Principles of MRI

EE225E / BIO265

Chapter 04 MR Physics

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Spin, Magnetic Moment and Magnetization

- Nuclei with Odd # protons/Neutrons posses spin angular momentum

\[ S = \hbar \hat{I} \]

- Associated with S is a magnetic moment

\[ \mu = \gamma \hat{S} = \gamma \hbar \hat{I} \]
Demonstration of Magnetic Resonance

https://www.youtube.com/watch?v=5r_aXiCKlhw
Spin, Magnetic Moment and Magnetization

- In a strong magnetic field $B_0$, spins align with $B_0$ giving a net magnetization
- Magnetic Moment is produced

$$M = \sum \mu$$
Energy

\[ E = -\mathbf{\mu} \cdot \mathbf{B} = -\mu_B B_0 = -\hbar s_z B_0 \]

- Potential energy
- No reverse component

- \( s_z \) is quantized to \( \hbar I_z \Rightarrow I_z = \pm \frac{\ell}{2} \)

\[ \Delta E = \frac{\Delta}{2\hbar} \hbar B_0 \]
Energy Splitting

- Zeeman splitting
  
  No field $\Rightarrow$ singlet state

- Two populations $\mu_+$ or $\mu_-$

- Jump between states do to thermal energy
  
  $\frac{\mu_-}{\mu_+} = e^{-\frac{\Delta E}{kT}} \approx 0.999993$
Magnetization

- Magnetization:

\[ M_0 = \frac{N \gamma^2 \hbar^2 I_z (I_z + 1) B_0}{3kT} \]
Increase in signal-to-noise ratio of >10,000 times in liquid-state NMR

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Slide: Simon Hu, UCSF

Solid material doped with unpaired electrons

P_e = 94% and P_C = 0.086%

Microwaves transfer the polarization from electrons to nuclei

3.35T, ~1.2K

electron

proton

carbon

Temperature (K)
https://www.youtube.com/watch?v=MzlDg-zT48c
Magnetism

• Most Objects Exhibit **induced magnetism**

\[ M_{\text{induced}} = \mu_0^{-1} V \times B \]

- **magnetic permeability**
- **volume**
- **magnetic susceptibility**
- **magnetic field**

\[ X > 0 \quad \text{Paramagnetism} \]
\[ X < 0 \quad \text{Diamagnetism} \]
Magnetism

• Where does it come from?
  1. Circulation of electric currents
  2. Magnetic moment of electrons
  3. Magnetic moment of nuclei

\[1 + 2 >> 3\]
Magnetism

- (1) is explained by classical Physics

Contributes to Diamagnetism

Orbit Fluctuations
Magnetism

• (2) + (3) are intrinsic magnetism

\[ \hat{\mu} = \gamma \hat{S} \]

\[ |\gamma_e| >> |\gamma_p| \]

• Effect (2) cancels in most materials due to paired electrons (only exists in stable free radicals)
Magnetic Susceptibility

• Effect (1) is huge !!!
  – Macroscopically, it just changes the bulk magnetic field

\[ \chi_{\text{water}} \approx -9 \cdot 10^{-6} \quad \chi_{\text{proton}} \approx 4 \cdot 10^{-9} \]

Diamagnetic Levitation example:
Quantitative Susceptibility

• Indirectly can be observed

*Tian Liu, Cornell (from Wikipedia)
Susceptibility Mapping

- Indirectly can be observed

http://mri.kennedykrieger.org/nationalresource/trd2.html
CHEMICAL SHIFT

But (1) has microscopic effect

\[ B_{\text{eff}} = B_0 - B_{\text{ax}} = B_0 (1 - \alpha) \]

\[ \omega_{\text{eff}} = \omega_0 (1 - \alpha) \]

\[ \delta \rightarrow \text{CHEMICAL SHIFT} = \frac{\omega_2 - \omega_{\text{reference}}}{\omega_{\text{reference}}} \times 10^6 \]

For example:

ACETIC ACID \( \text{CH}_3\text{COOH} \)

Oxygen attracts electrons

Direction is

Historical
PRECESSION

\[ \text{\( \mathbf{P} \times \mathbf{B} \)}\]

\( \text{FORCE} = \text{TORQUE} = \mathbf{P} \times \mathbf{B} \)

\( \text{TORQUE} = \frac{\partial \mathbf{S}}{\partial t} \)  

\( \text{Change of Angular Momentum} \)

\( \mathbf{S} = \frac{d\mathbf{S}}{dt} = \mathbf{P} \times \mathbf{B} \Rightarrow \frac{d\mathbf{P}}{dt} = \mathbf{P} \times \mathbf{B} \)

\( \text{Volume}\)

\( \frac{d\mathbf{M}}{dt} = \mathbf{M} \times \mathbf{B} \)

\( \text{or} \)

\( \frac{d\mathbf{M}}{dt} = -\mathbf{B} \times \mathbf{M} \)

**Solution for Precession**  
\( f = \frac{\partial}{\partial t} \mathbf{B}, \ \omega = \mathbf{B} \)

\( \frac{d\mathbf{M}}{dt} = \mathbf{M} \times \mathbf{B} \)

\( \text{Spatial Distribution}\)

\( \text{We have control!} \)
Mathematical Description of MRI

• Three Elements:
  – Precession about $\tilde{B}$ (all fields)
  – Transverse decay
  – Longitudinal recovery
Precession

\[ \vec{S} + \vec{\mu} + \vec{B} \Rightarrow \text{Precession} \]

\[ \frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \]

Solution: \( \omega = \gamma |B| \)
Mathematical Description of MRI

• Plan:
  1) Derive Math for each element
  2) Put together : e.g., the BLOCH equation
  3) Solve the Bloch eqn. for special cases
     a) Excitation          CH. 6 (later)
     b) Reception           CH. 5 (first)
        i) Derive k-space (AGAIN!!!)
        ii) Pulse sequence
        iii) Sampling
Precession

- We apply fields: $B_0$, $B_1$, $G$
Precession

• We apply fields: $B_0$, $B_1$, $G$

$$\vec{B} = B_0 \hat{k} + \nabla \cdot \vec{G} + B_{ax} (\cos \omega t \hat{i} - \sin \omega t \hat{j}) + B_{ay} (-\sin \omega t \hat{i} - \cos \omega t \hat{j})$$

$\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors

$$\vec{x} = [x, y, z]^T$$

$$\vec{G} = [Gx, Gy, Gz]^T$$
Precession

Magnetization is:
\[ \vec{M} = [M_x, M_y, M_z]^T \]

- \( \vec{M} \) precesses around \( \vec{B} \)
- Frequency of rotation is \( \omega = \gamma |\vec{B}| \)
- Axis of rotation is \( \vec{n} = \frac{\vec{B}}{|\vec{B}|} \)

normal
Precession

\[ \mathbf{B}_0 + \mathbf{G} = \mathbf{E} \]

\[ B_{1r}(t) \quad \text{"small"} \]

\[ \text{LAB FRAME} \]
Precession

- Described by cross product

\[ \frac{d\vec{M}}{dt} = -\gamma \vec{B} \times \vec{M} \]

- "-" Due to negative gyromagnetic ratio of protons

or:

\[ \frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \]

- \( B_0 \) Dominates! Hard to see other terms
Rotating Frame

• Change coordinates:

\[ [\hat{i}_r, \hat{j}_r, \hat{k}_r]^T = [i \cos \omega_0 t, j \sin \omega_0 t, k]^T \]

• In the rotating frame at \( w_0 \):
Rotating Frame

- Change coordinates:
  \[
  [\hat{i}_r, \hat{j}_r, \hat{k}_r]^T = [\hat{i} \cos \omega_0 t, \hat{j} \sin \omega_0 t, \hat{k}]^T
  \]

- In the rotating frame at \( w_0 \):
  \[
  \vec{B}_{\text{Rot}} = (B_0 - \frac{\omega_0}{\epsilon}) \hat{k}_r + \vec{G} \cdot \vec{\omega} \hat{k}_r + B_{1x} \hat{i}_r + B_{1y} \hat{j}_r
  \]

  And
  \[
  \left( \frac{d\vec{M}}{dt} \right)_{\text{Rot}} = -\gamma \vec{B}_{\text{Rot}} \times \vec{M}_{\text{Rot}}
  \]
Rotating Frame

- For $\omega = \gamma B_0$, MAIN FIELD GOES AWAY!

$$\vec{B}_{\text{rot}} = \vec{B} \cdot \hat{z} k_r + B_{\text{rot},x} \hat{x}_r + B_{\text{rot},y} \hat{j}_r$$

much simpler!

- $\vec{M}_{\text{rot}}$ precesses about applied fields $\vec{B} \cdot \hat{z}$ and $B_{\text{rot},x}, B_{\text{rot},y}$
Examples

Excitation

\[ \omega = \gamma |B_1| \]

Precession

\[ \omega = \gamma |\overrightarrow{G} \cdot \overrightarrow{x}| \]
Examples

\[ \omega = \nabla |B_1 + \frac{\gamma}{2} \mathbf{v} \times \mathbf{B}_1 | \]

- Gradient
- Total field

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Relaxation

- **T2 Decay**
  - Transverse magnetization decays
  - Due to loss of coherence between spins
  - Also called spin-spin relaxation

- Not a strong function of $B_0$
- Dipole effect stronger in solids
Transverse Relaxation (T2)

• Let

\[ M_{xy} = M_x + iM_y \]

• Then

\[ \frac{dM_{xy}}{dt} = -\frac{1}{T_2} M_{xy} \]
Transverse Relaxation (T2)

- Solution:

\[ M_{xy}(t) = e^{-\frac{t}{T_2}} M_{xy}(0) \]

Major source of contrast
Transverse Relaxation (T2)

- Example: Brain @ 1.5T
  - white matter T2=92ms, Density=0.65
  - gray matter T2=100ms, Density = 0.75

Excite, wait 100ms, collect data
T2 Example

Gray matter lighter

white matter darker

CSF Bright!
Magic Angle \(~55\) degrees

- Longer T2 due to dipole decoupling
Relaxation

• **T1 Recovery**
  – Longitudinal relaxation
  – Due to Spin-Lattice interaction
  – Thermal bouncing of molecules - lose cone of precession - align with field
  – Strong dependency on $B_0$, since energy level depends on $B_0$
  – $B_0$ strong - hard to transition - T1 long
• Bias towards up - stable anisotropic dist.
T1 Recovery

- Magnetization recovers to equilibrium $M_0$

\[
\frac{dM_z}{dt} = - \frac{M_z - M_0}{T_1}
\]

- Solution:

\[
M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1}
\]
T1 Recovery

After 90° pulse, \( M_z = 0 \)

\[
M_z(t) = M_0 - M_0 e^{-\frac{t}{T_1}} = M_0 (1 - e^{-\frac{t}{T_1}})
\]

Major source of contrast as well!
T1 Contrast

- Brain at 1.5T
  - Gray Matter T1 = 900ms
  - White Matter T1 = 800ms

Excite 90, wait, excite again, image....
T1 Contrast Example

Gray matter darker
white matter lighter
CSF Dark!
Fat Bright!
Relaxation
The Bloch Equation

• Combine Precession and relaxation

\[
\frac{d\vec{M}}{dt} = -\gamma \vec{B} \times \vec{M} - \frac{M_x \hat{i} + M_y \hat{j}}{T_2} - \frac{M_z - M_0}{T_1} \hat{k}
\]

• Phenomenological: Fits observations
  – Describes most of MRI
  – Sometimes Fails.... J-coupling, Magn, transfer