Signal decay (\( T_2 \))

Field Inhomogeneity (\( T_2^* \))

Diffusion

Typical resolution: \( \leq 1 \text{ mm} \)

\( \approx 20 \mu \text{m in small animal scanners} \)
Example: Square RECT

\[ \omega(x) = \omega_0 + \gamma G_{xx} x \]

- What is the signal after excitation?
  - Signal proportional to \( m_{xy} \)
  - Comes from the entire volume
Example: Square RECT

\[
\begin{align*}
\omega(x) &= \omega_0 + \gamma G_x x \\
\end{align*}
\]

- Magnetization at position \( x = x_0 \)
  \[
  m_{xy}(x) = m(x) e^{-i(\omega_0 + \gamma G_x x)t}
  \]

- Signal from everything
  \[
  s(t) = \int m(x) e^{-i(\omega_0 + \gamma G_x x)t} \, dx
  \]
Example: Square RECT

\[
\begin{align*}
  s(t) & = \int_x m(x) e^{-i(\omega_0 + \gamma G_x x)t} \, dx \\
  & = e^{-i\omega_0 t} \int_x m(x) e^{-i\gamma G_x x t} \, dx \\
  & = e^{-i\omega_0 t} \int_x m(x) e^{-i2\pi(\frac{\gamma}{2\pi} G_x t) x} \, dx \\
  & = e^{-i\omega_0 t} \int_x m(x) e^{-i2\pi k_x x} \, dx \\
  & = e^{-i\omega_0 t} \mathcal{F}\{m(x)\} \bigg|_{k_x = \frac{\gamma}{2\pi} G_x t}
\end{align*}
\]

\[k_x = \frac{\gamma}{2\pi} G_x t\]
Example: Square RECT

\[ s(t) = e^{-i\omega_0 t} \mathcal{F} \{ m(x) \} \bigg|_{k_x = \frac{\gamma}{2\pi} G_x t} \]

\[ m(x) = \text{rect}(\cdot) \quad \Rightarrow \quad \mathcal{F}\{m(x)\} \sim \text{sinc}(\cdot) \]

Inverse FT of baseband signal yields 1D projection (Almost..)
9. How do we get an image?

Several key components:

1. Selective excitation (dimension reduction)
2. Spatial encoding (Freq/Phase)
   \[ \text{more later} \]
3. Signal decay, Mo recovery
4. Repeat N Times
5. Image Reconstruction
Selectivie Excitation

\[ f = \frac{q}{4\pi} G_z z \]

want to excite a slice

\[ F \{ B, H_I \} = \]

Gradient \((G_z)\) maps position to bandwidth

Bandlimited \((B, H)\) \(\Rightarrow\) Excites a slab in space
Excitation Field

\[ e^{-i\omega_0 t} \]

carrier

(center of)

slice

envelope

(shape of)

slice

Choose \( B_1(t) \)

\[ \mathcal{F}\{ B_1(t) \} \approx \text{rect}(\frac{f}{A}) \]

\[ B_1(t) \approx A \text{ sinc}(\pi t) \]

finite in time

ripple in slice
We draw RF excitation as a pulse sequence:

Same area

more later!
Quiz

\[ B_1(t) = e^{-i\omega_0 t} \text{sinc}(5000t) \]

\[ G_z = 1 \text{ } G/cm \]

What is the excited slice thickness?
Quiz

\[ B_1(t) = e^{-i\omega_0 t} \text{sinc}(5000t) \]

\[ G_z = 1 \, \text{G/cm} \]

Q: What is the excited slice thickness?

A:

\[ F\{B_1(t)\} \propto \Pi\left(\frac{f}{5000}\right) \]

\[ \frac{\Delta B_z}{\Delta z} = 4257 \, \text{H}_z/\text{cm} \]

\[ \frac{\partial}{\partial z} B_z \Delta z = 5000 \, \text{Hz} \]

\[ \Delta z = \frac{5000}{4257} \approx 1.17 \, \text{cm} \]
2D SPATIAL ENCODING

\[ s(t) = \iint m(x,y) e^{-i2\pi (k_x x + k_y y)} \, dx \, dy \]

\[ k_x = \frac{\delta}{2\pi} G_{xt} \quad k_y = 0 \]

Need \( k_x - k_y \) coverage:

Try: \( G_x = G \cos \Theta_n \) \quad \Theta_n = \frac{\pi}{N} n \quad n=[0,N-1] \)

\[ G_y = G \sin \Theta_n \]
The Central Section Theorem

(Projection Slice)

R. Bracewell

\[ \mathcal{F}_{\text{2D}} \{ m(x,y) \} \]

Image Space

\[ \mathcal{F}_{\text{2D}} \{ f_0(x) \} \]

k-space

Transform (2D) of projection of an object

\[ \Rightarrow \text{Diameter through transform (2D) of object} \]

Very cool! ... and useful...
Projection slice $\Rightarrow$ basis for CT

Proof (for $\Theta = 0$)

$$p(x) = \int_{-\infty}^{\infty} m(x,y) \, dy$$

$$M(k_x, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-i2\pi (k_x x + k_y y)} \, dx \, dy =$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x,y) e^{-i2\pi k_x x} \, dx \, dy =$$

$$= \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} m(x,y) \, dy \right] e^{-i2\pi k_x x} \, dx =$$

$$= \int_{-\infty}^{\infty} p(x) e^{-i2\pi k_x x} \, dx = \mathcal{F} \{ p(x) \}$$
Let's see if it makes sense...

\[ \mathcal{P}_0 (r) \]

\[ \mathcal{P}_\text{d} (r) \]

\[ \mathcal{P}_\text{m} (r) \]

\[ \mathcal{H} (x, y) \]

How to reconstruct?
1. filtered back-projection
2. Interpolation (in k-space)

**Good class project**
Projection Reconstruction (Lauterber)

\[ \widetilde{F}_{id} \{ \text{Diameter} \} \Rightarrow \text{Projection} \]

However... not quite diameter yet...
Better Idea

RF

G\_z

phase encode

G\_y

G\_x

A/D

m(x,y)

(still not perfect)

(\text{still not perfect})

(-) Make "horizontal projections"

(-) Encode phase in y prior to readout in x

"2DFT"

"SPIN-WARP"
Other approaches to Imaging

(1) Localization is the key!
   (a) B0 localization
   (b) Saddle point
   (c) Local magnet

(2) RF
   (a) Local coils
   (b) Local excitation

(3) Non linear gradients

Great Project!
Complex Samples: IQ Demodulation

• With IQ demodulation we look at part of the spectrum

• Example:

![Diagram showing IQ demodulation]

samples represent this freq. band
IQ Demodulation

- How is it implemented?

\[ e^{-i2\pi f_0 t} \]

\[ \text{RF} \xrightarrow{x} \text{LPF} \xrightarrow{\text{ADC}} \text{complex samples} \]
IQ Demodulation

• How is it physically implemented?

\[ e^{-i2\pi f_0 t} = \cos(2\pi f_0 t) - i \sin(2\pi f_0 t) \]
IQ Demodulation

• How is it physically implemented?

\[ e^{-i2\pi f_0 t} = \cos(2\pi f_0 t) - i \sin(2\pi f_0 t) \]
**Complex Value & Demodulation**

**Physical Signal:**

\[ S_p(t) = A(t) \cos(\omega_0 t + \phi(t)) \]

**Real** \[ real \rightarrow A(t) \cos(\theta(t)) \cos(\omega_0 t) + [A(t) \sin(\phi(t))] \sin(\omega_0 t) \]

\[ I(t) \quad \text{in phase} \]

\[ Q(t) \quad \text{quadrature} \]

**Analytic Signal (convenient)**

\[ S_r(t) = A(t) e^{-i(\omega_0 t + \phi(t))} \]

\[ S_p = \text{Re}\{S_r\} \]

**Baseband Signal**

\[ s(t) = S_r(t) e^{+i\omega_0 t} = A(t) e^{-i\phi(t)} = I(t) + iQ(t) \]
Quadature Phase-Sensitive Detection:

\[ s(t) \rightarrow \begin{cases} \cos(\omega t) \\ \sin(\omega t) \end{cases} \rightarrow \text{LPF} \rightarrow I(t) \rightarrow Q(t) \]

\[ s(t) = I(t) + iQ(t) \]

I(t) \rightarrow M_x \text{ in rotating frame}

Q(t) \rightarrow M_y \text{ in rotating frame}