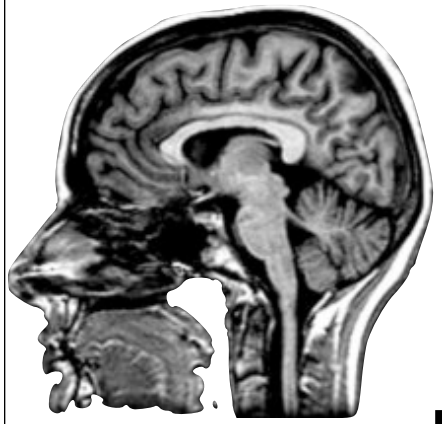
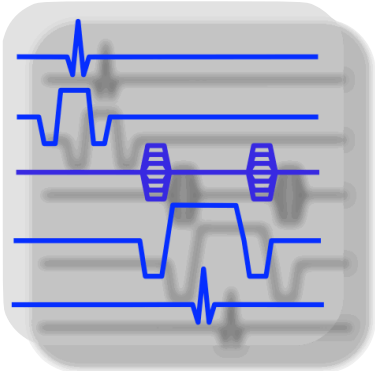




Principles of MRI

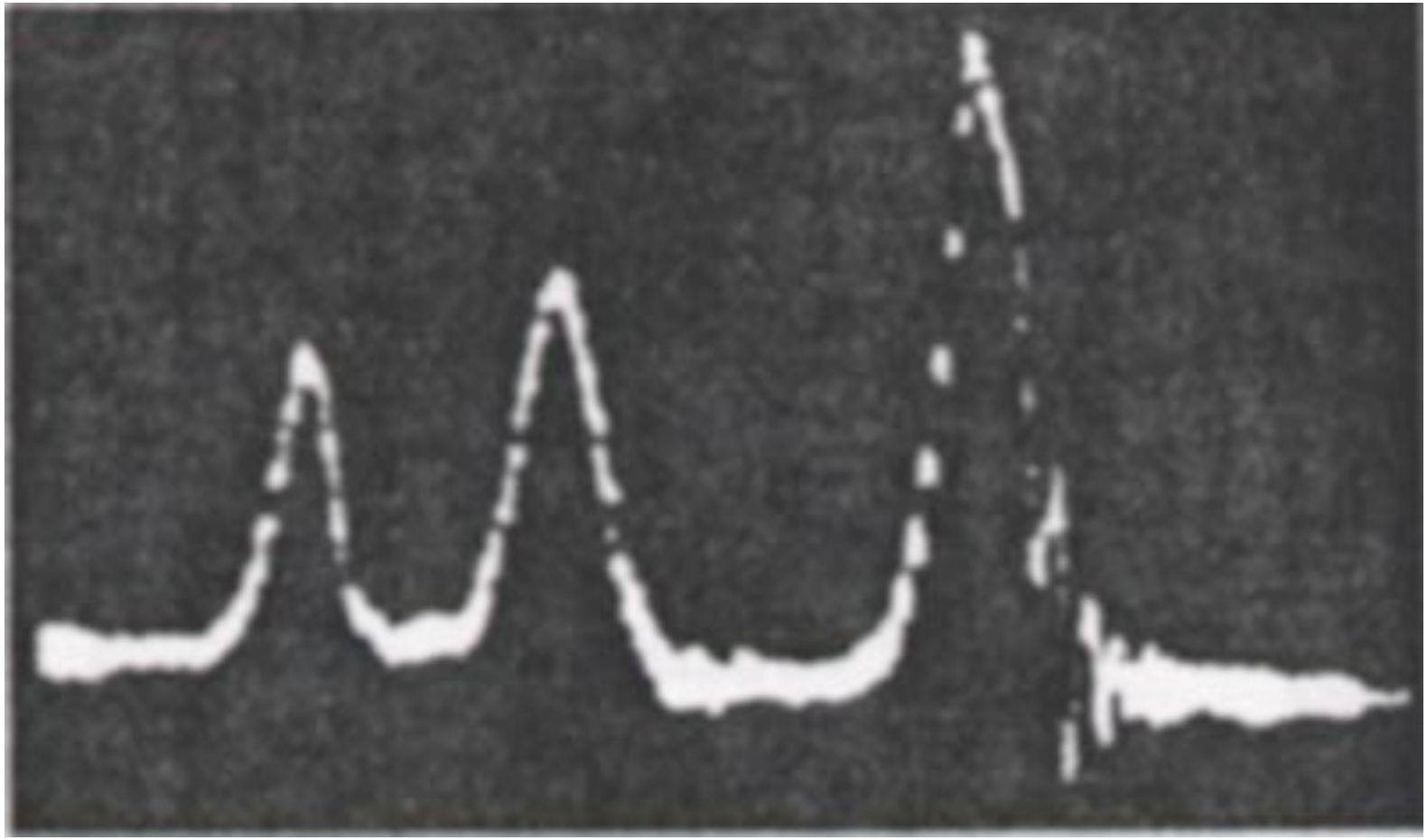
EE225E / BIO265

Chapter 03

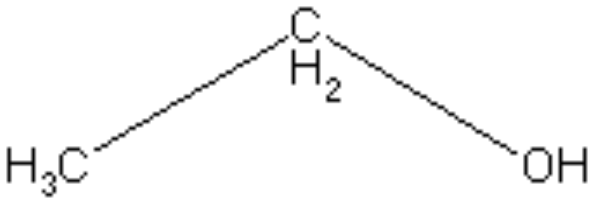
Instructor: Miki Lustig
UC Berkeley, EECS

What is this?

- `

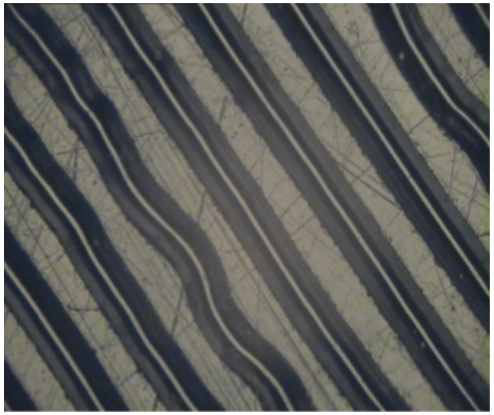


The first NMR spectrum of ethanol 1951.

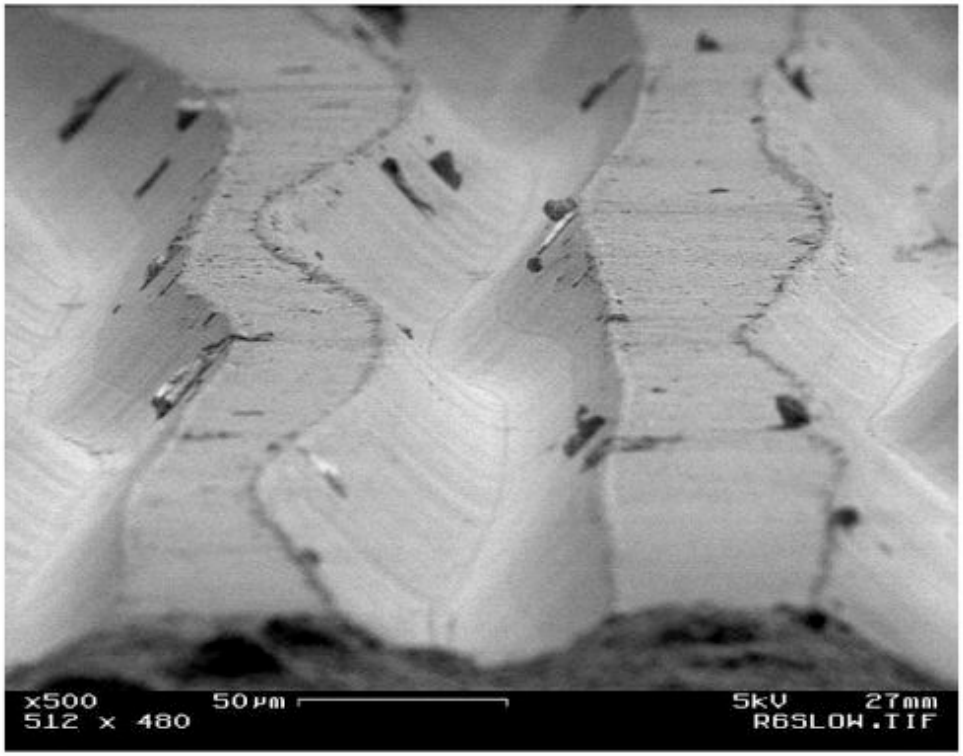


Spatial Frequency

- Vinyl Record
  - Transforms a temporal signal to a spatial signal



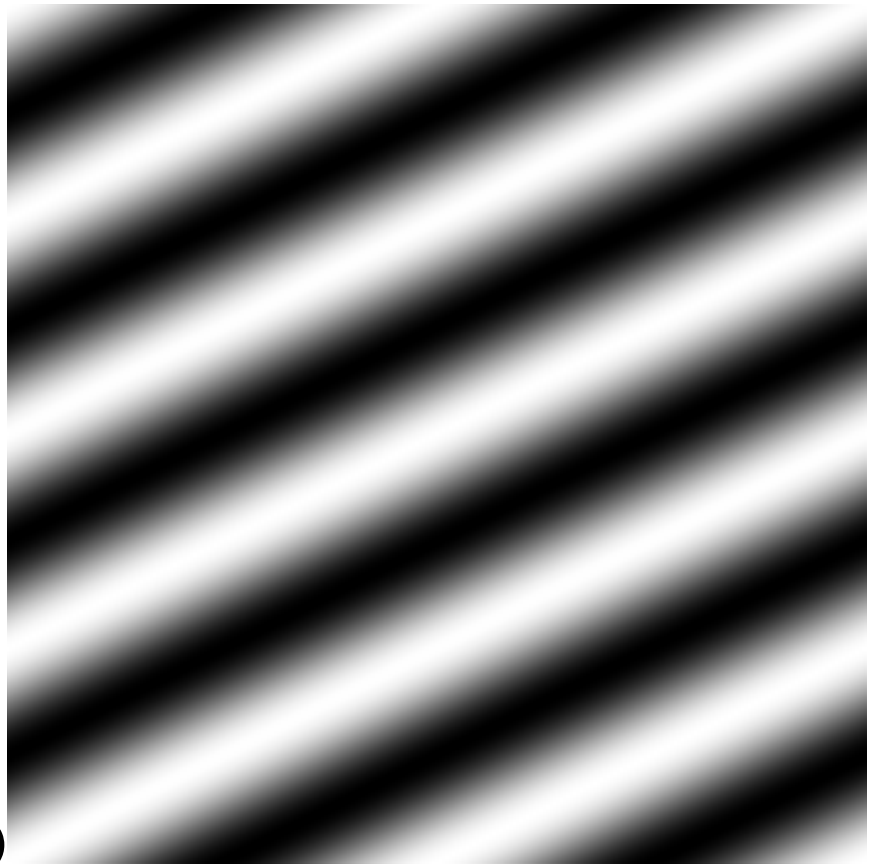
http://offtosognefjord.tumblr.com



What is the frequency?

$$\cos(2\pi(f_x x + f_y y))$$

(-1,1) (1,1)



(-1,-1) (1,-1)

a) f_x=1, f_y=2

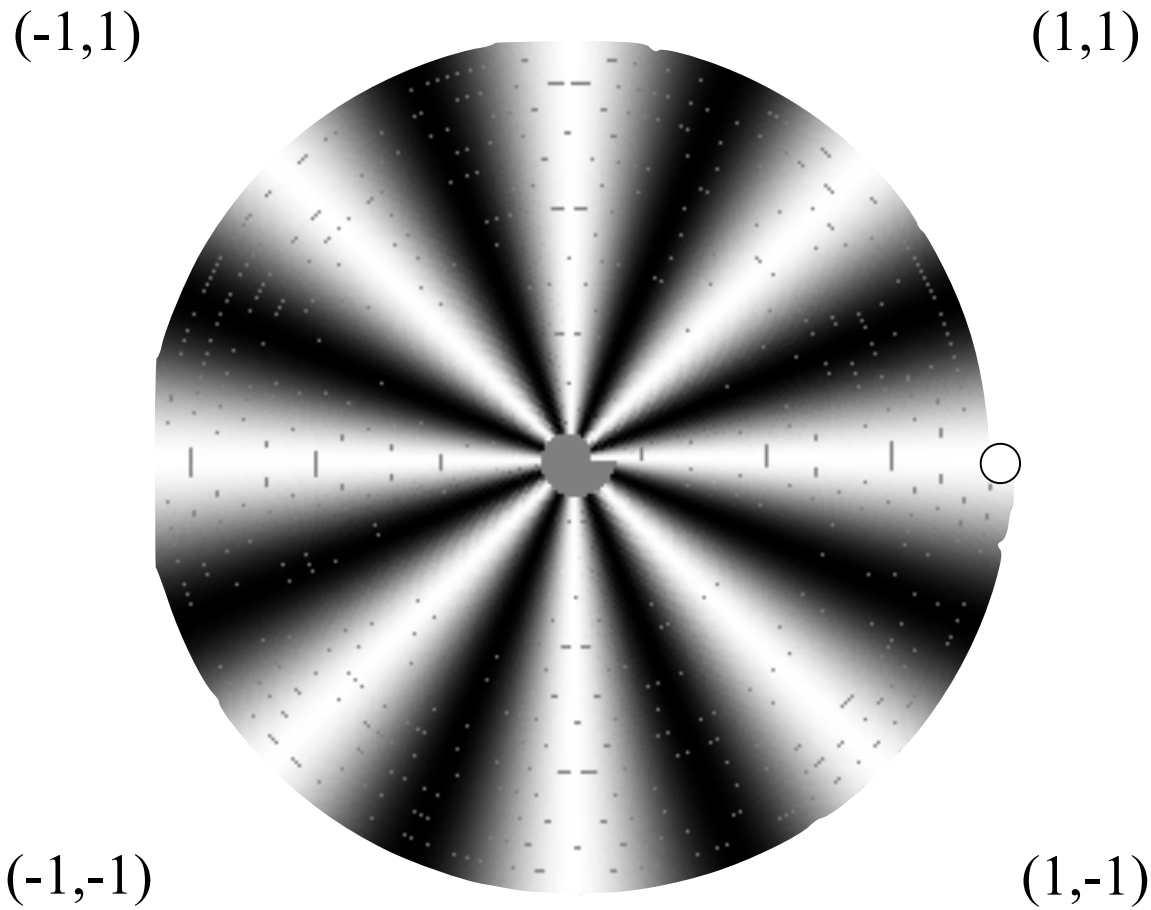
b) f_x=2, fy=1

c) f_x = 4, f_y=2

d) f_x=2, fy=4

What is the Temporal Frequency?

Vinyl rotates at 1 Hz



a) $\cos(2\pi 8t)$

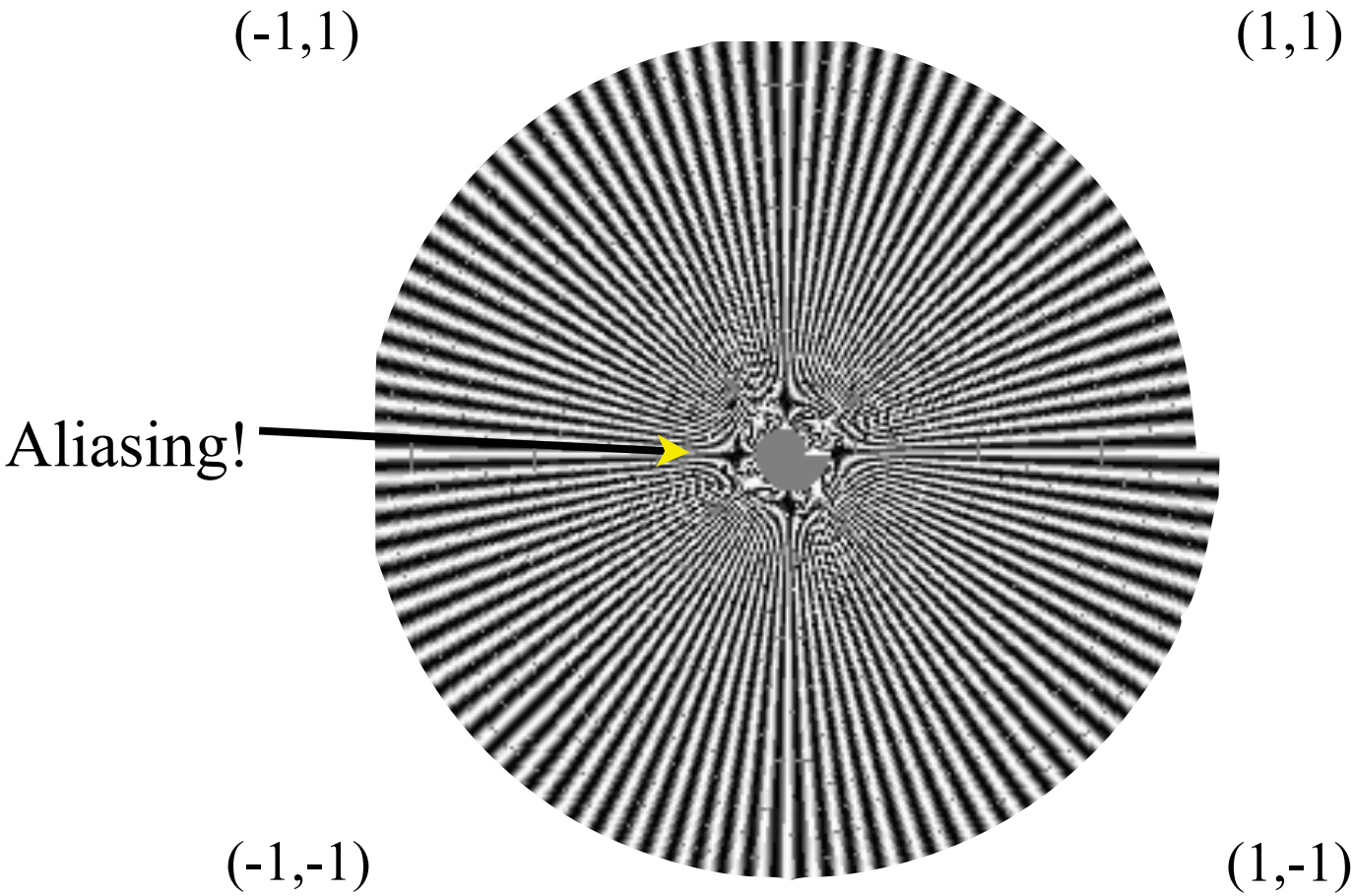
b) $\cos(2\pi 8t^2)$

c) $\cos(2\pi 4t)$

d) $\cos(2\pi 4t^2)$

What is the Temporal Frequency?

Vinyl rotates at 1 Hz



a) $\cos(2\pi 100t)$

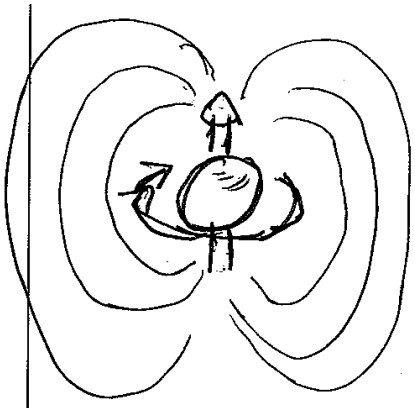
b) $\cos(2\pi 100t^2)$

c) $\cos(2\pi 40t)$

d) none of the answers

Classical Description of MR

- Atoms with odd # of protons/neutrons have nuclear spin angular momentum
  - Intrinsic QM property (ch. 4)
  - Also intrinsic magnetic moment



- Like Spinning magnetic dipoles
- In biological tissue:
  - Mostly ^{1}H in H_2O
  - Sometimes ^{31}P, ^{13}C, ^{23}Na - <u>Exotic</u>

Classical Description of MR

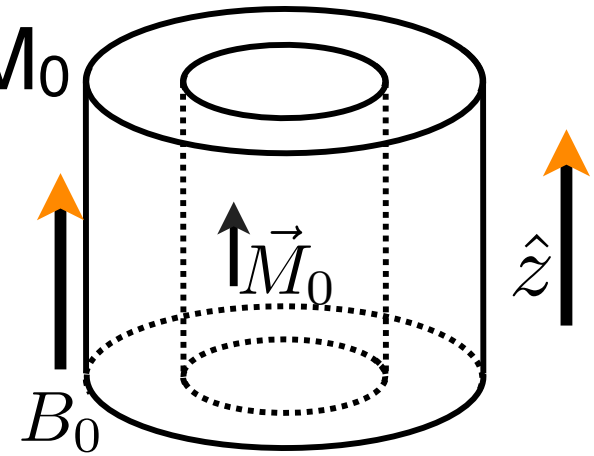
- Interaction of magnetization with 3 fields
  - B0 - Main field ⇒ Polarization and resonance
  - B1 - RF field ⇒ Signal production + reception
  - G - Gradient fields ⇒ Spatial encoding

B0 - Main Field

- Produces polarization of sample M_0
- Resonance at Larmor frequency

$$\omega = -\gamma B_0$$

Gyromagnetic Ratio



- For Protons:

$$\frac{\gamma}{2\pi} = 4.257 \quad \mathrm{KHz/G}$$

$$\gamma = 2\pi 4.257 \quad \mathrm{Krad/G}$$

- Others:
  - ^{23}Na $\quad \frac{\gamma}{2\pi} = 1.127 \quad \mathrm{KHz/G}$
  - ^{13}C $\quad \frac{\gamma}{2\pi} = 1.071 \quad \mathrm{KHz/G}$
  - ^{15}N $\quad \frac{\gamma}{2\pi} = -0.43 \quad \mathrm{KHz/G}$

Typical B_0

0.1T	4.2MHz	Very Low!
0.5T	21MHz	Low (permanent/ resistive)
1.5T	63MHz	“High” Diagnostic (superconducting)
3T	127MHz	“High” Diagnostic (superconducting)
4T	170MHz	Rare
7/9.4T	300/400 MHz	Very High - research only

B0 Field

- For Spatial/Spectral Localization we require homogeneity

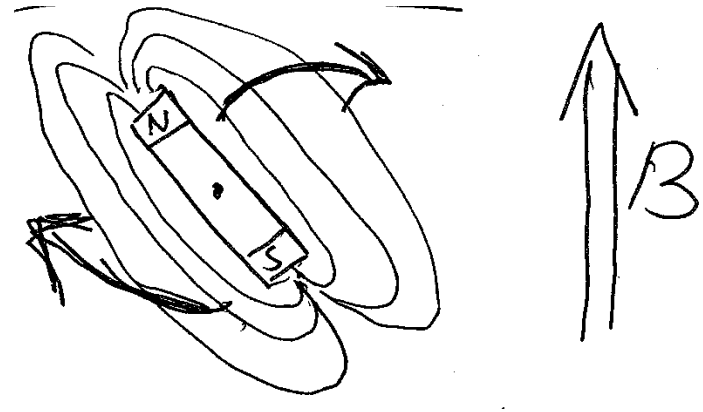
$$\Delta B_0 \sim 1\mathrm{ppm}$$

over 40cm^3 FOV

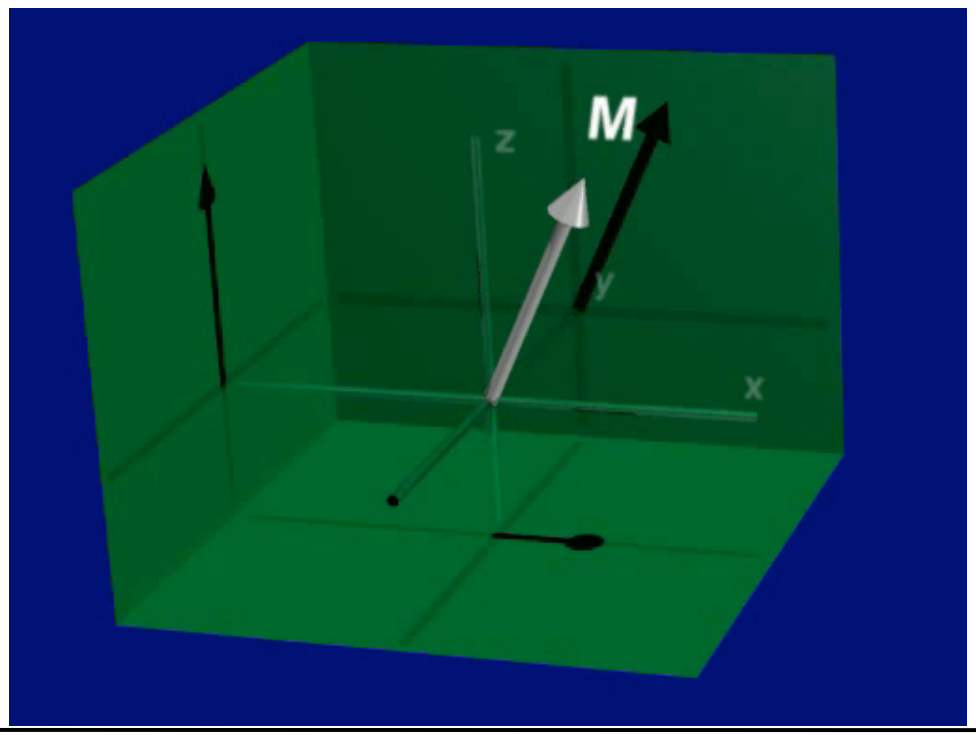
- This is: 64Hz @ 1.5T
  - Pretty remarkable!

Why Resonance

- For a bar magnet
  - Torque, but no resonance
  - Missing angular momentum



- Resonance is like a spinning-top



B_1 - RF Field

- Can't Directly Detect M_0
  - $M_{\text{induced}} = \mu_0^{-1} V \chi B \qquad \chi \approx 4 \cdot 10^{-9}$

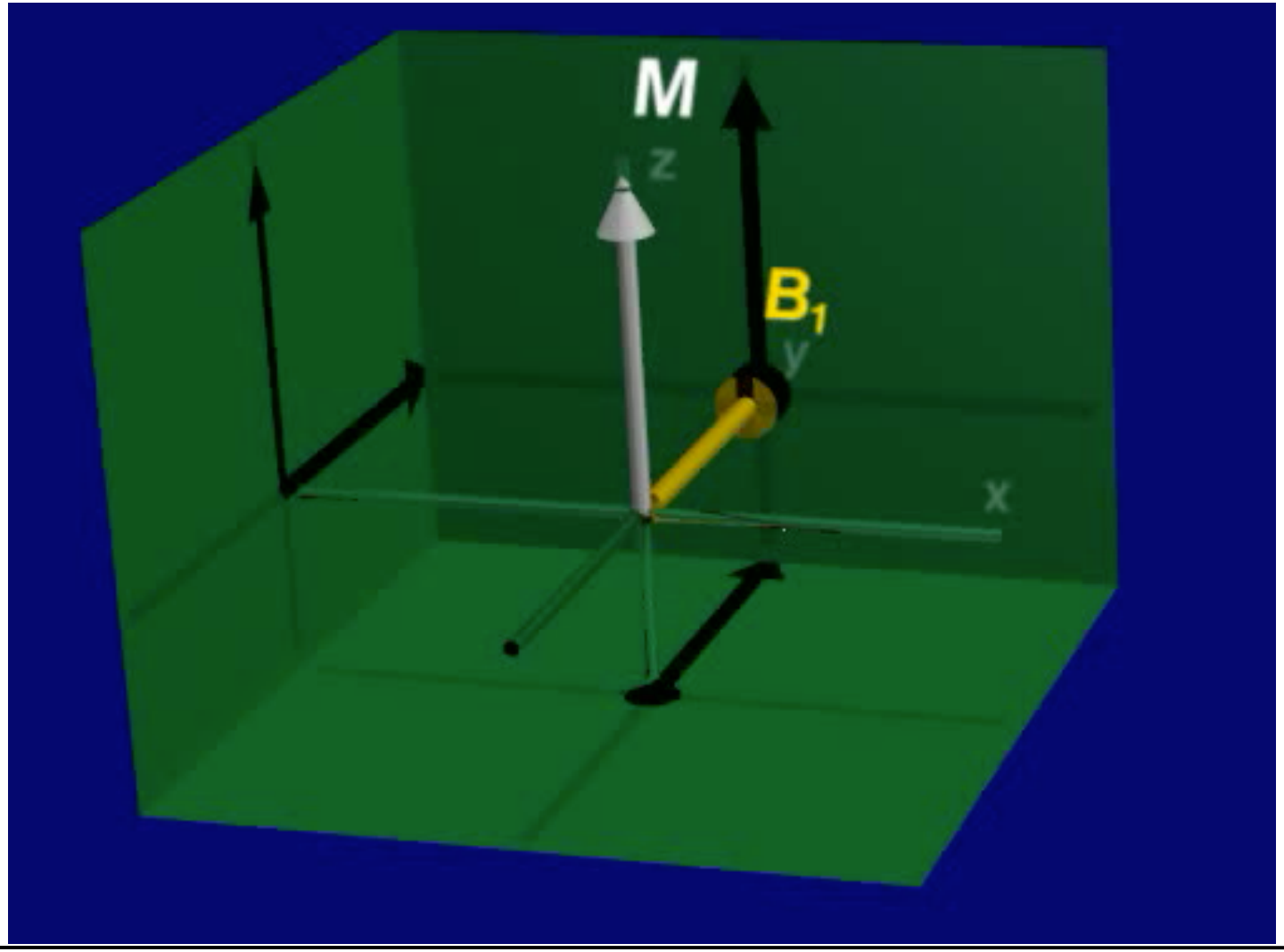
Q: Why?
A: Huge field!

- Resonance is the key!
  - B0 is DC while spins resonate ⇒ Detection!
  - Sample resonates at $\omega_0 = -\gamma B_0$
- Excite magnetization off the z direction
  - Apply RF field at $\omega_0 = -\gamma B_0$ in the x-verse plane
  - Has to be resonant to do something

RF Excitation

- In the lab frame

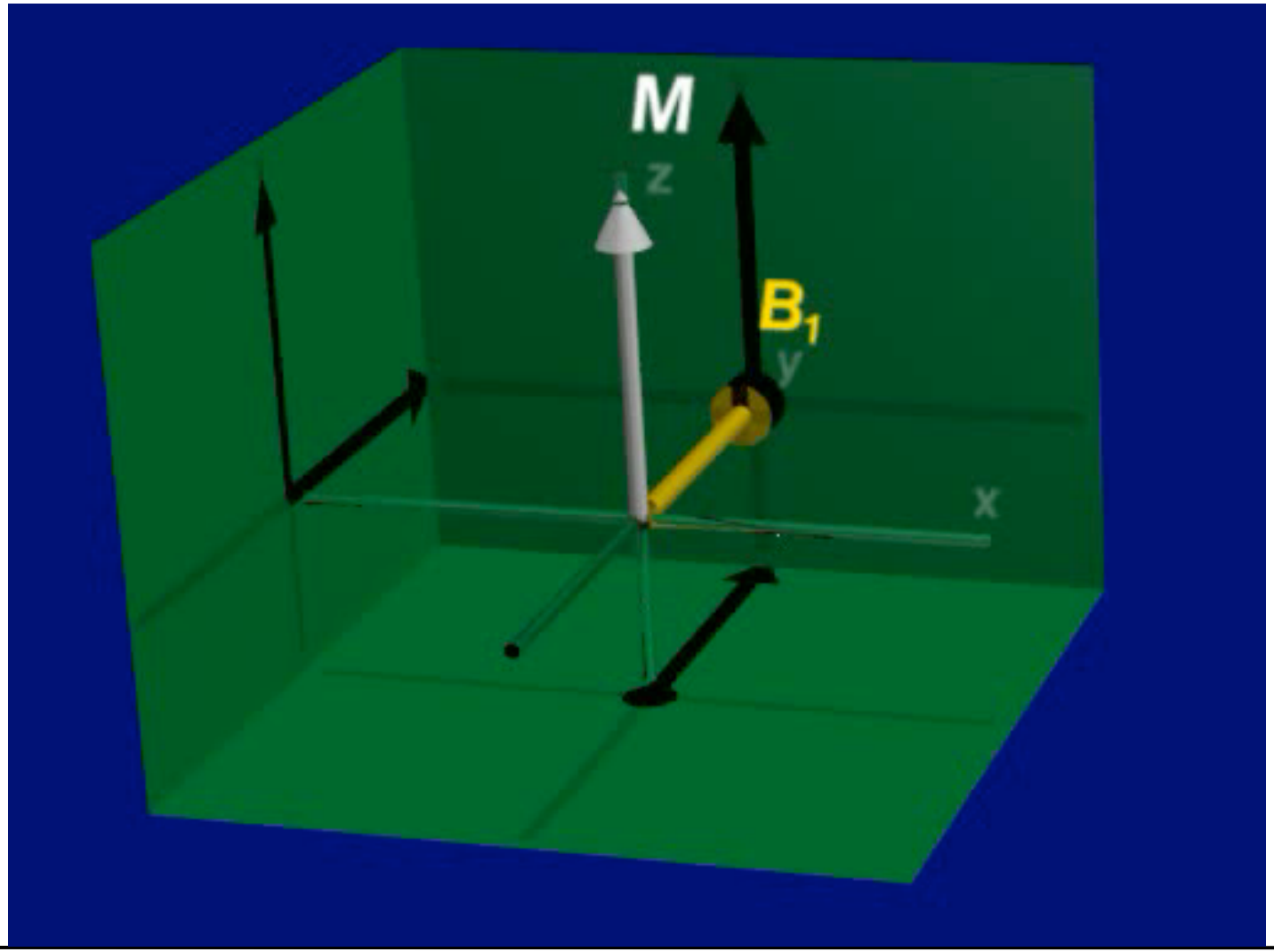
$$B_1(t) = A\hat{y}$$



RF excitation

- In the lab frame

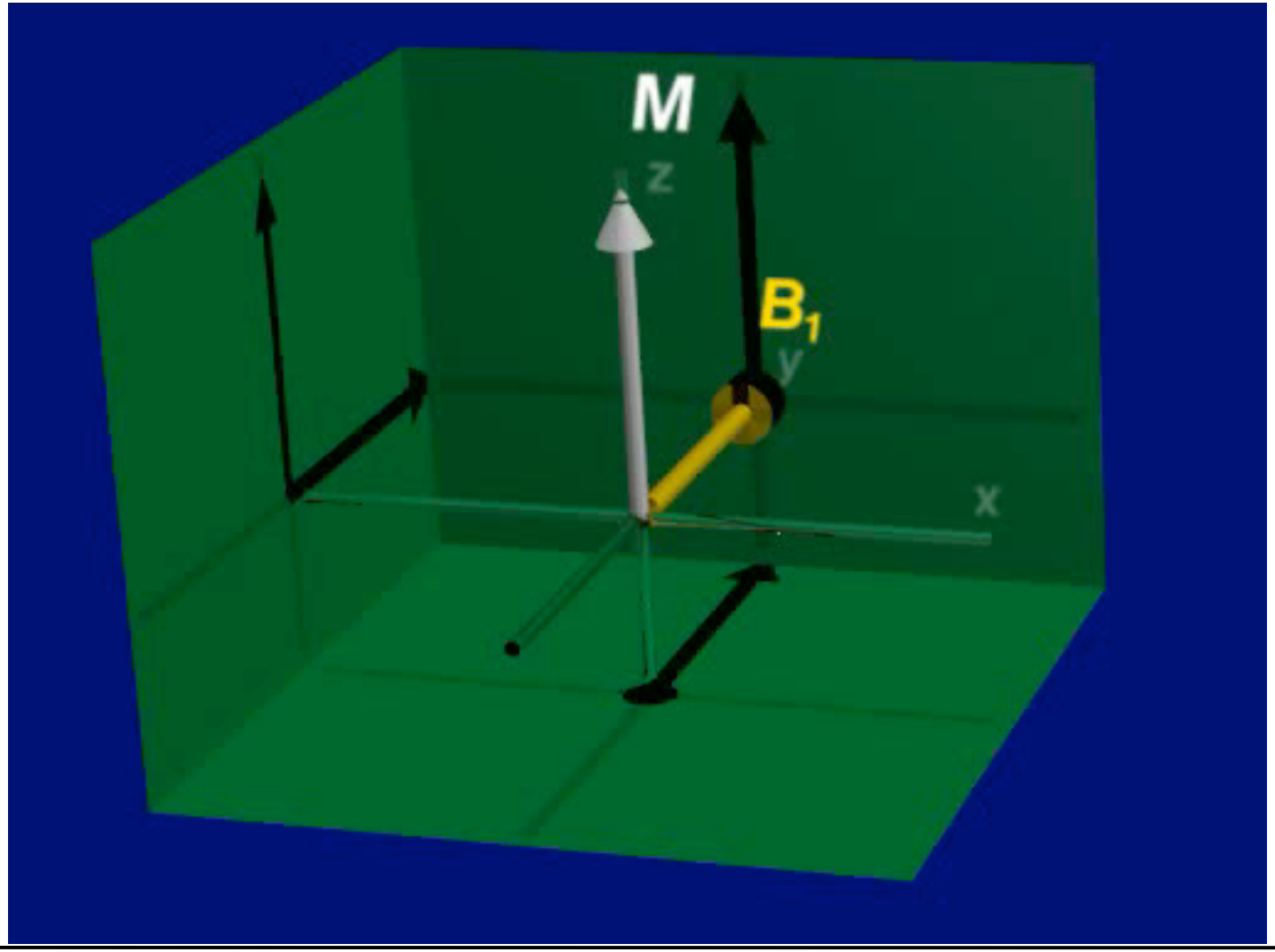
$$B_1(t) = Ae^{-i\omega_0 t}$$



RF excitation

- In the rotating frame @ω_0

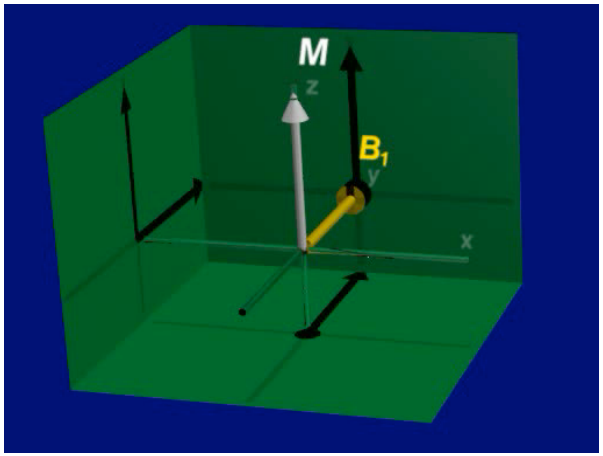
$$B_1(t) = A\hat{y}_{\mathrm{rot}}$$



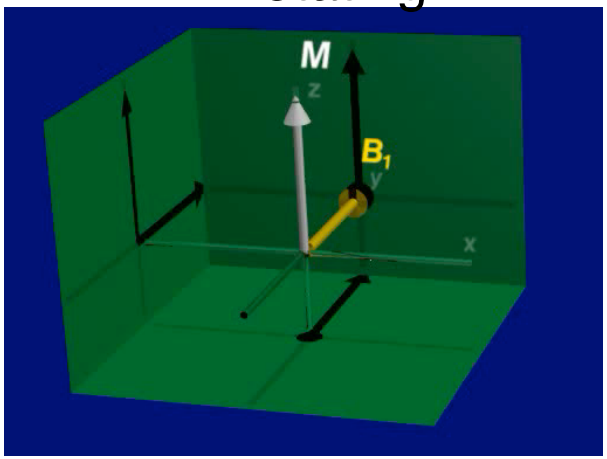
RF Excitation

- In the rotating frame: Precession about B_1

lab



rotating



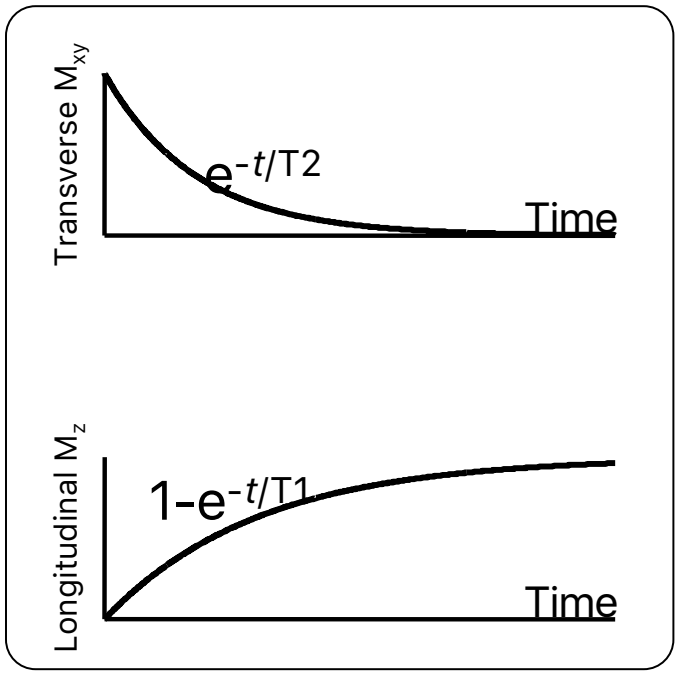
- Typical B_1's: 0.14 - 0.35G ± (10%/20%) accuracy @(1.5T/3T)
- Duration 1-3ms which is a long time at 64MHz!
- Peak power 20KW!

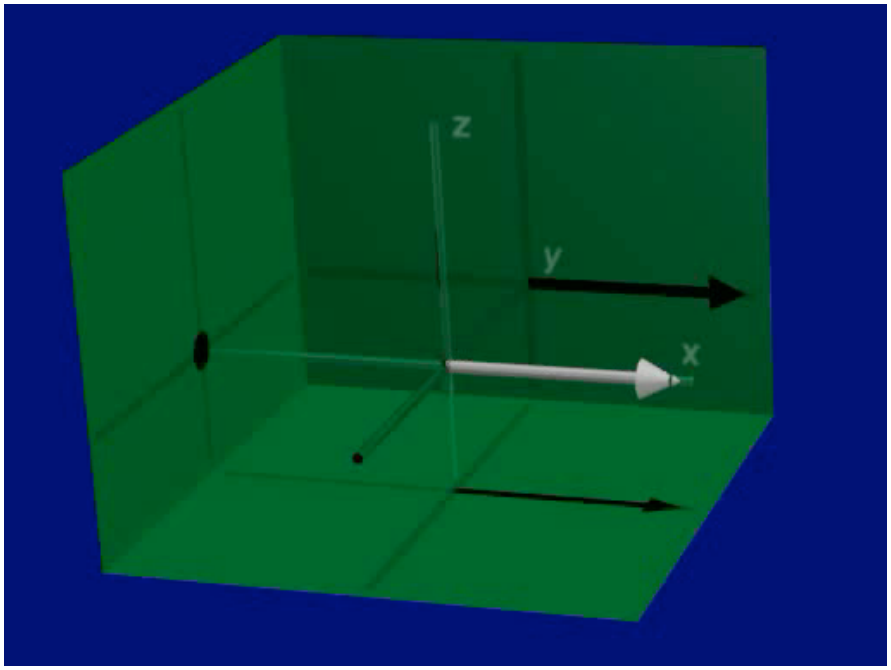
$$\omega_0 = \gamma B_1 = 4.257 \cdot 0.16 \quad = \quad 0.68 \text{ KHz}$$

$$0.367\text{ms } 90^\circ \quad \Rightarrow \quad 23000 \text{ rotations}$$

Relaxation

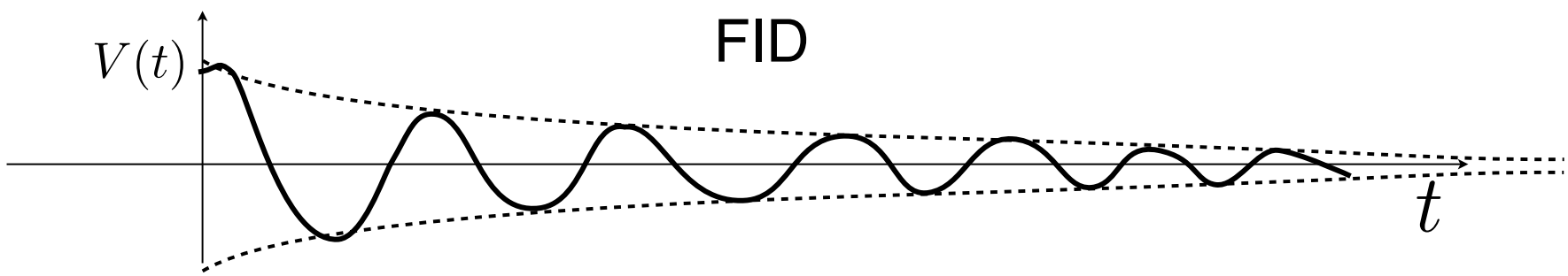
- T_1 : Longitudal relaxation ~ 10-2000ms
- T_2 : Transverse relaxation ~ 10-300ms
- Main source of contrast (Later!)
- $T_2 < T_1$ Always!

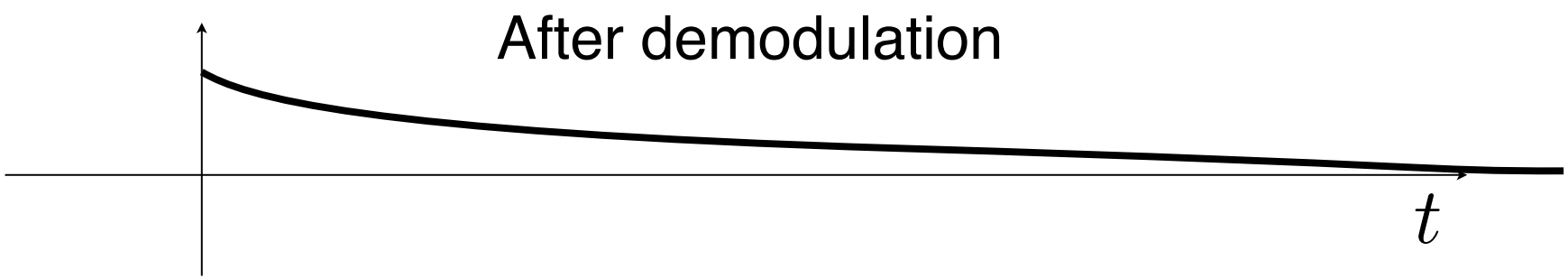




RF Reception

- Precession enduces EMF in coil:
  Faraday's law ⇒ Free induction decay



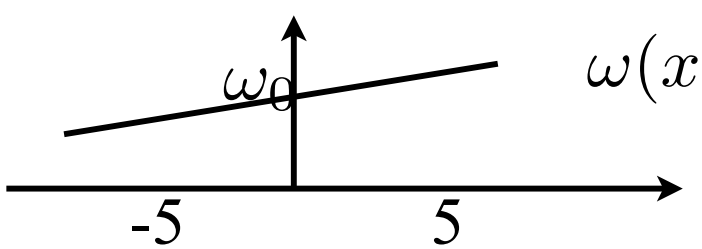


Gradient Fields

- B1 has poor localization λ@64MHz ~ 0.5m in tissue
- Instead encode position in **frequency**

$$\vec{G} = \left[\frac{\partial B_z}{\partial x}, \ \frac{\partial B_z}{\partial y}, \ \frac{\partial B_z}{\partial z} \right]$$

- Small concomitant fields Bx, By are also created. These do not contribute much to precession - fields are NOT oscillating at Larmor freq.



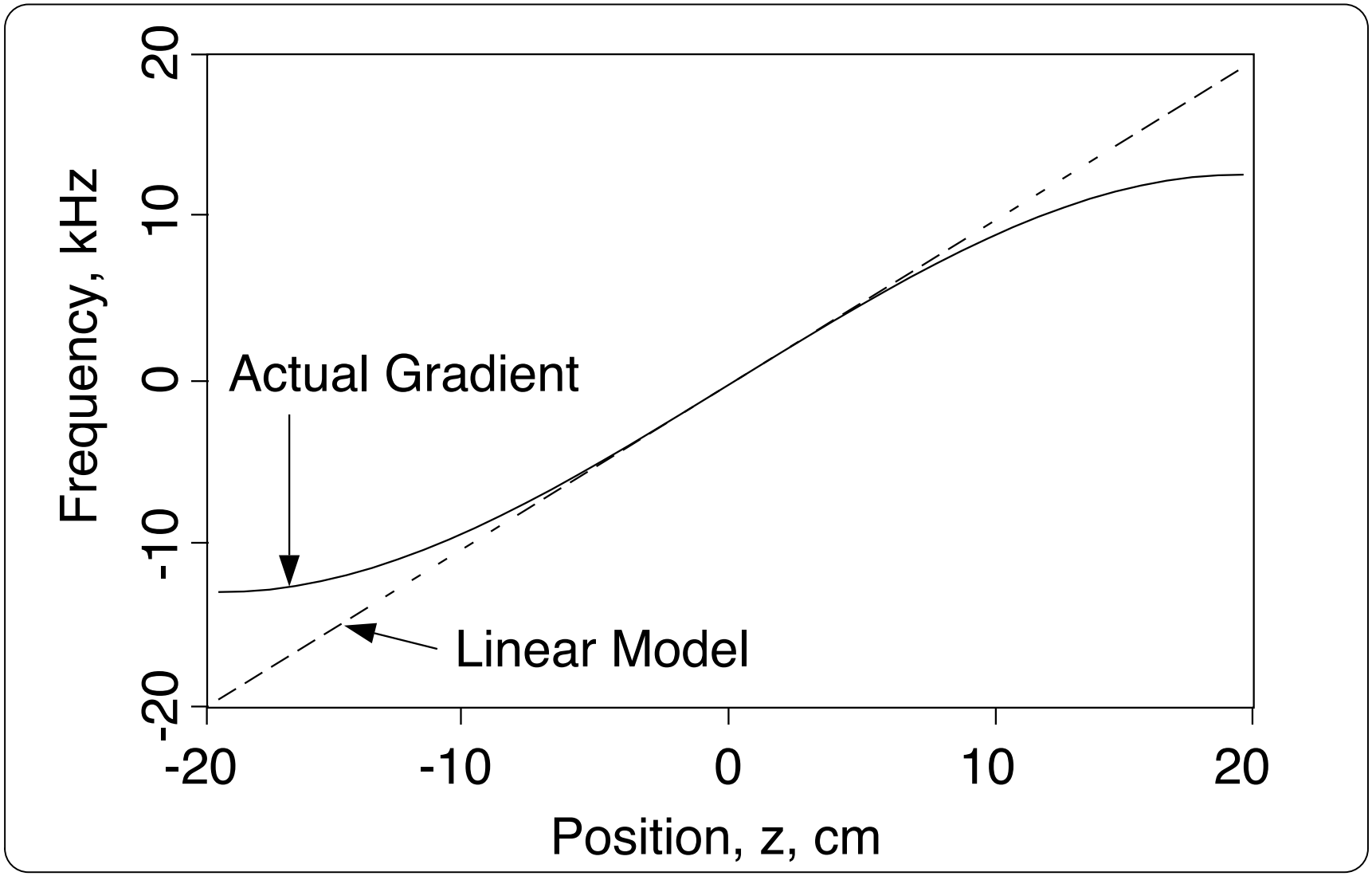
$$\omega(x) = \omega_0 + \gamma G_x x$$

Gradient Fields

- Typical #
  - G = 1-10 G/cm = 10-100 mT/m = 4.2-42 KHz/cm
  - Waveforms in audio frequency
  - Slew-Rate = 15-20 G/cm/ms
    - Safety concern is in dB/dt
    - Peripheral nerve stimulation can happen
  - Big Amplifiers: 1200 Volts, 200 Amps

Followup on Homework

- Gradient non-Linearities

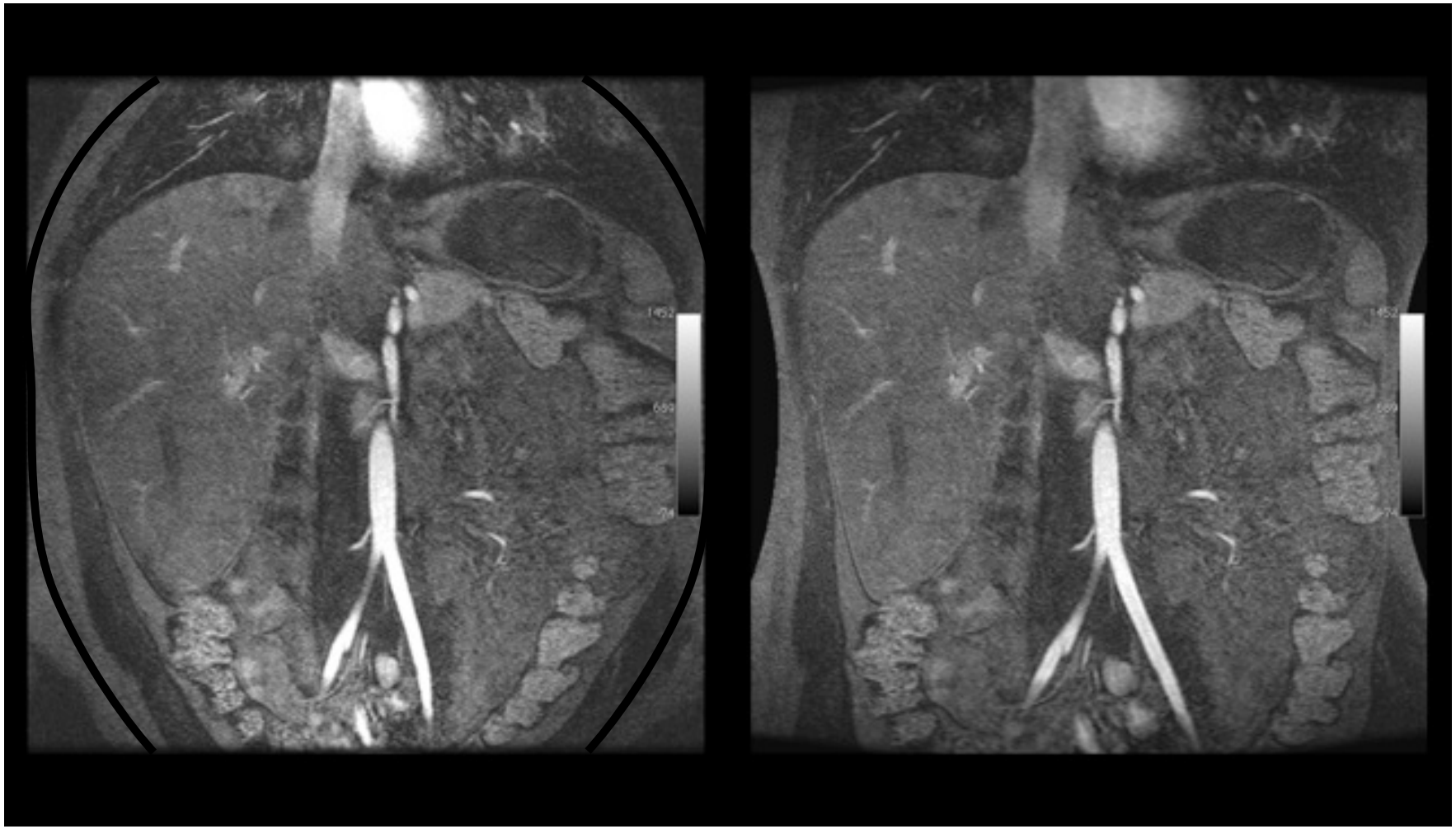


Followup on Homework

- Gradient non-Linearities

Before correction

Grad-warp correction



Imaging

(1) place sample in B_0

M_z Develops $\sim 5T_1$

(2) Excite using $B_1(t)$

creates transverse magn.

(3) Instantaniouse precession of M_{xy}

Induces EMF in Coil

(4) Encode Position in Freq. using gradients

1-D "Projection"

Some Limitations

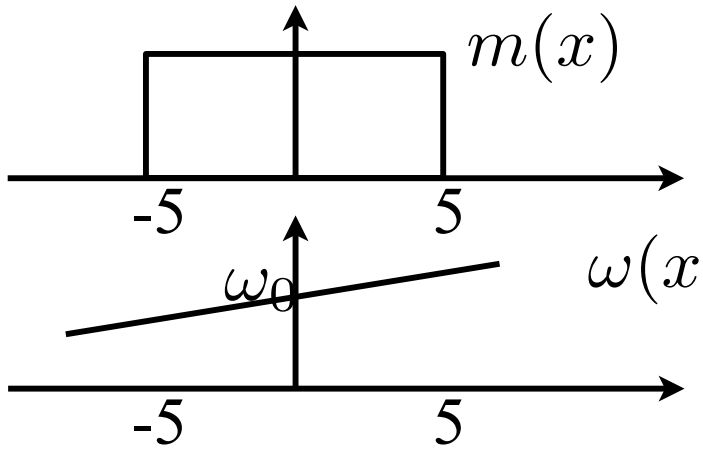
Gradient strength
Gradient duration } Resolution

$\Downarrow$

signal decay (T_2)
Field Inhomogeneity (T_2^*)
DIFFUSION

TYPICAL RESOLUTION: ≤ 1 mm
(~20 μm in small animal scanners)

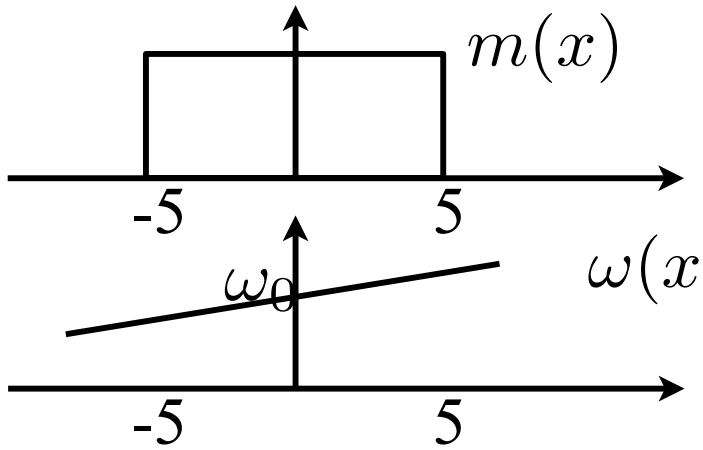
Example: Square RECT



$$\omega(x) = \omega_0 + \gamma G_x x$$

- What is the signal after excitation?
  - Signal proportional to m_{xy}
  - Comes from the entire volume

Example: Square RECT



$$\omega(x) = \omega_0 + \gamma G_x x$$

- Magnetization at position $x=x_0$

$$m_{xy}(x) = m(x)e^{-i(\omega_0+\gamma G_x x)t}$$

- Signal from everything

$$s(t) = \int_x m(x)e^{-i(\omega_0+\gamma G_x x)t}dx$$

Example: Square RECT

$$
\begin{aligned}
s(t) &= \int_x m(x) e^{-i(\omega_0 + \gamma G_x x)t} dx \\
&= e^{-i\omega_0 t} \int_x m(x) e^{-i\gamma G_x x t} dx \\
&= e^{-i\omega_0 t} \int_x m(x) e^{-i2\pi(\frac{\gamma}{2\pi} G_x t)x} dx \\
&= e^{-i\omega_0 t} \int_x m(x) e^{-i2\pi k_x x} dx \\
&= e^{-i\omega_0 t} \mathcal{F}\left\{m(x)\right\}\Big|_{k_x = \frac{\gamma}{2\pi} G_x t}
\end{aligned}
$$

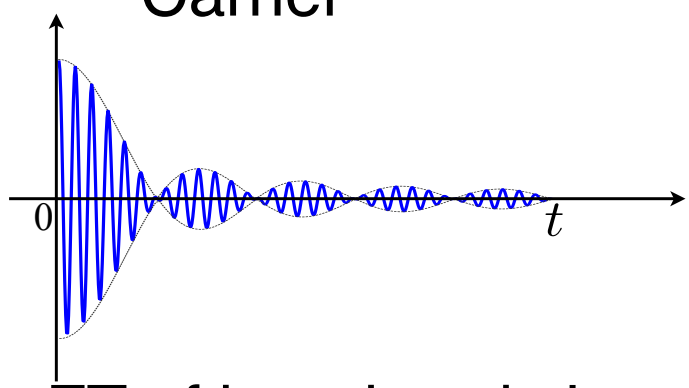
$$k_x = \frac{\gamma}{2\pi} G_x t$$

Example: Square RECT

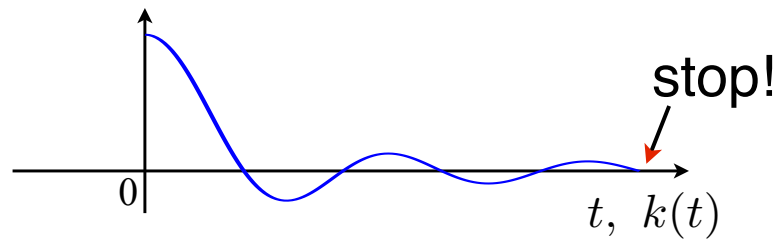
$$s(t) \quad = \quad e^{-i\omega_0 t}\mathcal{F}\left\{m(x)\right\}\Big|_{k_x=\frac{\gamma}{2\pi}G_x t}$$

$$m(x) = \mathrm{rect}(\cdot) \quad \Rightarrow \quad \mathcal{F}\{m(x)\} \sim \mathrm{sinc}(\cdot)$$

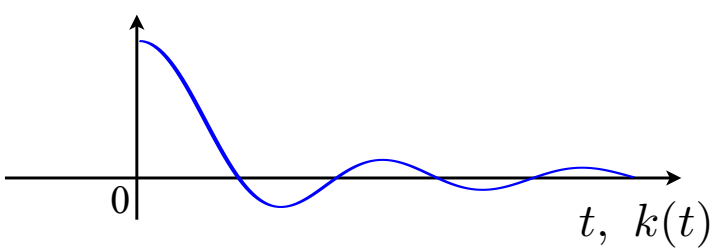
Carrier

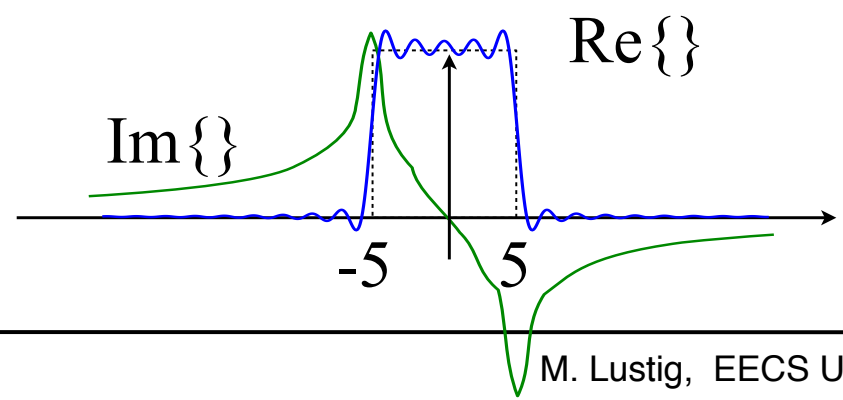


Baseband



Inverse FT of baseband signal yields 1D projection (Almost..)





Imaging

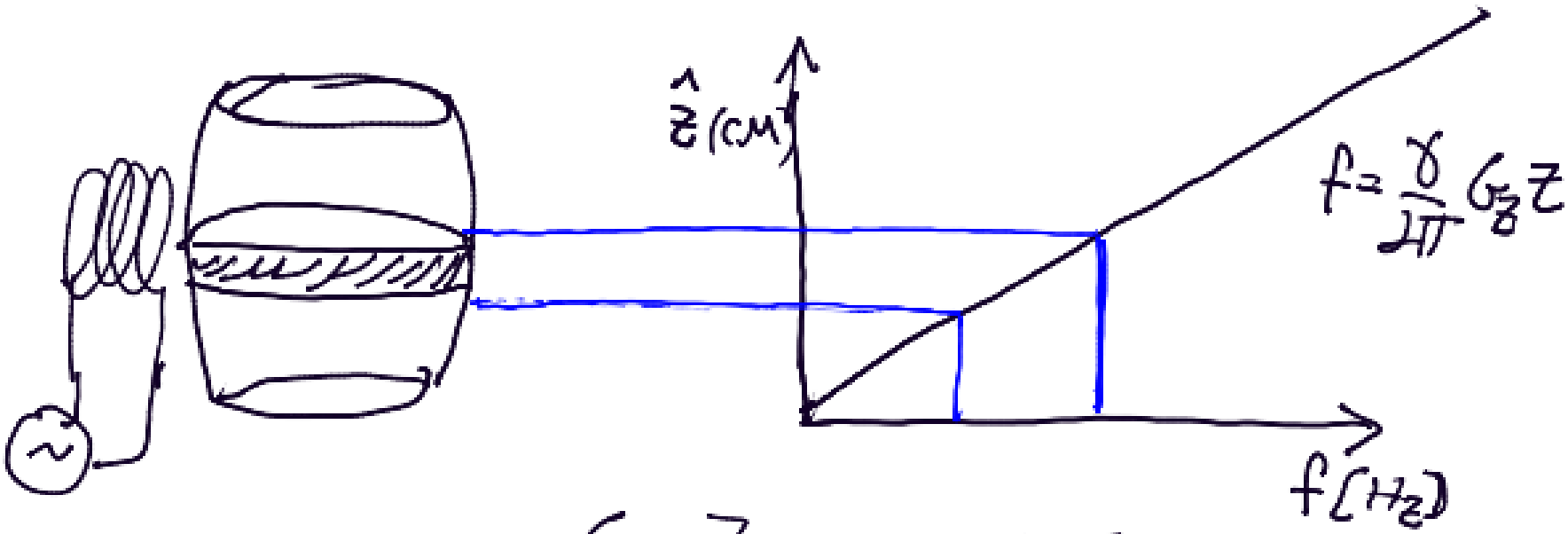
Q. How do we get an image?

several key components:

1. Selective excitation (Dimension reduction)
2. Spatial encoding (Freq/phase) ← more later
3. Signal decay, M_0 recovery
4. Repeat N Times
5. Image Reconstruction

Selective Excitation

want to excite a slice



$\mathcal{F}\{B_1(t)\} =$ 

Gradient (G_z) maps position to bandwidth

Bandlimited $B_1(t) \Longrightarrow$ Excites a slab in space

Excitation Field

$$\underbrace{e^{-i\omega_0 t}}_{\text{carrier}} \quad \underbrace{B_1(t)}_{\text{envelope}}$$

(center of slice) (shape of slice)

choose $B_1(t)$

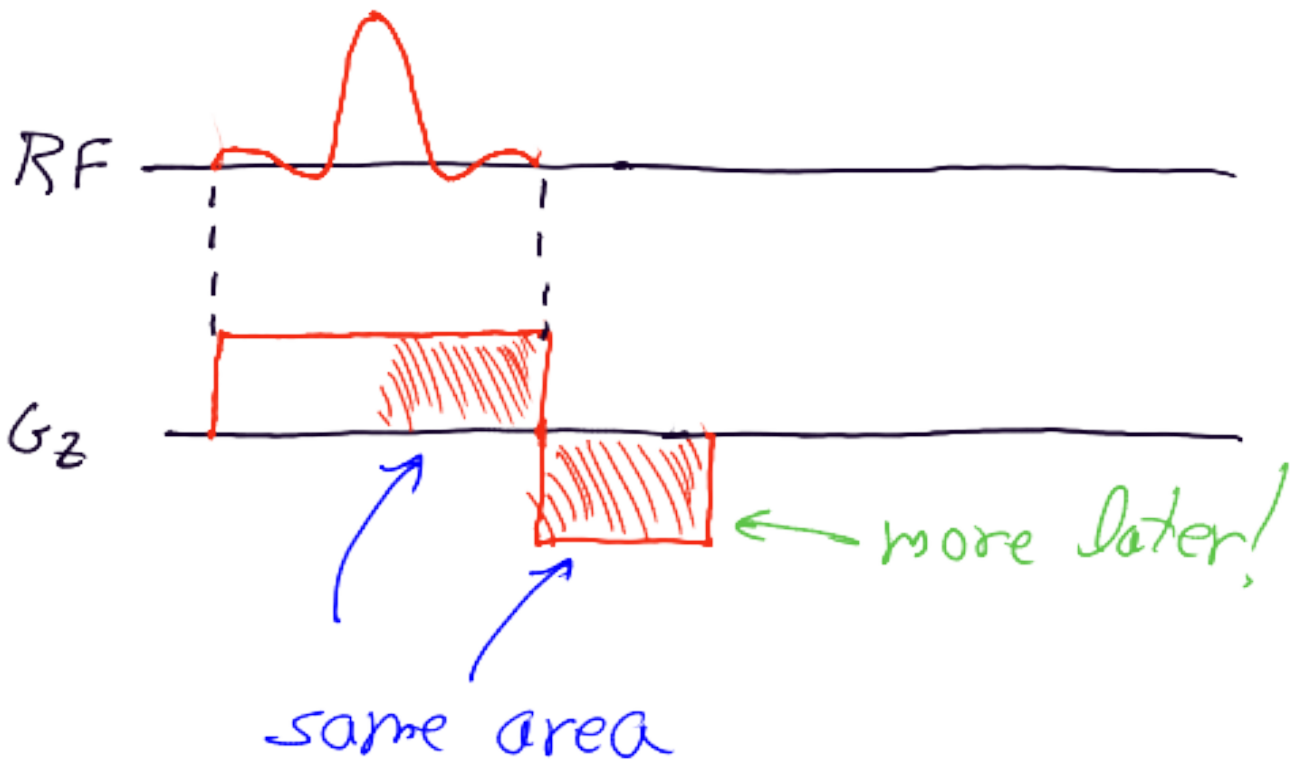
$$\mathcal{F}\{B_1(t)\} \simeq \mathrm{rect}\left(\frac{f}{A}\right)$$

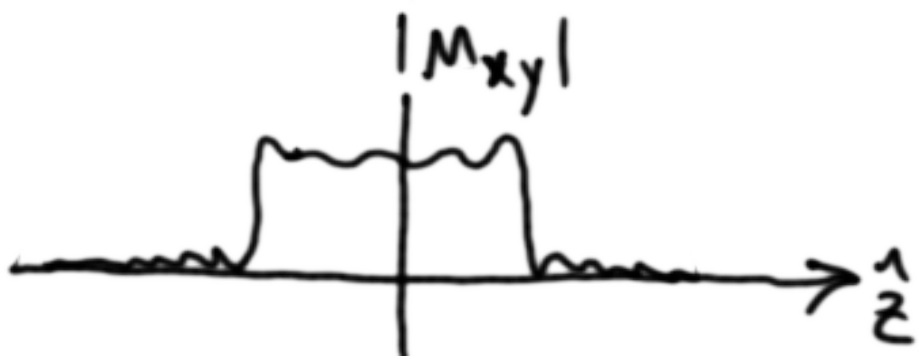
$$B_1(t) \simeq A\,\mathrm{sinc}(At)$$

finite in time

ripple in slice

we draw RF excitation as a pulse sequence:





Quiz

$$B_1(t) = e^{-i\omega_0 t}\mathrm{sinc}(5000t)$$

$$G_z = 1\ G/cm$$

What is the excited slice thickness?

Quiz

$$B_1(t) = e^{-i\omega_0 t}\mathrm{sinc}(5000t)$$

$$G_z = 1\ G/cm$$

Q: What is the excited slice thickness?

A:

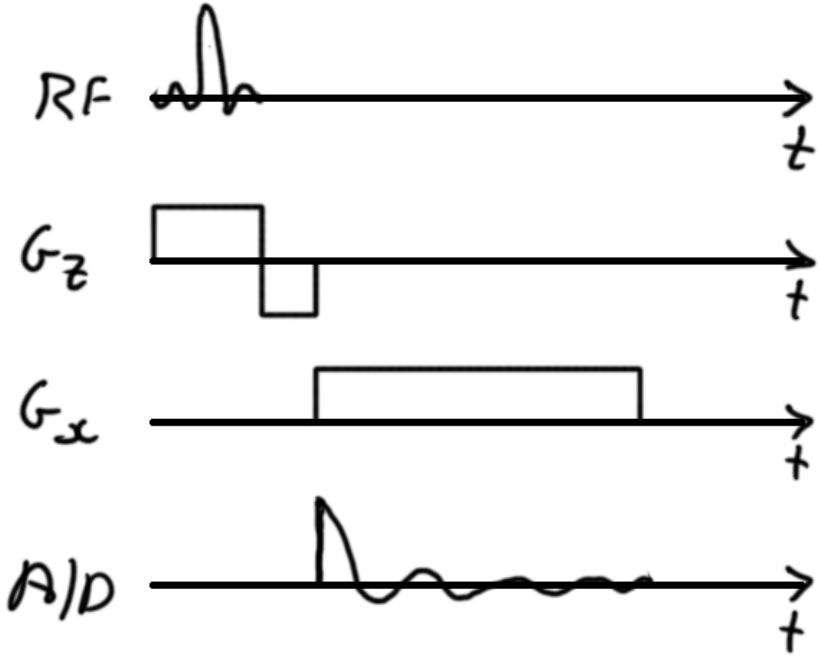
$$\mathcal{F}\{B_1(t)\} \propto \Pi\left(\frac{f}{5000}\right)$$

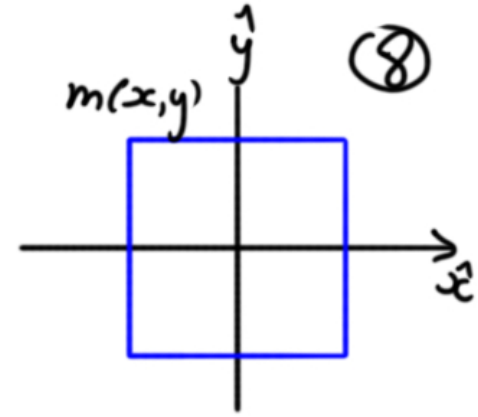
$$\frac{\gamma}{2\pi} G_z = 4257\ Hz/cm$$

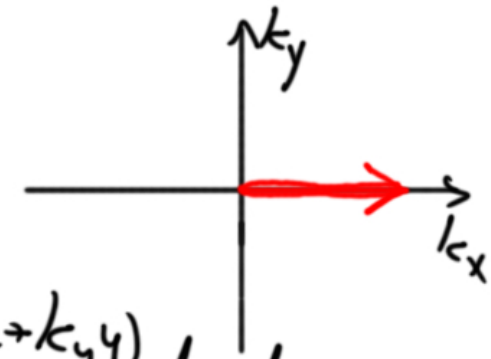
$$\frac{\gamma}{2\pi} G_z \Delta z = 5000\ Hz$$

$$\Delta z = \frac{5000}{4257} \simeq 1.17\ cm$$

2D SPATIAL ENCODING







$$s(t) = \iint m(x,y)\, e^{-i2\pi(k_x x + k_y y)}\, dx\, dy$$

$$k_x = \frac{\gamma}{2\pi} G_x t \qquad k_y = 0$$

Need k_x–k_y coverage:

Try: $G_x = G\cos\theta_n$ $\quad \theta_n = \frac{\pi}{N} n \quad n = [0, N-1]$

$G_y = G\sin\theta_n$

The Central Section Theorem

(Projection slice)

R. Bracewell

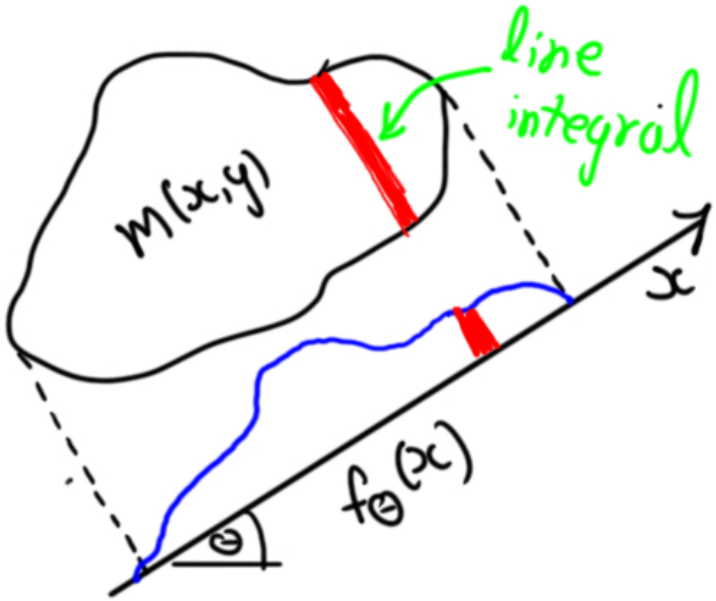


Image Space

k-space

Transform (1D) of projection of an object

$\Longrightarrow$ Diameter through transform(2D) of object

Very cool! ... and useful...

Projection slice $\Rightarrow$ basis for CT

Proof (for $\theta = 0$)

$$p(x) = \int_{-\infty}^{\infty} m(x,y)\,dy$$

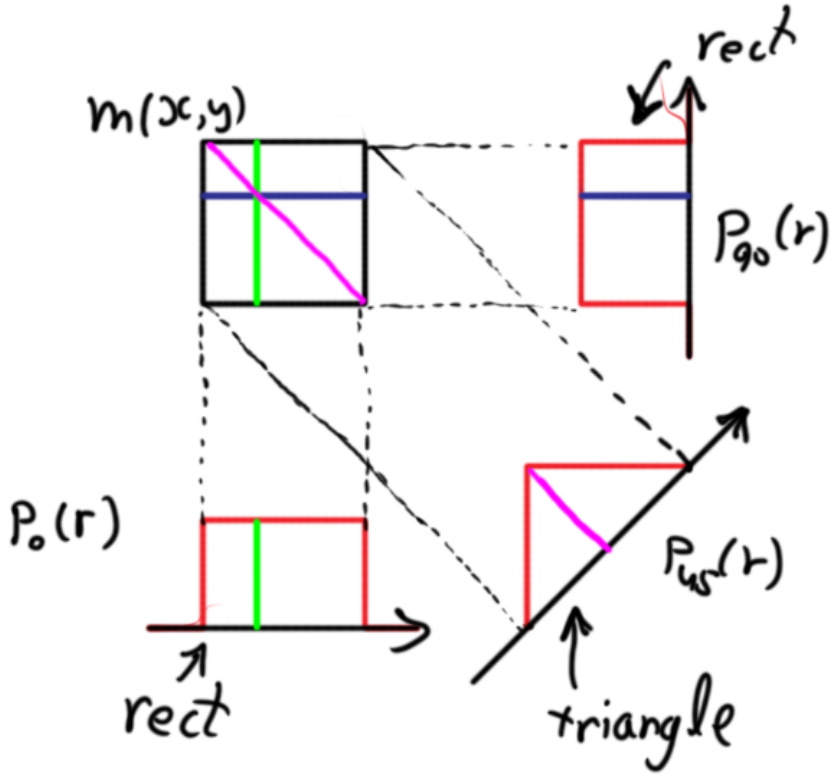
$$M(k_x, 0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\, e^{-i2\pi(k_x x + k_y y)}\,dx\,dy =$$

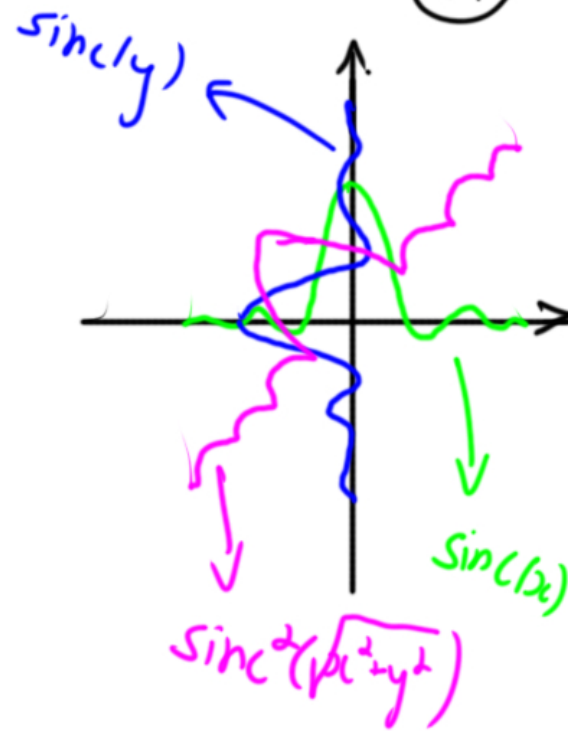
$$= \iint_{-\infty}^{\infty} m(x,y)\, e^{-i2\pi k_x x}\,dx\,dy =$$

$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} m(x,y)\,dy\right] e^{-i2\pi k_x x}\,dx =$$

$$= \int_{-\infty}^{\infty} p(x)\, e^{-i2\pi k_x x}\,dx = \mathcal{F}\{p(x)\}$$

Let's see if it makes sense...

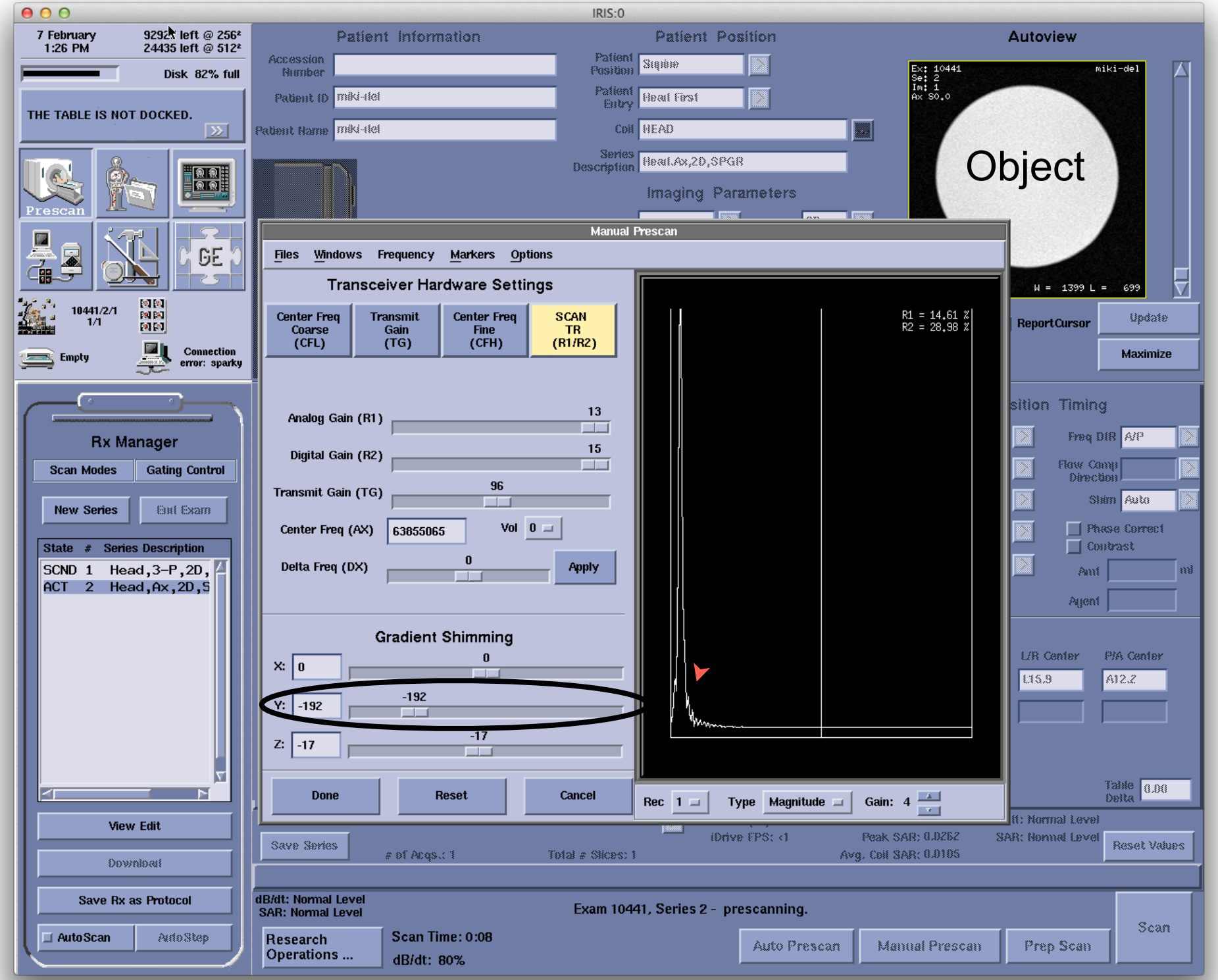


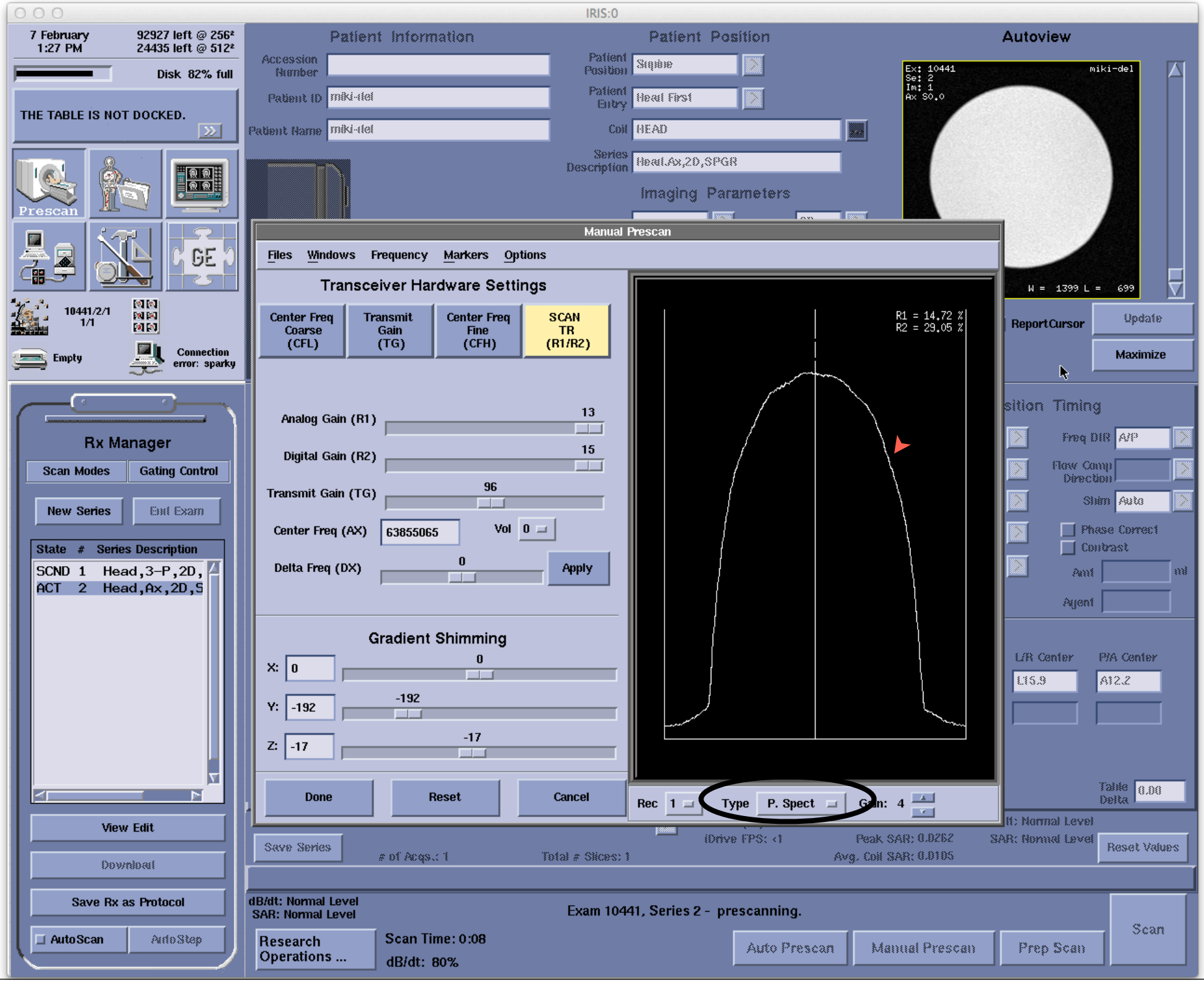


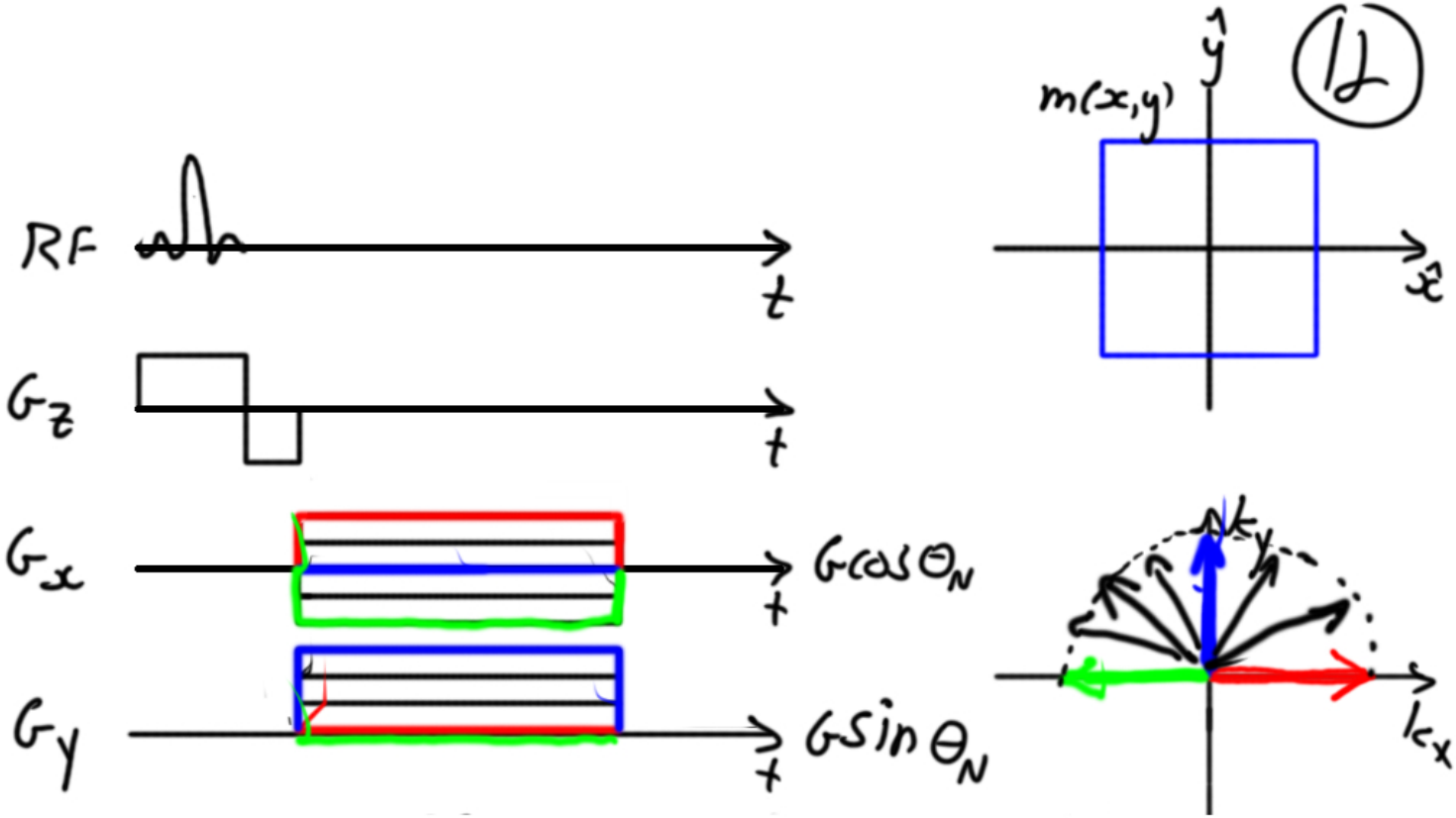
How to reconstruct?

- filtered back-projection
- Interpolation (in k-space)

Good class project


IRIS:0
7 February
1:26 PM
9292 left @ 256²
24435 left @ 512²
Disk 82% full
THE TABLE IS NOT DOCKED.
Prescan
GE
10441/2/1
1/1
Empty
Connection error: sparky
Rx Manager
Scan Modes
Gating Control
New Series
Exit Exam
State # Series Description
SCND 1 Head,3-P,2D,
ACT 2 Head,Ax,2D,S
View Edit
Download
Save Rx as Protocol
AutoScan
AutoStep
Patient Information
Accession Number
Patient ID
Patient Name
Patient Position
Patient Position
Supine
Patient Entry
Head First
Coil
HEAD
Series Description
Head,Ax,2D,SPGR
Imaging Parameters
Autoview
Object
ReportCursor
Update
Maximize
Manual Prescan
Files
Windows
Frequency
Markers
Options
Transceiver Hardware Settings
Center Freq Coarse (CFL)
Transmit Gain (TG)
Center Freq Fine (CFH)
SCAN TR (R1/R2)
Analog Gain (R1)
13
Digital Gain (R2)
15
Transmit Gain (TG)
96
Center Freq (AX)
63855065
Vol
0
Delta Freq (DX)
0
Apply
Gradient Shimming
X:
0
Y:
-192
Z:
-17
Done
Reset
Cancel
R1 = 14.61 %
R2 = 28.98 %
Rec
1
Type
Magnitude
Gain:
4
Freq DIR
A/P
Flow Comp Direction
Shim
Auto
Phase Correct
Contrast
Amt
Agent
L/R Center
P/A Center
L15.9
A12.2
Table Delta
0.00
Save Series
# of Acqs.: 1
Total # Slices: 1
iDrive FPS: <1
Peak SAR: 0.0262
Avg. Coil SAR: 0.0105
SAR: Normal Level
Reset Values
dB/dt: Normal Level
SAR: Normal Level
Exam 10441, Series 2 - prescanning.
Research Operations ...
Scan Time: 0:08
dB/dt: 80%
Auto Prescan
Manual Prescan
Prep Scan
Scan


IRIS:0
7 February
1:27 PM
92927 left @ 256²
24435 left @ 512²
Disk 82% full
THE TABLE IS NOT DOCKED.
Prescan
10441/2/1
1/1
Empty
Connection error: sparky
Rx Manager
Scan Modes
Gating Control
New Series
Exit Exam
State # Series Description
SCND 1 Head,3-P,2D,
ACT 2 Head,Ax,2D,S
View Edit
Download
Save Rx as Protocol
AutoScan
AutoStep
Patient Information
Accession Number
Patient ID
Patient Name
Patient Position
Patient Position
Supine
Patient Entry
Head First
Coil
HEAD
Series Description
Head,Ax,2D,SPGR
Imaging Parameters
Autoview
Ex: 10441
Se: 2
Im: 1
Ax S0.0
miki-del
W = 1399 L = 699
ReportCursor
Update
Maximize
Manual Prescan
Files
Windows
Frequency
Markers
Options
Transceiver Hardware Settings
Center Freq Coarse (CFL)
Transmit Gain (TG)
Center Freq Fine (CFH)
SCAN TR (R1/R2)
Analog Gain (R1)
13
Digital Gain (R2)
15
Transmit Gain (TG)
96
Center Freq (AX)
63855065
Vol
0
Delta Freq (DX)
0
Apply
Gradient Shimming
X:
0
Y:
-192
Z:
-17
Done
Reset
Cancel
R1 = 14.72 %
R2 = 29.05 %
Rec
1
Type
P. Spect
Gain:
4
Freq DIR
A/P
Flow Comp Direction
Shim
Auto
Phase Correct
Contrast
Amt
Agent
L/R Center
P/A Center
L15.9
A12.2
Table Delta
0.00
Save Series
# of Acqs.: 1
Total # Slices: 1
iDrive FPS: <1
Peak SAR: 0.0262
Avg. Coil SAR: 0.0105
SAR: Normal Level
Reset Values
dB/dt: Normal Level
SAR: Normal Level
Exam 10441, Series 2 - prescanning.
Research Operations ...
Scan Time: 0:08
dB/dt: 80%
Auto Prescan
Manual Prescan
Prep Scan
Scan

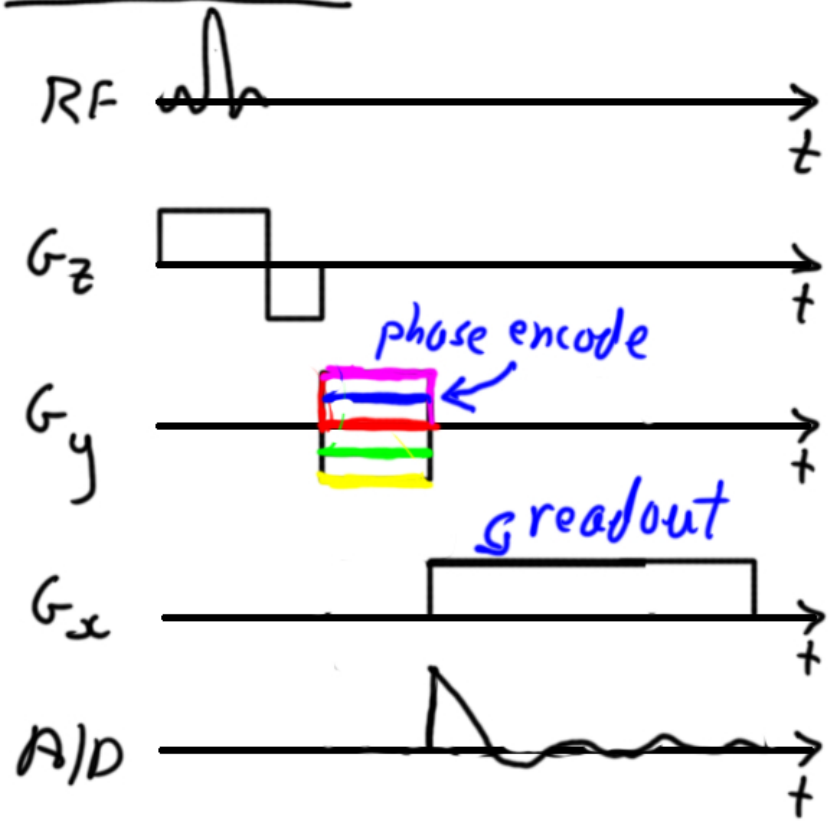


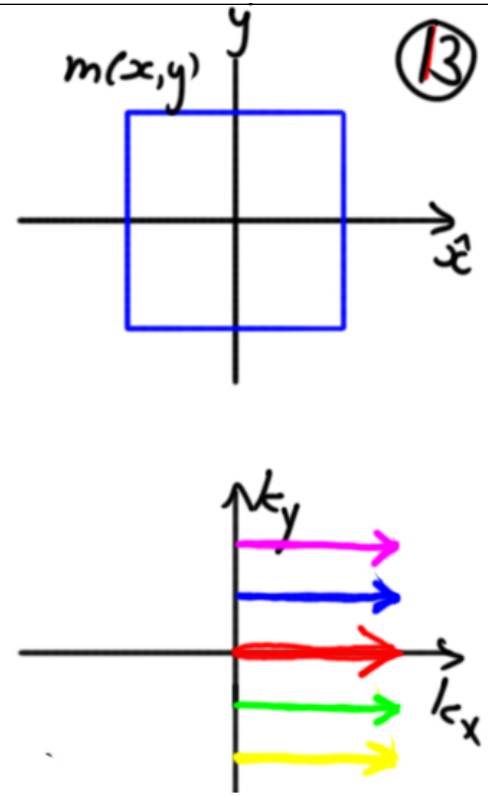
Projection Reconstruction (Lauterber)

$\mathcal{F}_{1D}^{-1}\{\text{Diameter}\} \Rightarrow \text{Projection}$

However... not quite diameter yet...

Better Idea





"2DFT"

"SPIN-WARP"

(still not perfect)

(-) Make "horizontal projections"

(-) Encode phase in y prior to readout in x

Other approaches to Imaging

→ Localization is the key!

(1) B_0 localization

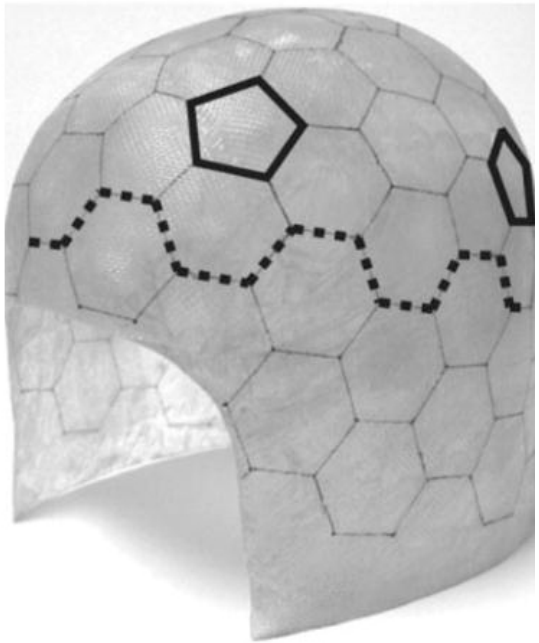
→ saddle point

→ local magnet

(2) RF

→ Local coils

→ Local Excitation

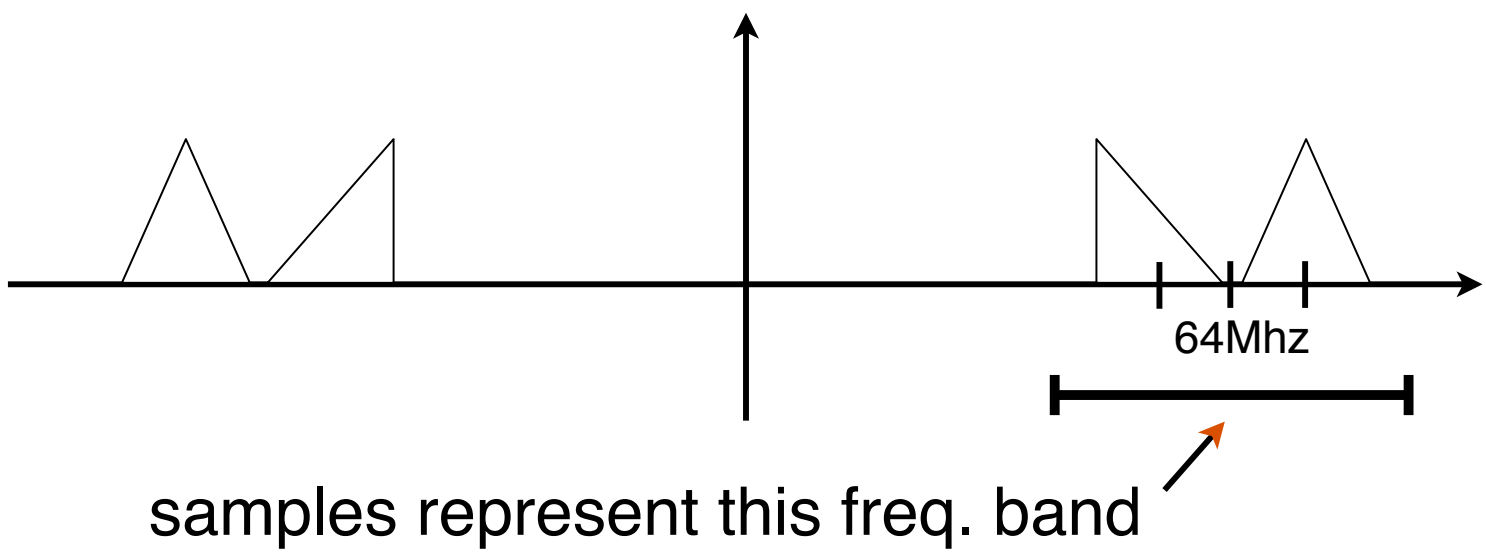


(3) Non linear gradients

Great Project!

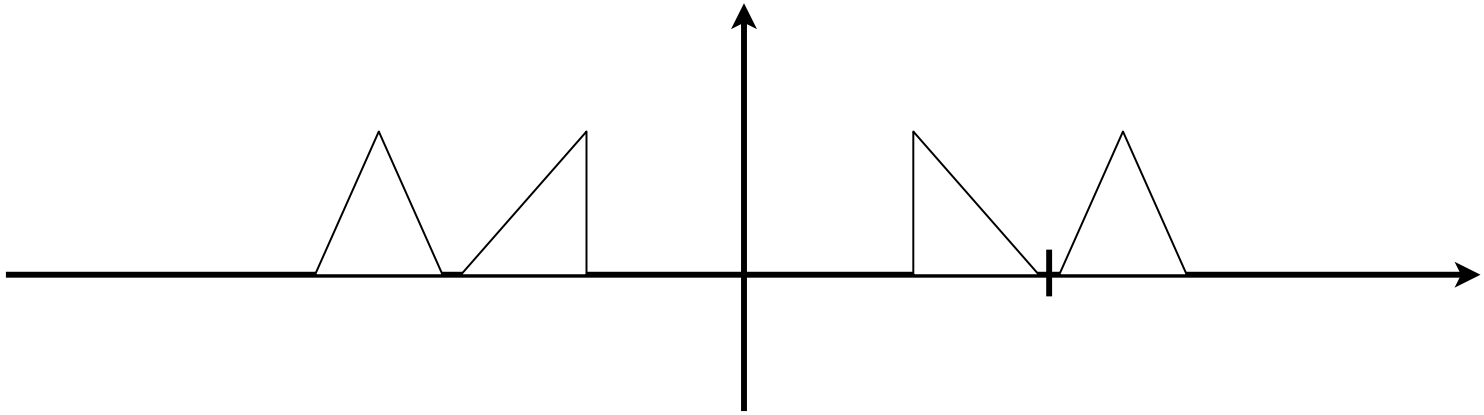
Complex Samples: IQ Demodulation

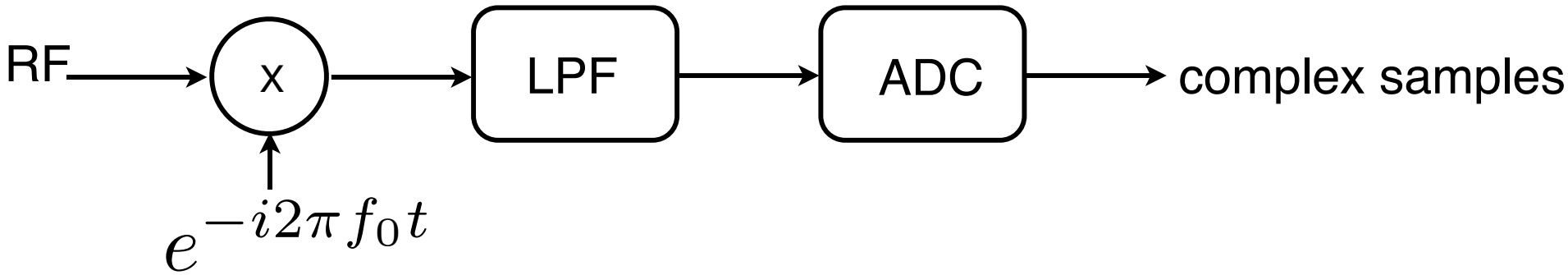
- With IQ demodulation we look at <u>part</u> of the spectrum
- Example:



IQ Demodulation

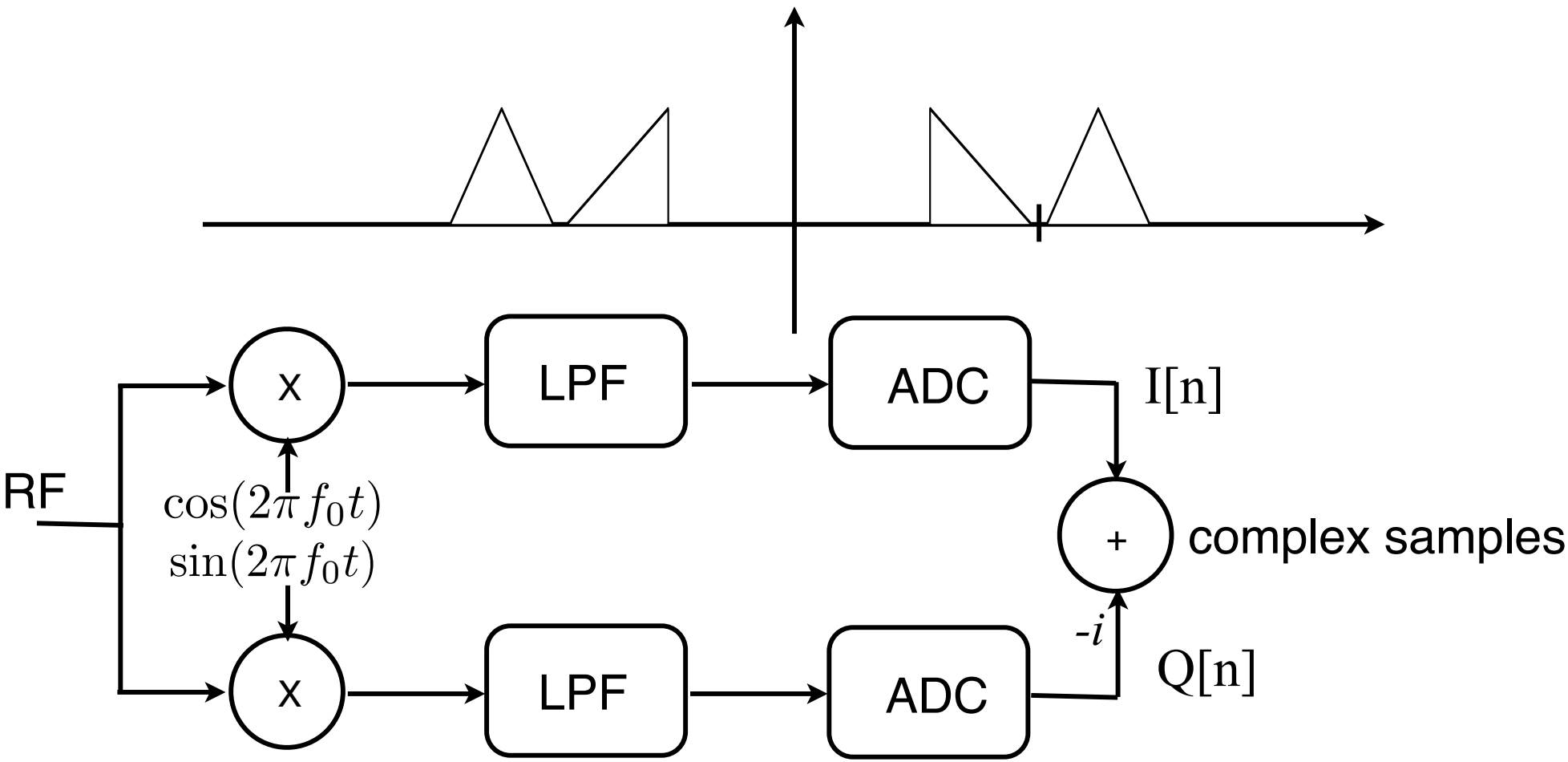
- How is it implemented?





IQ Demodulation

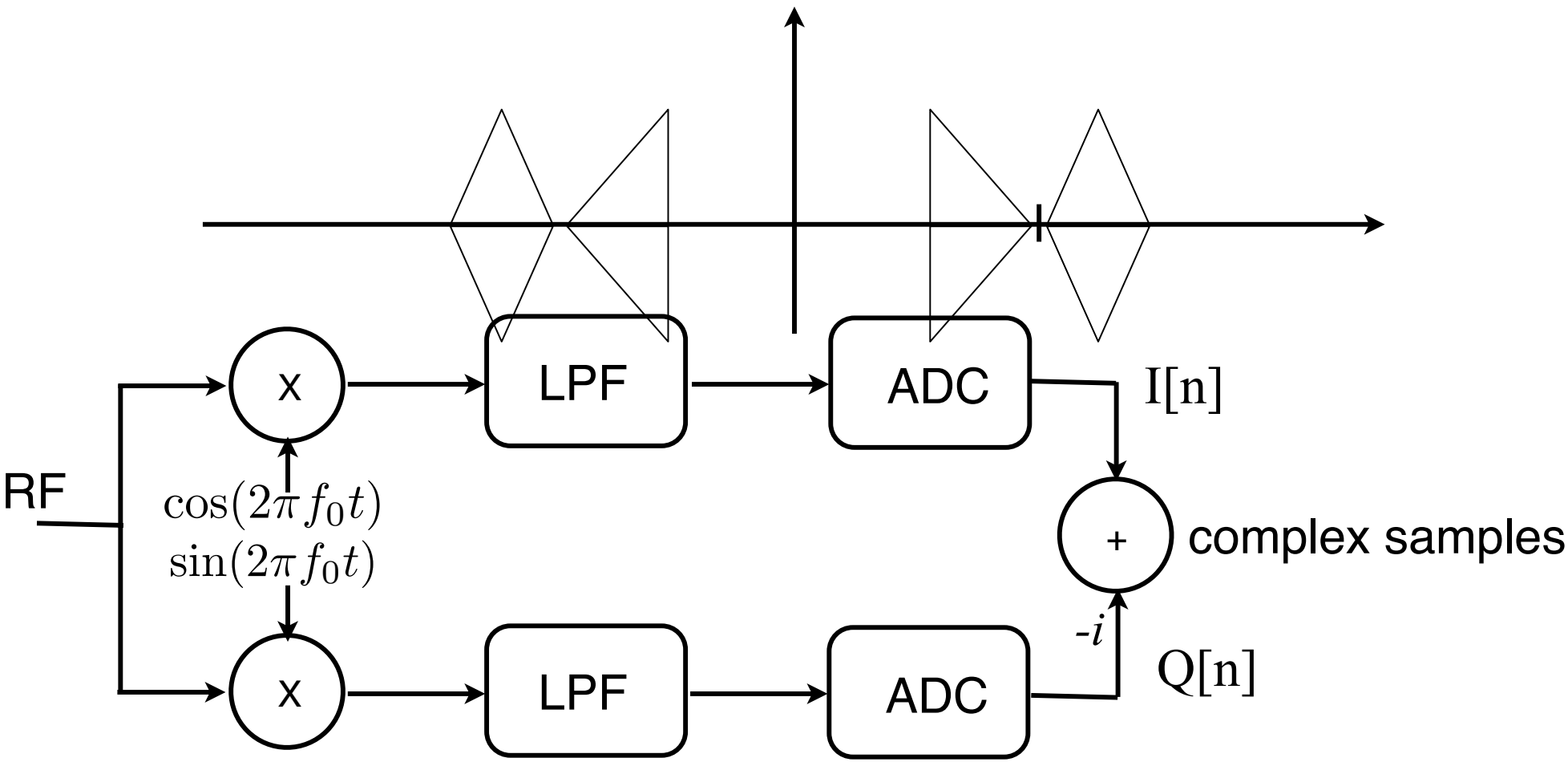
- How is it physically implemented?



$$e^{-i2\pi f_0 t} = \cos(2\pi f_0 t) - i\sin(2\pi f_0 t)$$

IQ Demodulation

- How is it physically implemented?



$$e^{-i2\pi f_0 t} = \cos(2\pi f_0 t) - i\sin(2\pi f_0 t)$$

Complex value & Demodulation

Physical signal:

$$s_p(t) = A(t)\cos(\omega_0 t + \phi(t)) =$$

real

trigo

$$\Longrightarrow \underbrace{A(t)\cos(\phi(t))}_{I(t)}\cos(\omega_0 t) + \underbrace{\left[-A(t)\sin(\phi(t))\right]}_{Q(t)}\sin(\omega_0 t)$$

$I(t)$ in phase

$Q(t)$ quadrature

Analytic signal (convinient)

$$s_r = A(t)e^{-i(\omega_0 t + \phi(t))}$$

complex

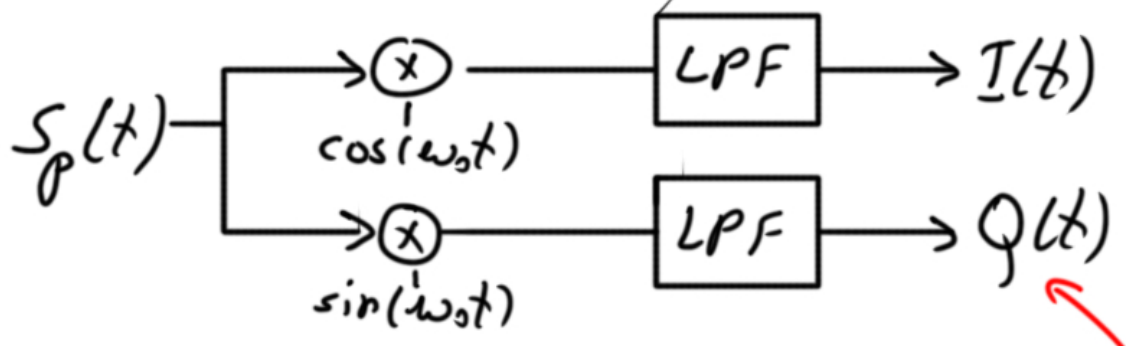
real

$$s_p = \mathrm{Re}\{s_r\}$$

Baseband signal

$$s(t) = s_r(t)e^{+i\omega_0 t} = A(t)e^{-i\phi(t)} = I(t) + iQ(t)$$

Quadrature Phase-Sensitive Detection:



often used in communication

$$s(t) = I(t) + i\,Q(t)$$

$I(t) \rightarrow M_x$ in rotating frame

$Q(t) \rightarrow M_y$ in rotating frame