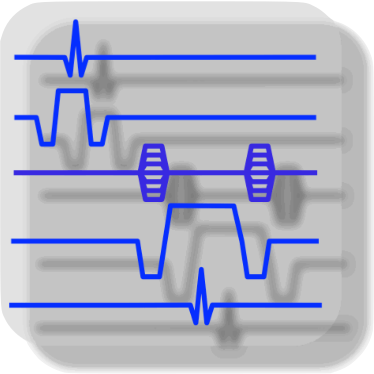


Principles of MRI

EE225E / BIO265

Chapter 04 MR Physics

Instructor: Miki Lustig
UC Berkeley, EECS

Spin, Magnetic Moment and Magnetization

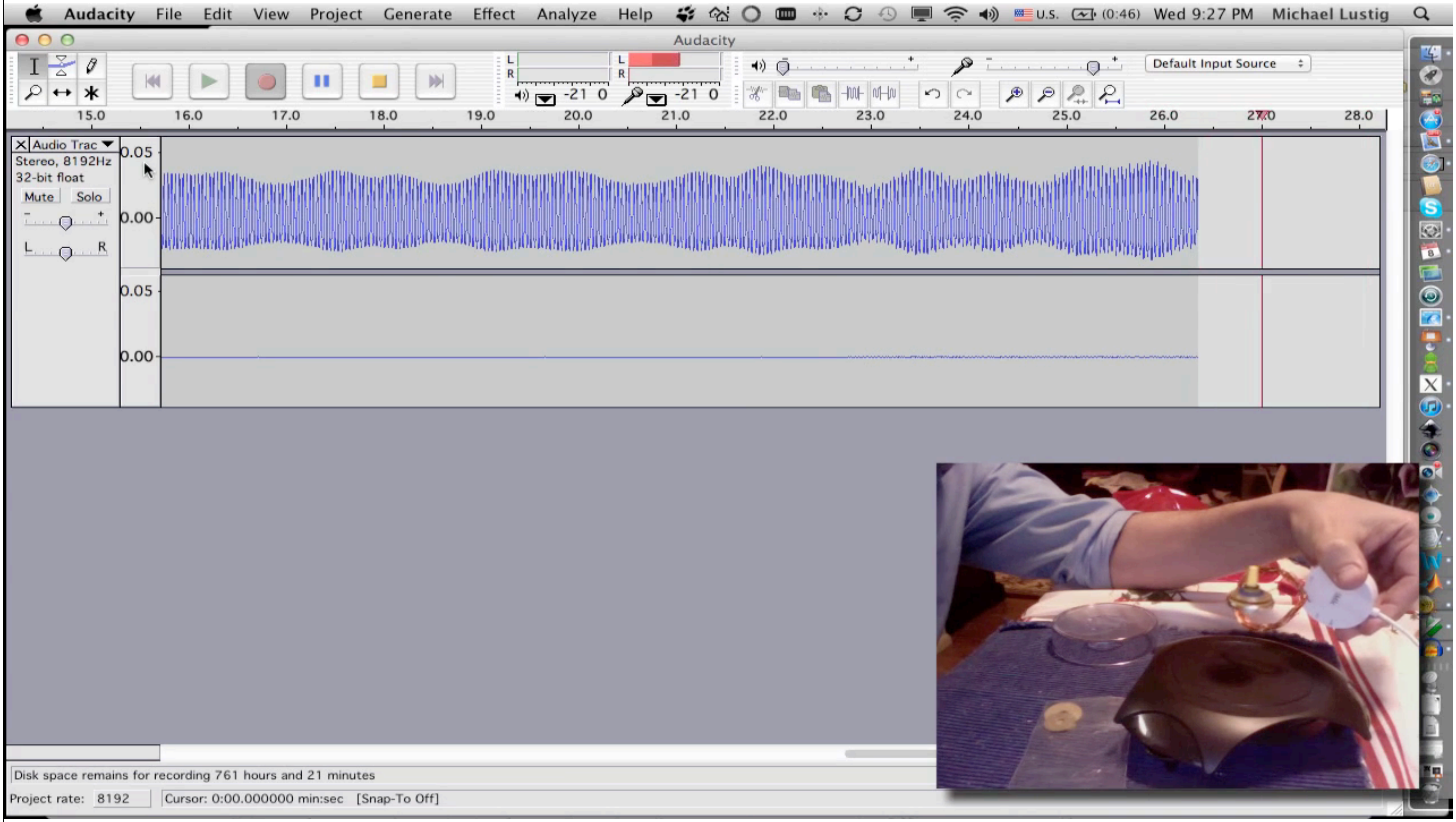
- Nuclei with Odd # protons/Neutrons posses spin angular momentum

$$\boxed{S = \hbar \hat{I}}$$

- Associated with S is a magnetic moment

$$\boxed{\mu = \gamma \hat{S} = \gamma \hbar \hat{I}}$$

Demonstration of Magnetic Resonance



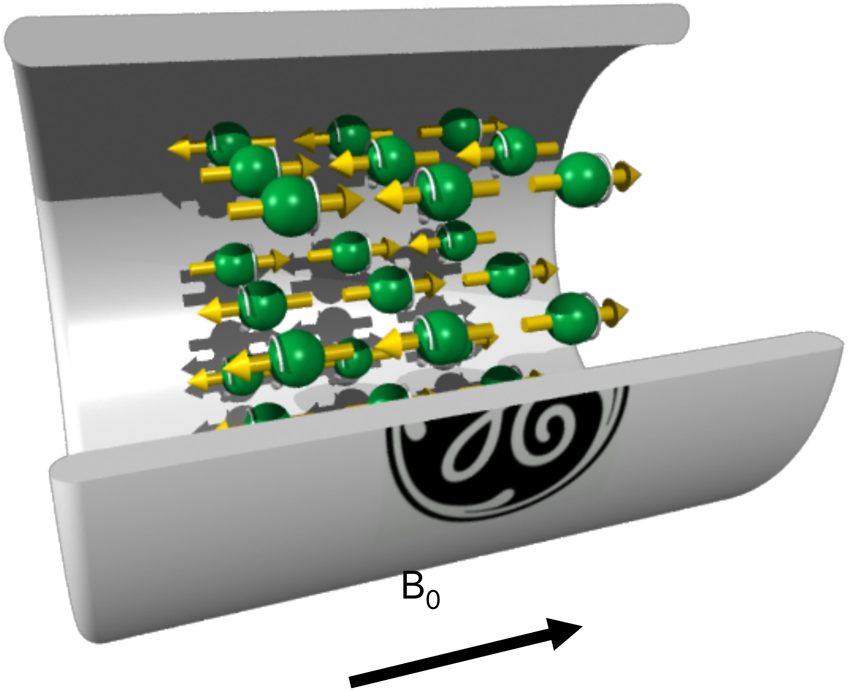
Disk space remains for recording 761 hours and 21 minutes

Project rate: 8192 Cursor: 0:00.000000 min:sec [Snap-To Off]

https://www.youtube.com/watch?v=5r_aXiCKlhw

Spin, Magnetic Moment and Magnetization

- In a strong magnetic field B0, spins align with B0 giving a net magnetization
- Magnetic Moment is produced
- $$M = \sum \mu$$



*Graphic rendering Bill Overall

Energy

- $E = -\vec{\mu} \cdot \vec{B} = -\mu_z B_0 = -\gamma S_z B_0$

potential energy

No xverse component

- S_z is quantized to $\hbar I_z \Longrightarrow I_z = \pm \frac{1}{2}$

$$\Delta E = \frac{\gamma}{2\pi} h B_0$$

Energy Splitting

- Zeeman splitting

  No field ⇒ singlet state

- Two populations ρ_+ or ρ_-

- Jump between states do to thermal energy

$$\frac{\rho_-}{\rho_+} = e^{-\frac{\Delta E}{KT}} \approx 0.999993$$

Magnetization

- Magnetization:

$$M_0 = \frac{N\gamma^2\hbar^2 I_z(I_z+1)B_0}{3kT}$$

linear

quadratic

linear

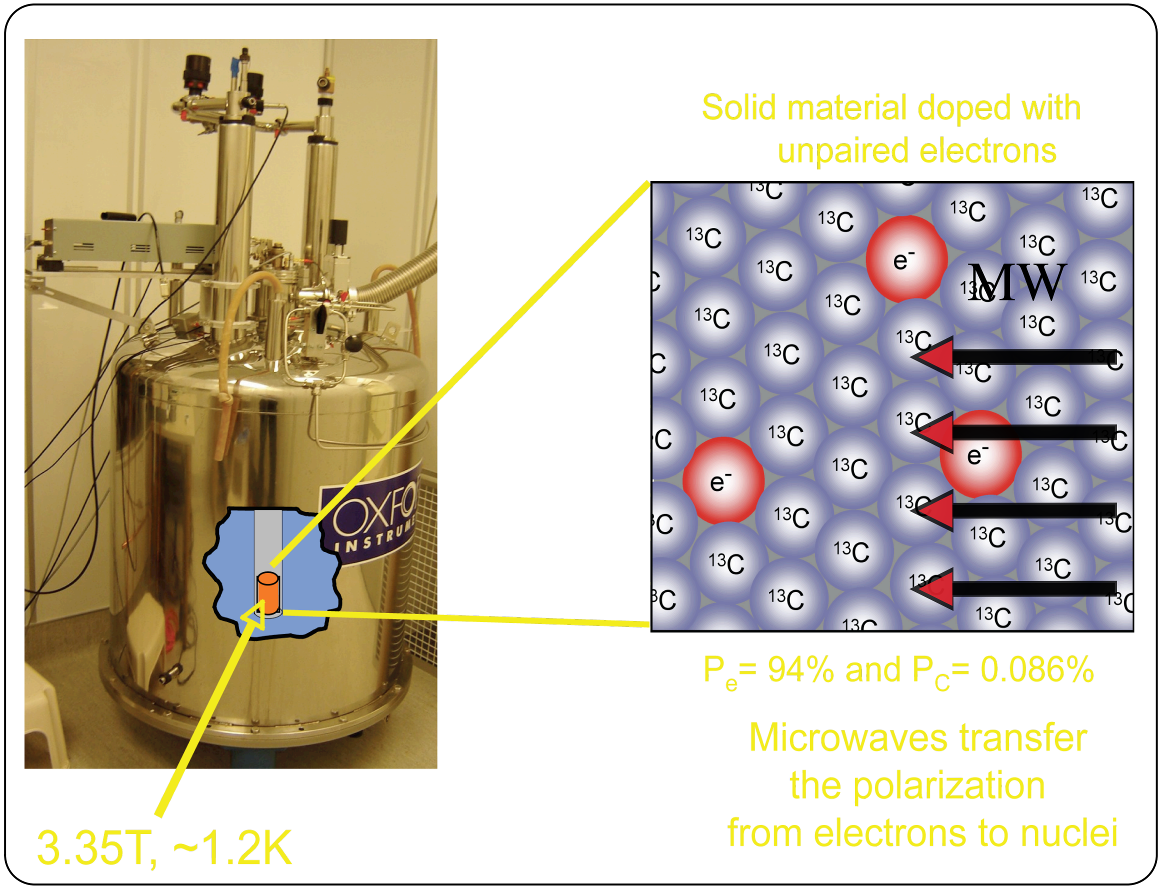
Increase in signal-to-noise ratio of >10,000 times in liquid-state NMR

Jan H. Ardenkjær-Larsen*, Björn Fridlund, Andreas Gram, Georg Hansson, Lennart Hansson, Mathilde H. Lerche, Rolf Servin, Mikkel Thaning, and Klaes Golman

Amersham Health Research and Development AB, Medeon, SE-205 12 Malmö, Sweden

Communicated by Albert W. Overhauser, Purdue University, West Lafayette, IN, June 20, 2003 (received for review April 16, 2003)

PNAS **September 2, 2003** vol. 100 no. 18 **10158–10163**

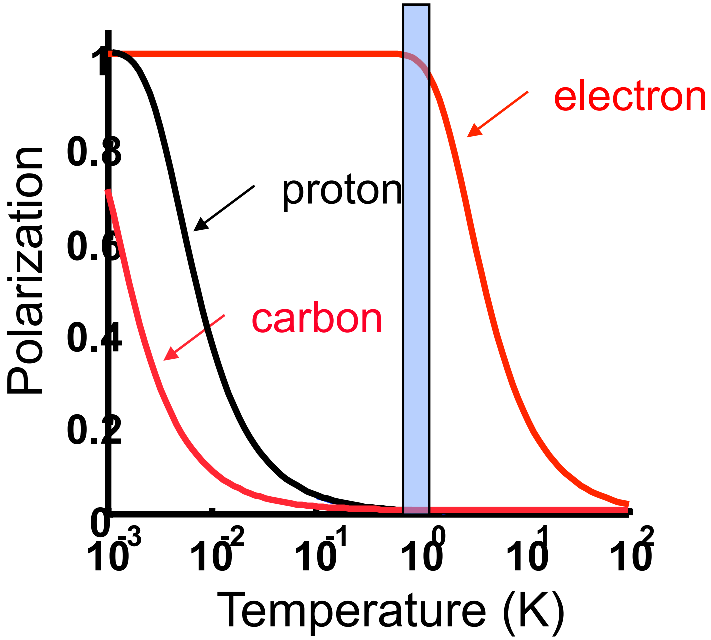


Solid material doped with unpaired electrons

P_e= 94% and P_C= 0.086%

Microwaves transfer the polarization from electrons to nuclei

3.35T, ~1.2K



Slide: Simon Hu, UCSF

https://www.youtube.com/watch?v=MzlDg-zT48c

Magnetism

- Most Objects Exhibit induced magnetism

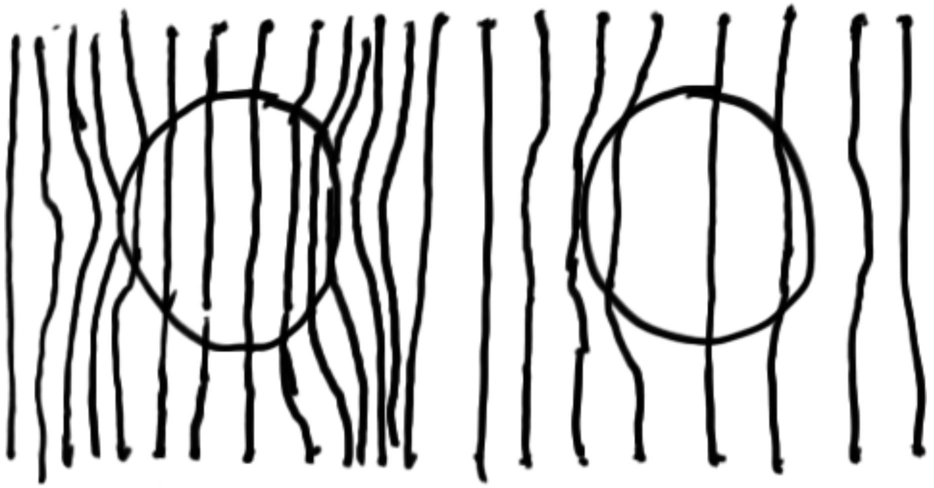
$$M_{induced} = \mu_0^{-1} V \chi B$$

magnetic permiability

volume

magnetic susceptibility

magnetic field



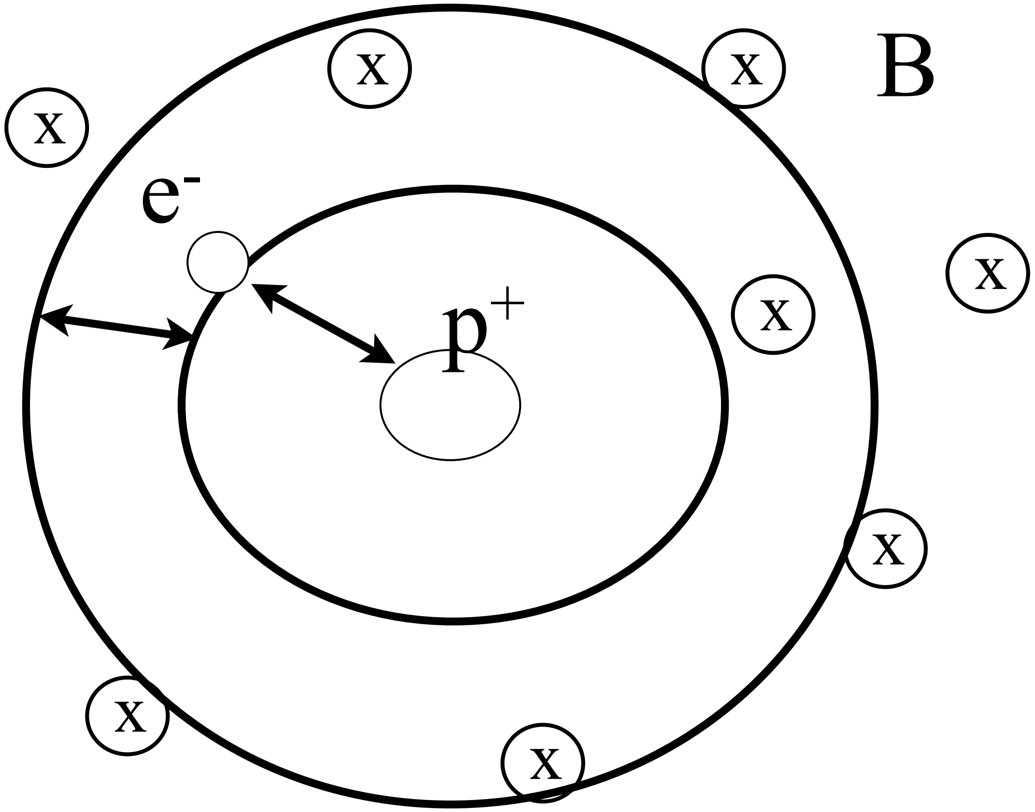
Magnetism

- Where does it come from?
  1. Circulation of electric currents
  2. Magnetic moment of electrons
  3. magnetic moment of neuclei

$$1 + 2 >>> 3$$

Magnetism

- (1) is explained by classical Physics



Orbit
Fluctuations

Contributes to Diamagnetism

Magnetism

- (2) + (3) are intrinsic magnetism

$$\hat{\mu} = \gamma \hat{S}$$

$$|\gamma_e| >> |\gamma_p|$$

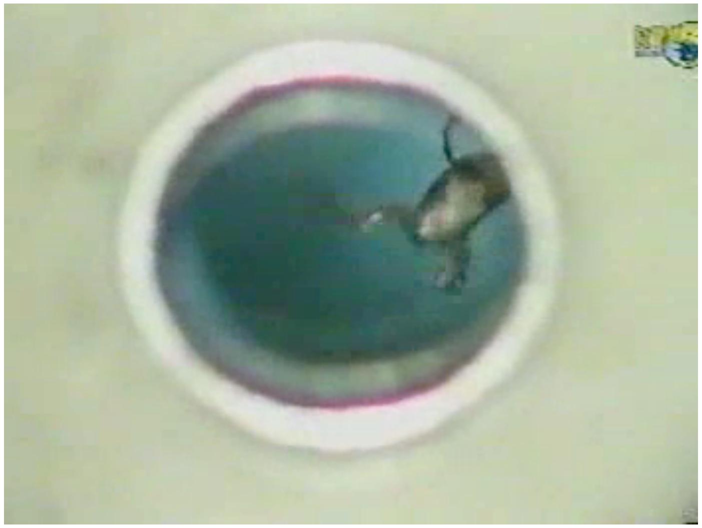
- Effect (2) cancels in most materials due to paired electrons (only exhists in stable free radicals

Magnetic Susceptibility

- Effect (1) is huge !!!
  - Macroscopically, it just changes the bulk magnetic field

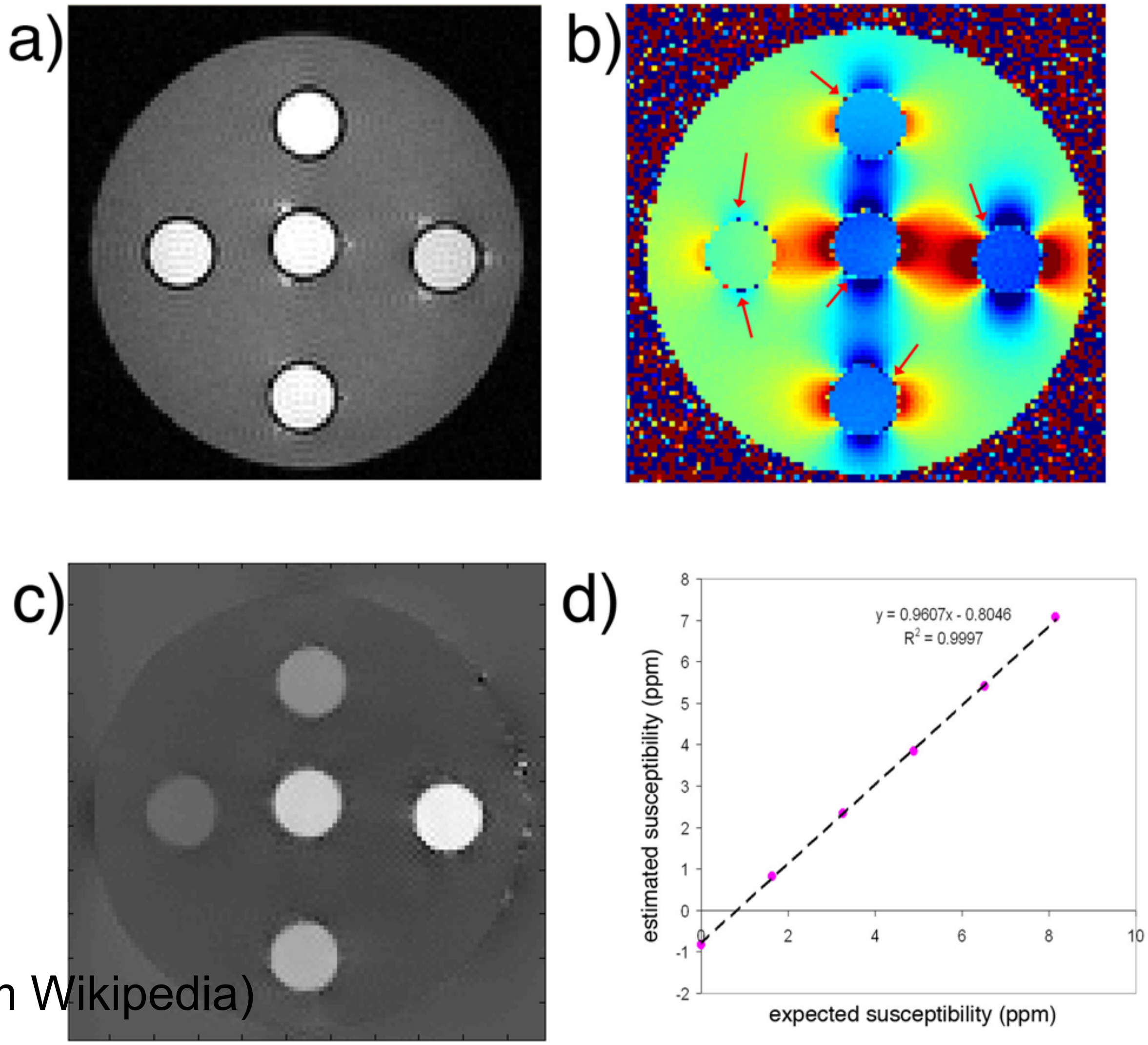
$$\chi_{\text{water}} \approx -9 \cdot 10^{-6} \qquad \chi_{\text{proton}} \approx 4 \cdot 10^{-9}$$

Diamagnetic Levitation example:



Quantitative Susceptibility

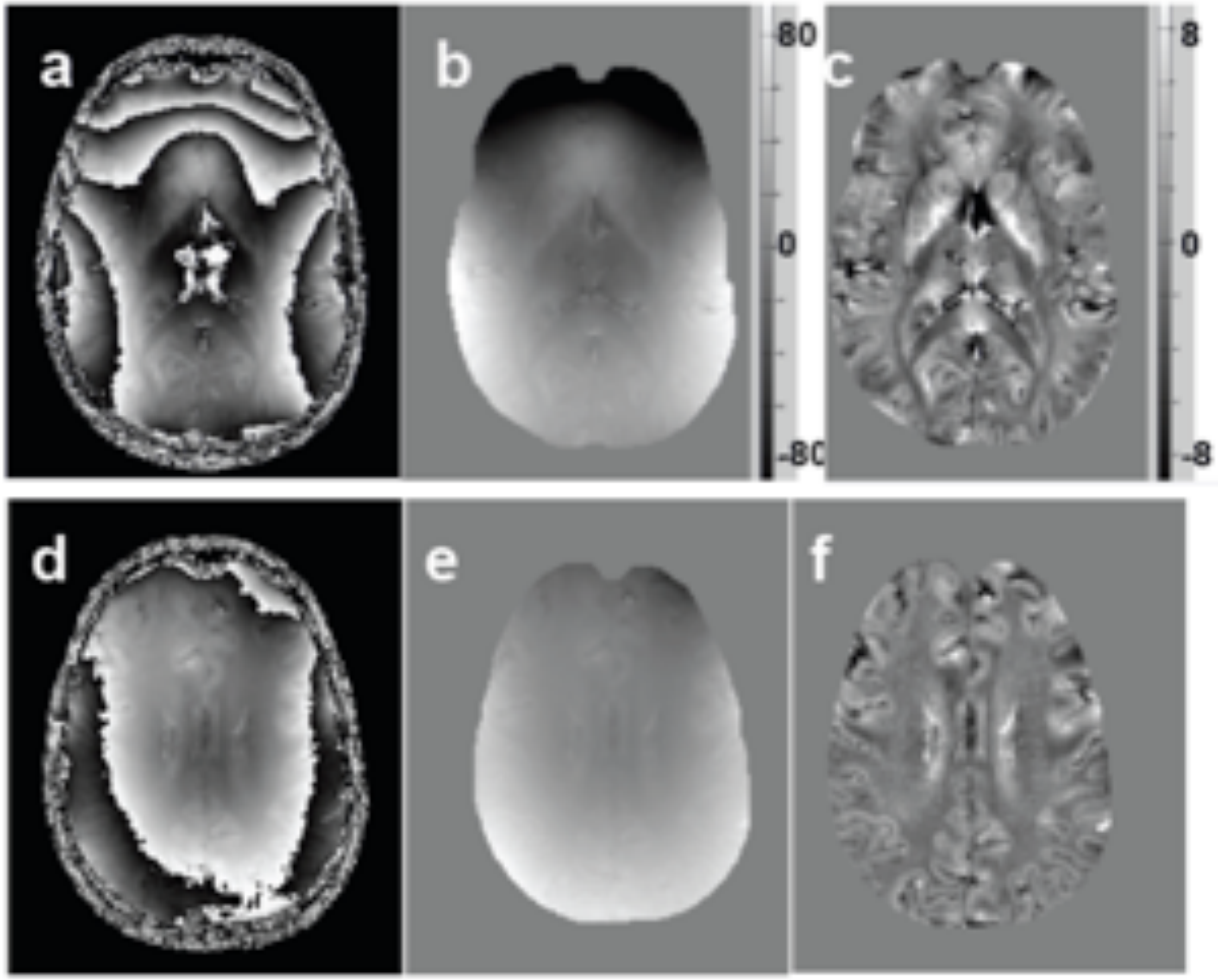
- Indirectly can be observed



*Tian Liu, Cornell (from Wikipedia)

Susceptibility Mapping

- Indirectly can be observed



http://mri.kennedykrieger.org/nationalresource/trd2.html

CHEMICAL shift

↳ BUT $(\hat{I})$ Has MICROSCOPIC EFFECT

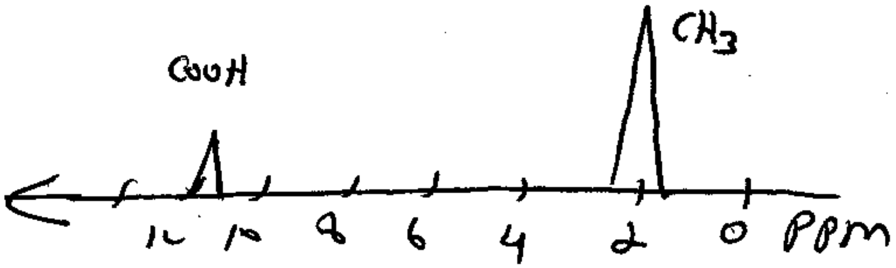
$$B_{eff} = B_0 - B_0\cancel{\sigma} = B_0(1-\cancel{\sigma})$$

$$\omega_{eff} = \omega_0(1-\cancel{\sigma})$$

$$\delta \rightarrow \text{CHEMICHAL SHIFT} = \frac{\omega_s - \omega_{reference}}{\omega_{reference}} \times 10^6$$

FOR EXAMPLE!

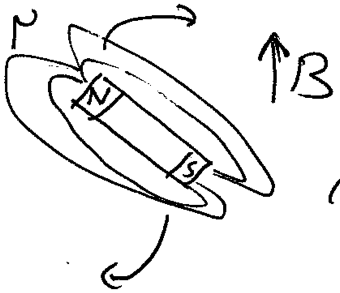
ACETIC ACID CH_3COOH



DIRECTION IS ← HISTORICAL

OXIGEN ATTRACTS ELECTRONS

PRECESSION



(1) TORQUE = $\mu \times B$

(2) TORQUE = $\frac{\partial S}{\partial t}$ change of ANGULAR MOMENTUM.

$$\gamma \times \frac{dS}{dt} = \mu \times B \;{}^{\times\gamma} \Rightarrow \quad \frac{d\mu}{dt} = \mu \times \gamma B$$

UNIT VOLUME ← $\frac{dM}{dt} = M \times \gamma B$

or

$$\frac{dM}{dt} = -\gamma B \times M$$

SOLUTION IS PRECESSION @ $f = \frac{\gamma}{2\pi}B$, $\omega = \gamma B$

$$\frac{dM}{dt} = \underset{\substack{\uparrow \\ \text{spatial} \\ \text{distribution}}}{M} \times \underset{\substack{\uparrow \\ \text{we have} \\ \text{control!}}}{\gamma B}$$

Mathematical Description of MRI

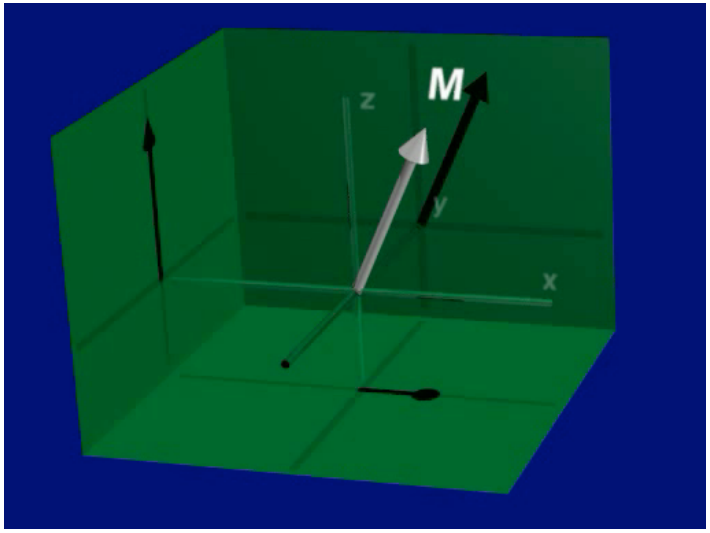
- Three Elements:
  - Precession about $\vec{B}$ (all fields)
  - Transverse decay
  - Longitudinal recovery

Precession

$$\vec{S} + \vec{\mu} + \vec{B} \Rightarrow \text{Precession}$$

Solution: ω=γ|B|

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$



Mathematical Description of MRI

- Plan:
  1) Derive Math for each element
  2) Put together : e.g., the BLOCH equation
  3) Solve the Bloch eqn. for special cases
     a) Excitation CH. 6 (later)
     b) Reception CH. 5 (first)
        i) Derive k-space (AGAIN!!!)
        ii) Pulse sequence
        iii)Sampling

Precession

- We apply fields: B_0, B_1, G

Precession

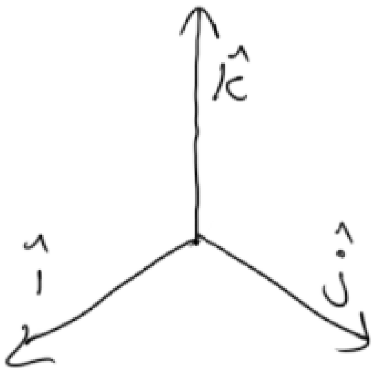
- We apply fields: B_0, B_1, G

$$\vec{B} = \underbrace{B_0\hat{k}}_{B_0} + \underbrace{\vec{G}\cdot x\hat{k}}_{\text{Gradient}} + \underbrace{B_{1x}(\cos\omega_0 t\,\hat{i} - \sin\omega_0 t\,\hat{j}) + B_{1y}(-\sin\omega_0 t\,\hat{i} - \cos\omega_0 t\,\hat{j})}_{RF}$$

$\hat{i}, \hat{j}, \hat{k}$ are unit vectors

$\vec{x} = [x, y, z]^T$

$\vec{G} = [G_x, G_y, G_z]^T$



Precession

Magnetization is:

$$\vec{M} = [M_x, M_y, M_z]^T$$

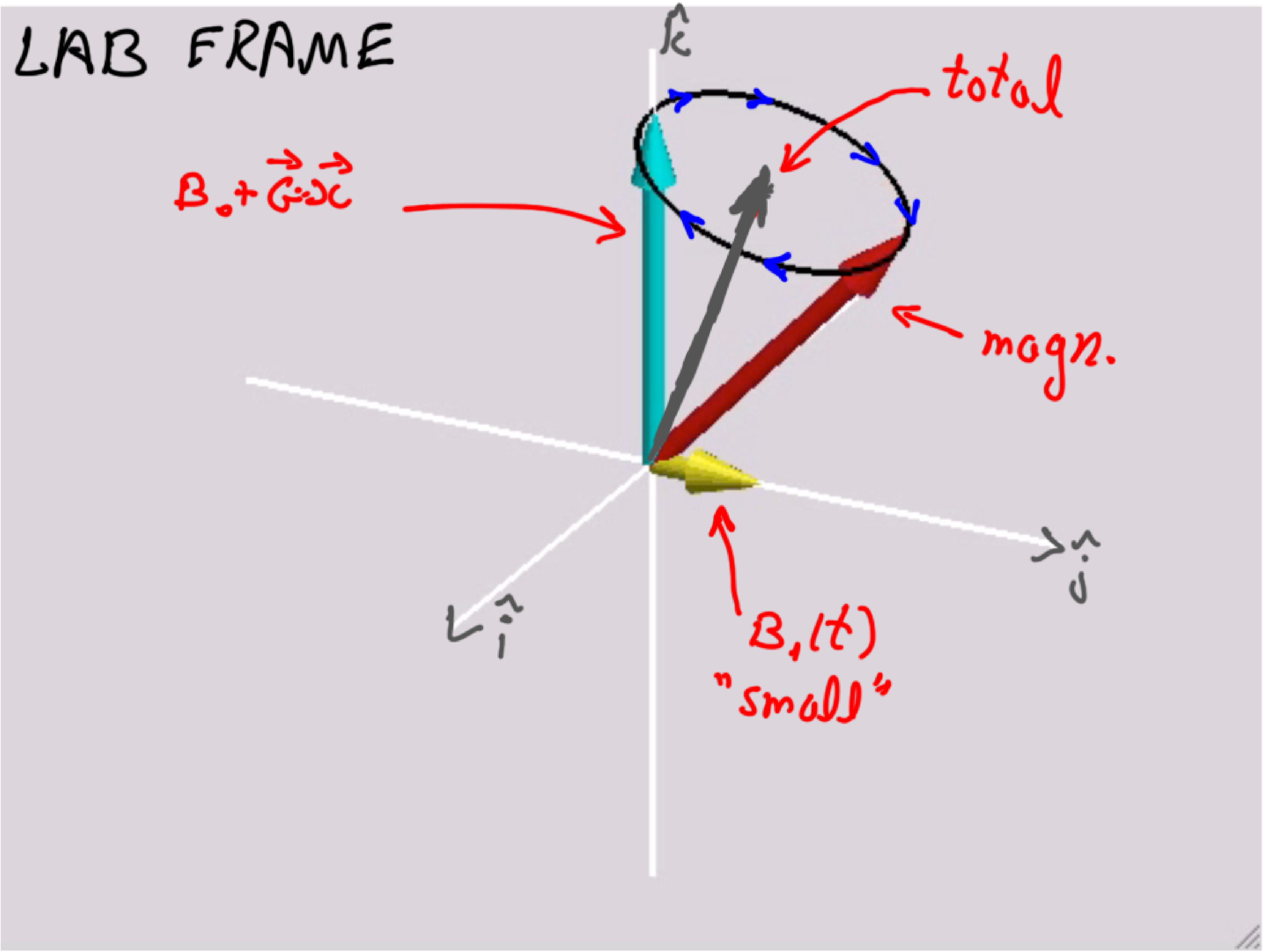
- $\vec{M}$ Precesses around $\vec{B}$
- Frequency of rotation is

$$\omega = \gamma |\vec{B}|$$

- Axis of rotation is $\vec{n}$ $\frac{\vec{B}}{|B|}$

normal

Precession

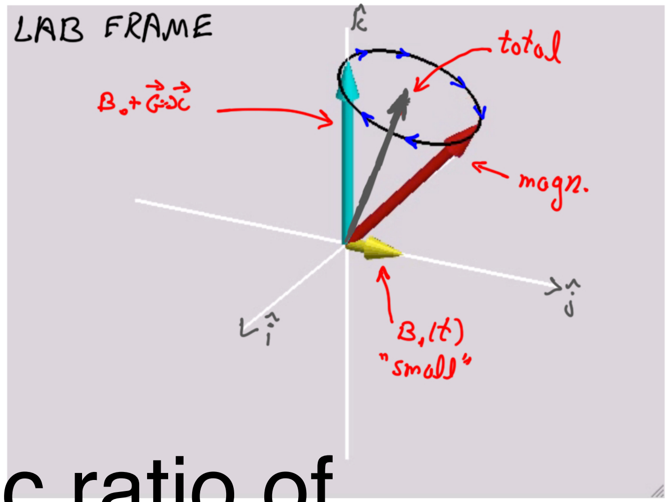


LAB FRAME
$\hat{k}$
total
$B_0 + \vec{G}\cdot\vec{x}$
magn.
$\hat{j}$
$\hat{i}$
$B_1(t)$
"small"

Precession

- Described by cross product

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B} \times \vec{M}$$



- "-" Due to negative gyromagnetic ratio of protons
  or:

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

- B_0 Dominates! Hard to see other terms

Rotating Frame

- Change coordinates:

$$[\hat{i}_r,\ \hat{j}_r,\ \hat{k}_r]^T \quad = [\hat{i}\cos\omega_0 t,\ \hat{j}\sin\omega_0 t,\ \hat{k}]^T$$

- In the rotating frame at w$_0$:

Rotating Frame

- Change coordinates:

$$[\hat{i}_r,\ \hat{j}_r,\ \hat{k}_r]^T = [\hat{i}\cos\omega_0 t,\ \hat{j}\sin\omega_0 t,\ \hat{k}]^T$$

- In the rotating frame at w_0:

$$\vec{B}_{ROT} = \left(B_0 - \frac{\omega_0}{\gamma}\right)\hat{k}_r + \vec{G}\cdot\vec{x}\,\hat{k}_r + B_{1,x}\hat{i}_r + B_{1,y}\hat{j}_r$$

And $$\left(\frac{d\vec{M}}{dt}\right)_{ROT} = -\gamma\vec{B}_{ROT} \times \vec{M}_{ROT}$$

Rotating Frame

- For $\omega_0 = \gamma B_0$ MAIN FIELD GOES AWAY!

$$\vec{B}_{ROT} = \vec{G}\cdot\vec{x}\,\hat{k}_r + B_{1,x}\hat{i}_r + B_{1,y}\hat{j}_r$$

Much simpler!

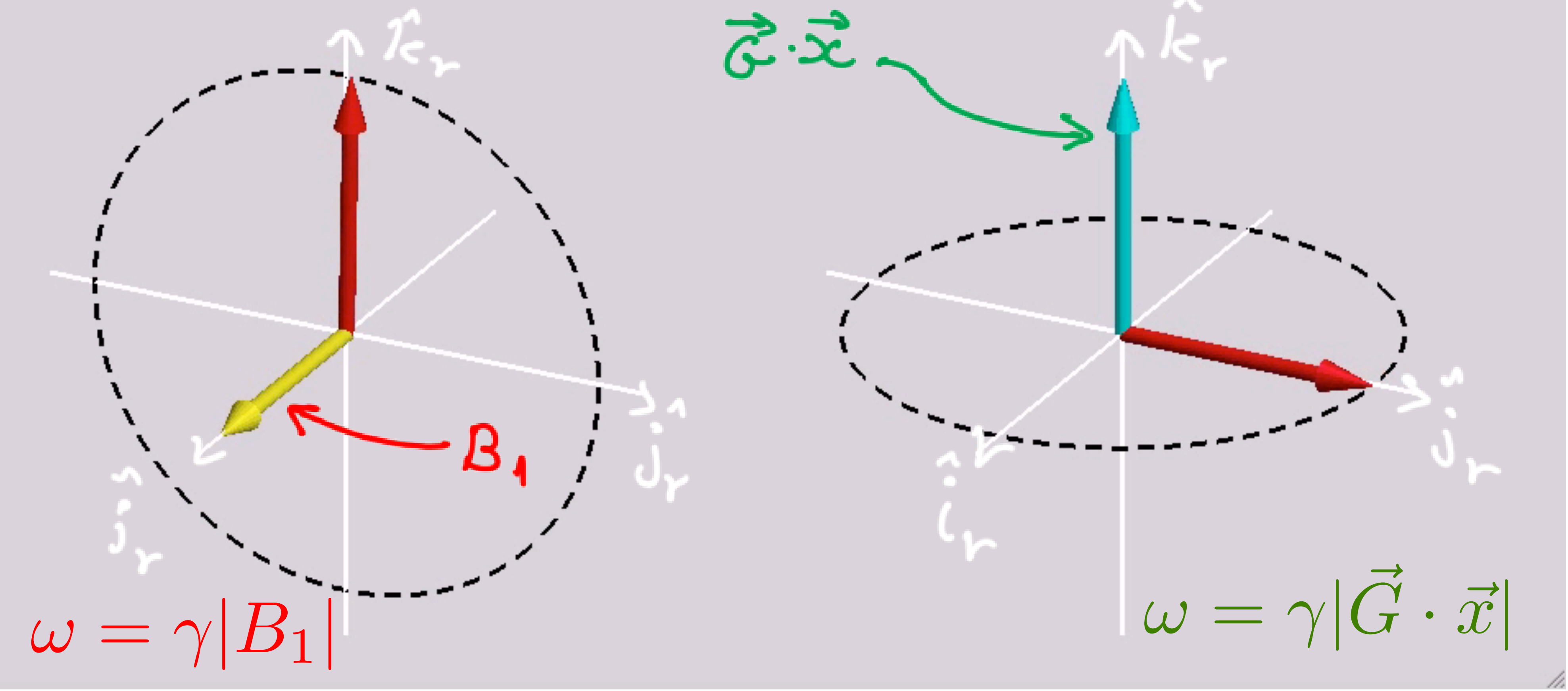
- $\vec{M}_{ROT}$ Precesses about applied fields

$\vec{G}\cdot\hat{x}$ and B_{1x}, B_{1y}

Examples

Excitation

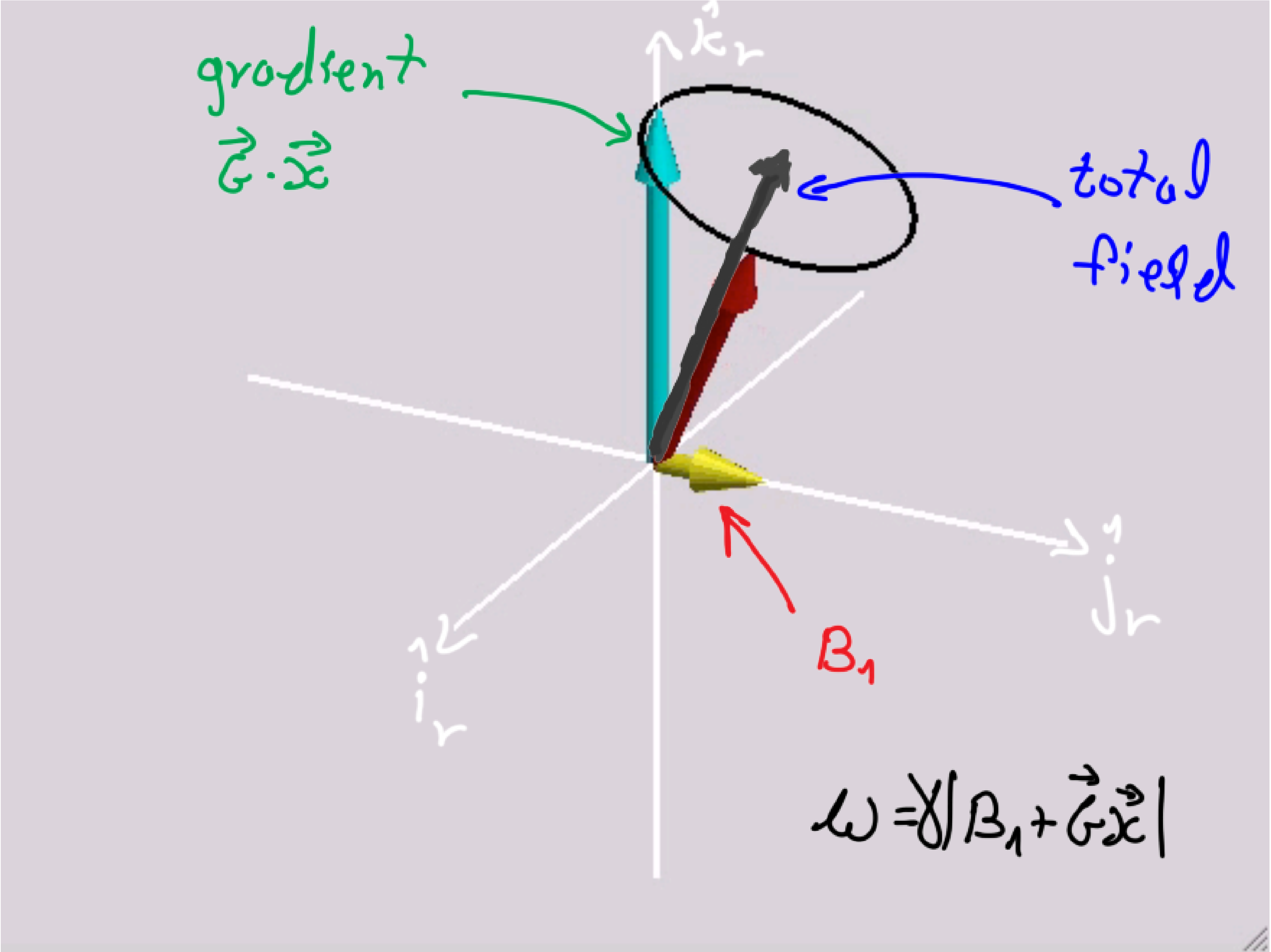
Precession



$\omega = \gamma|B_1|$

$\omega = \gamma|\vec{G}\cdot\vec{x}|$

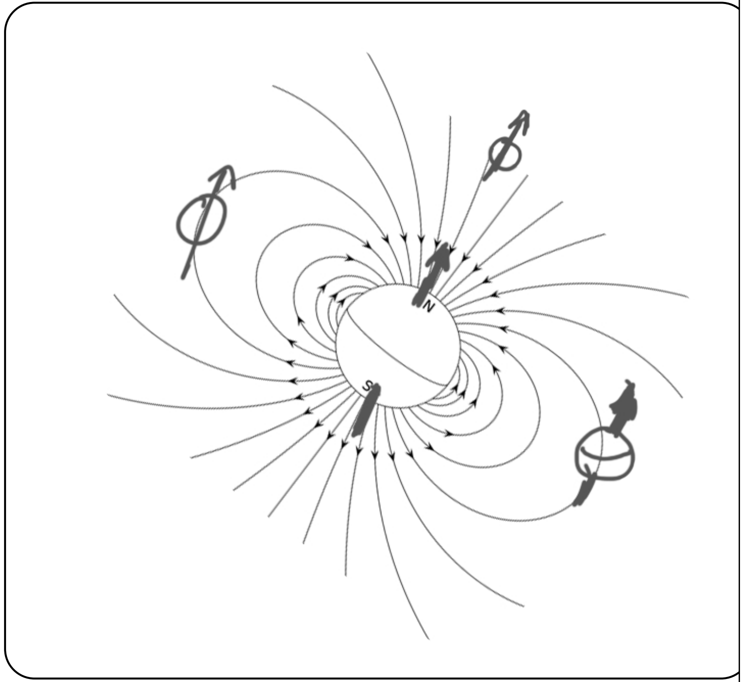
Examples


gradient
$\vec{G}\cdot\vec{x}$
k_r
total field
B_1
j_r
i_r
$\omega = \gamma|B_1 + \vec{G}\vec{x}|$

Relaxation

- T2 Decay
  - –Transverse magnetization decays
  - –Due to loss of coherence between spins
  - –Also called spin-spin relaxation

  - –Not a strong function of B_0
  - –Dipole effect stronger in solids



Transverse Relaxation (T2)

- Let

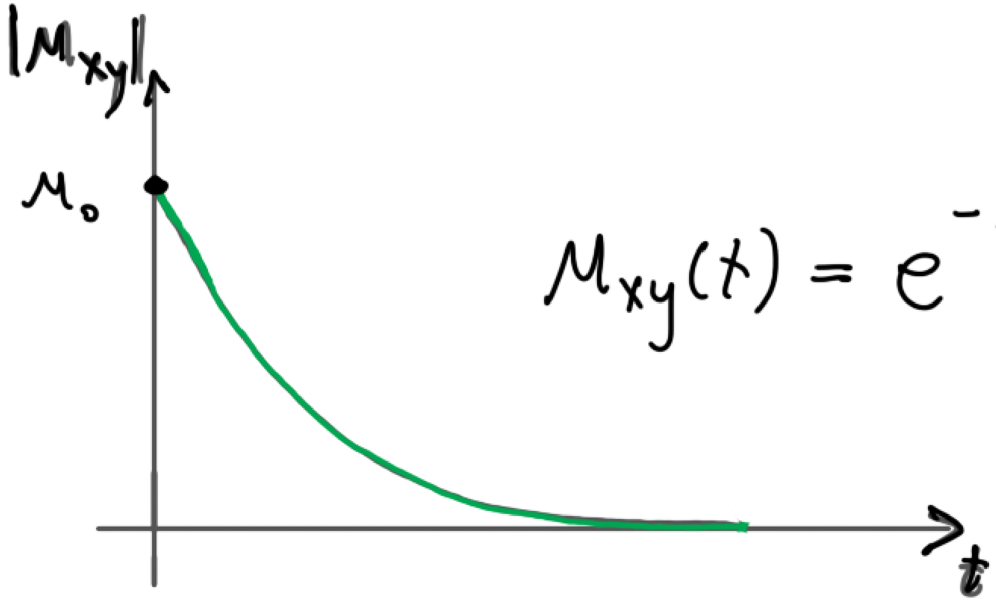
$$M_{xy} = M_x + iM_y$$

- Then

$$\frac{dM_{xy}}{dt} = -\frac{1}{T_2}M_{xy}$$

Transverse Relaxation (T2)

- Solution:



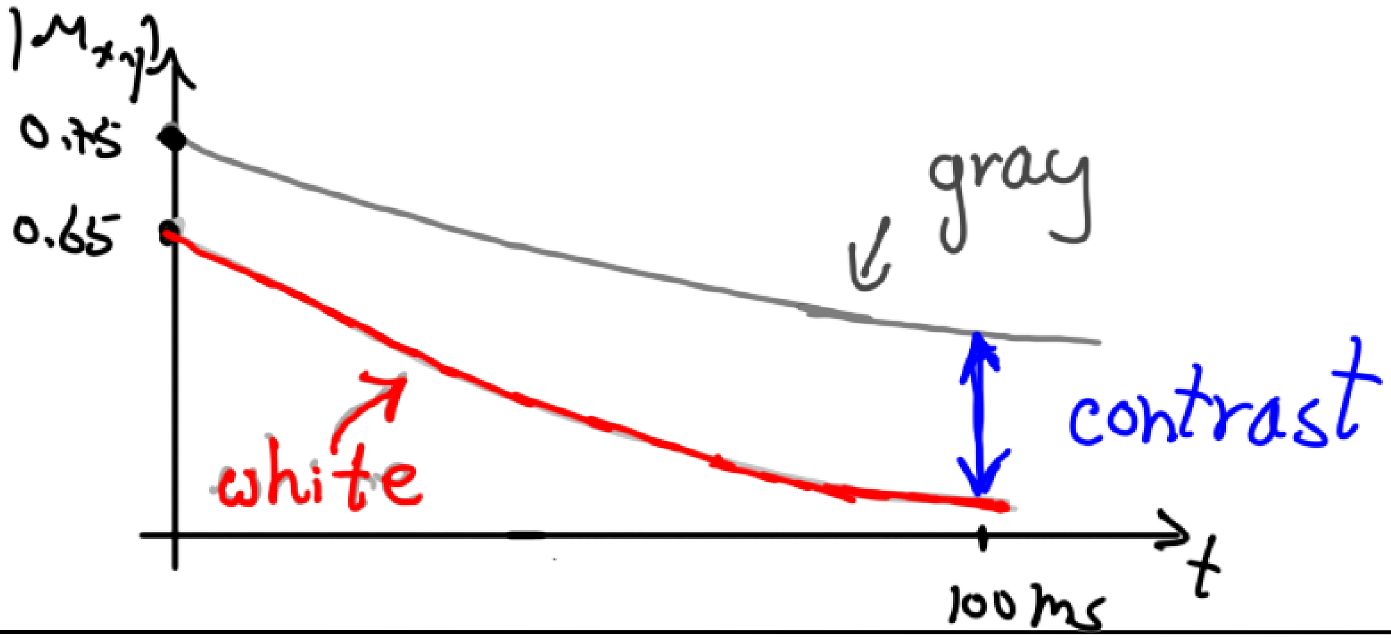
$$M_{xy}(t) = e^{-\frac{t}{T_2}} M_{xy}(0)$$

Major source of contrast

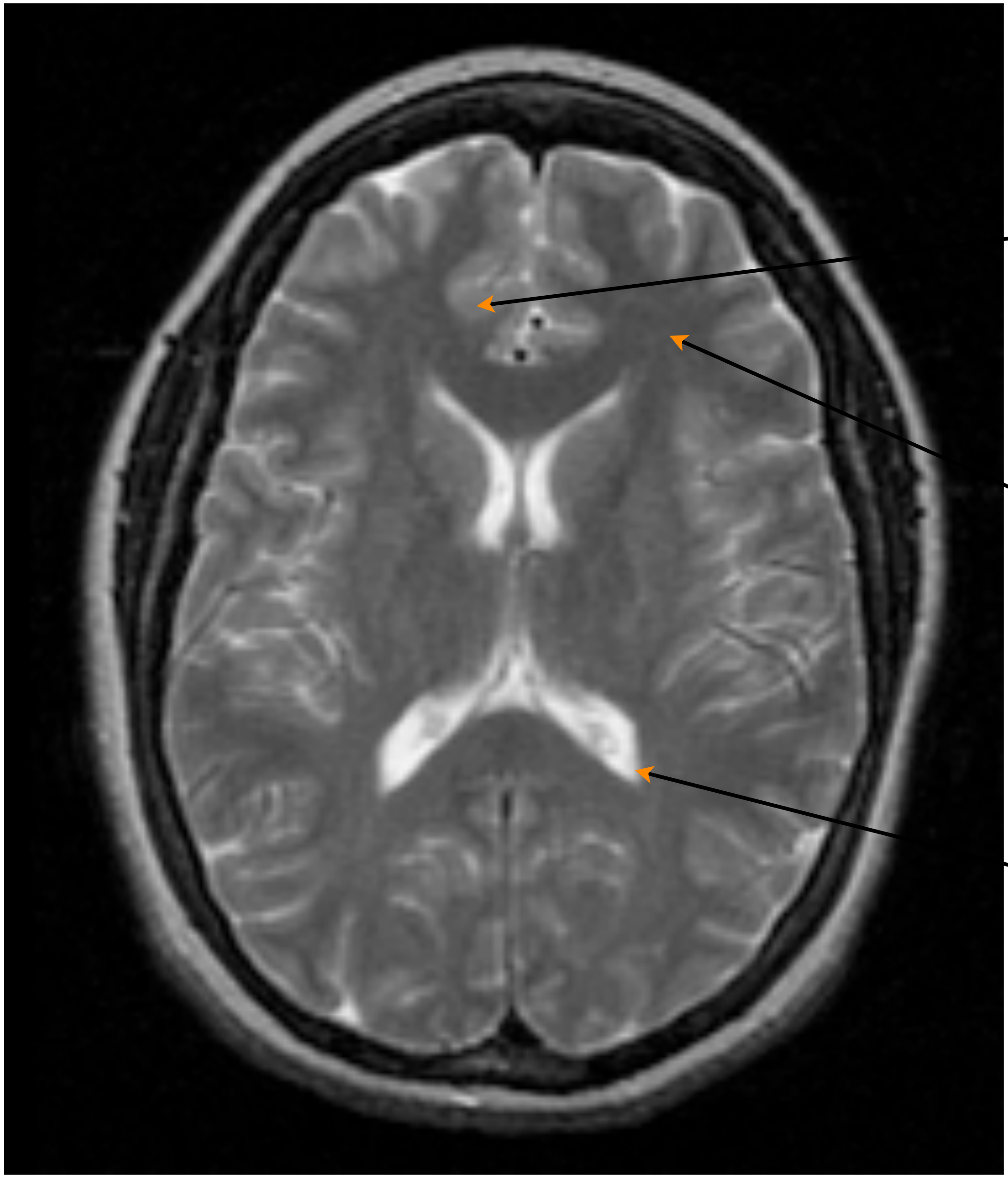
Transverse Relaxation (T2)

- Example: Brain @ 1.5T
  - –white matter T2=92ms, Density=0.65
  - –gray matter T2=100ms, Density = 0.75

Excite, wait 100ms, collect data



T2 Example



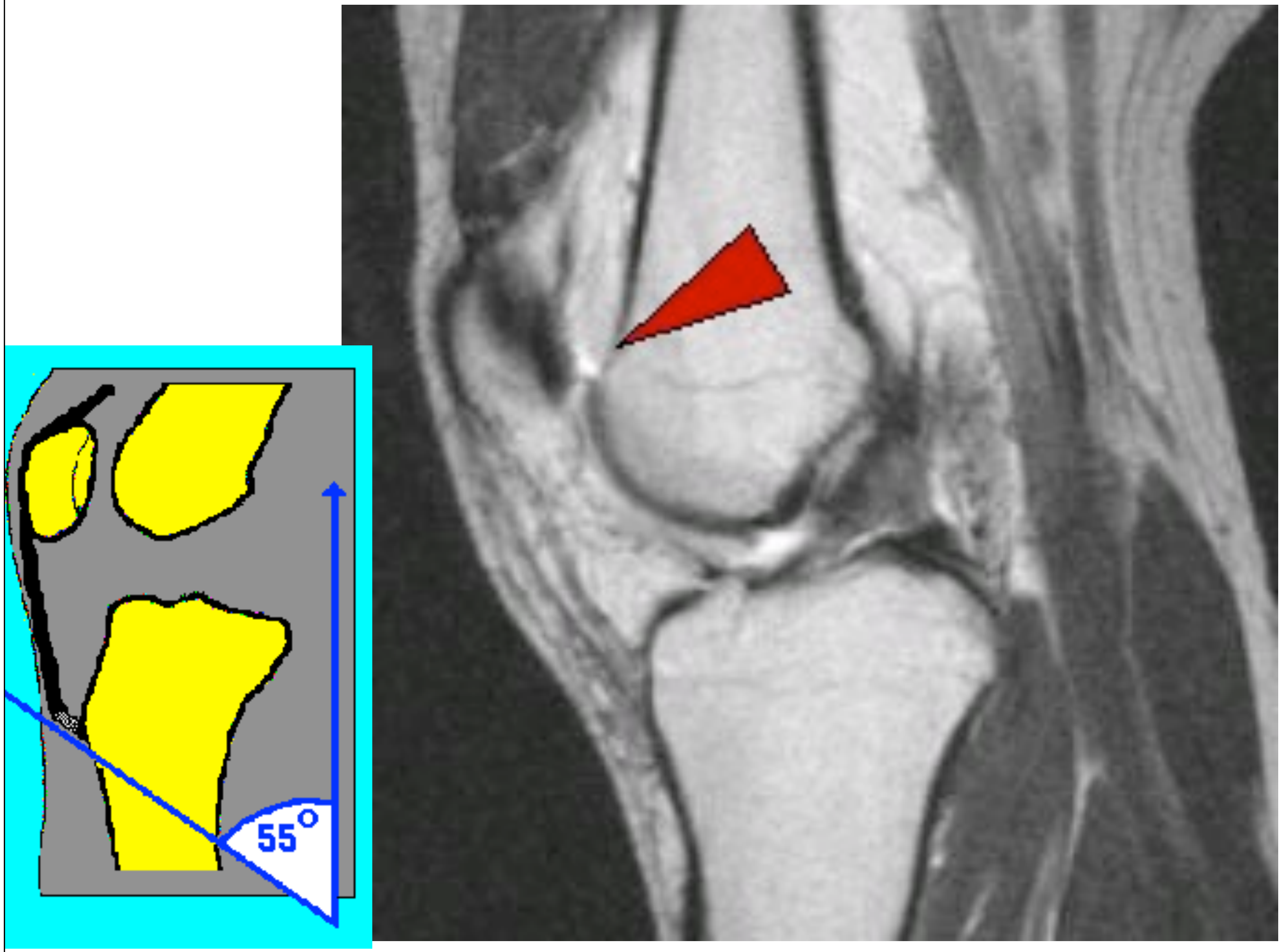
Gray matter lighter

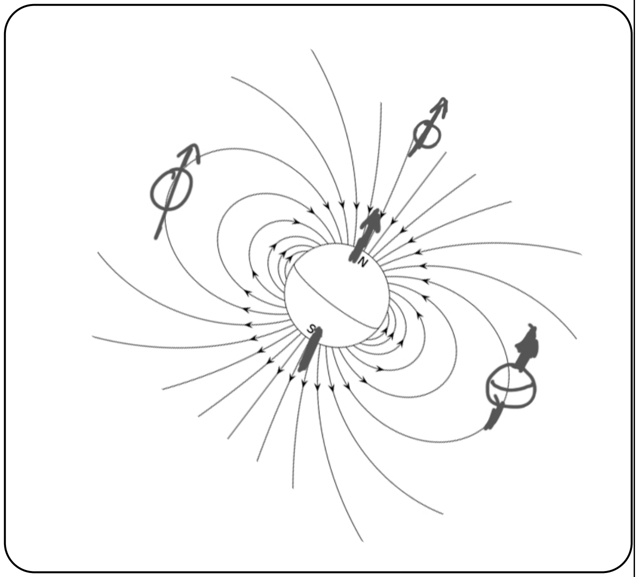
white matter darker

CSF Bright!

Magic Angle ~55 degrees

- Longer T2 due to dipole decoupling

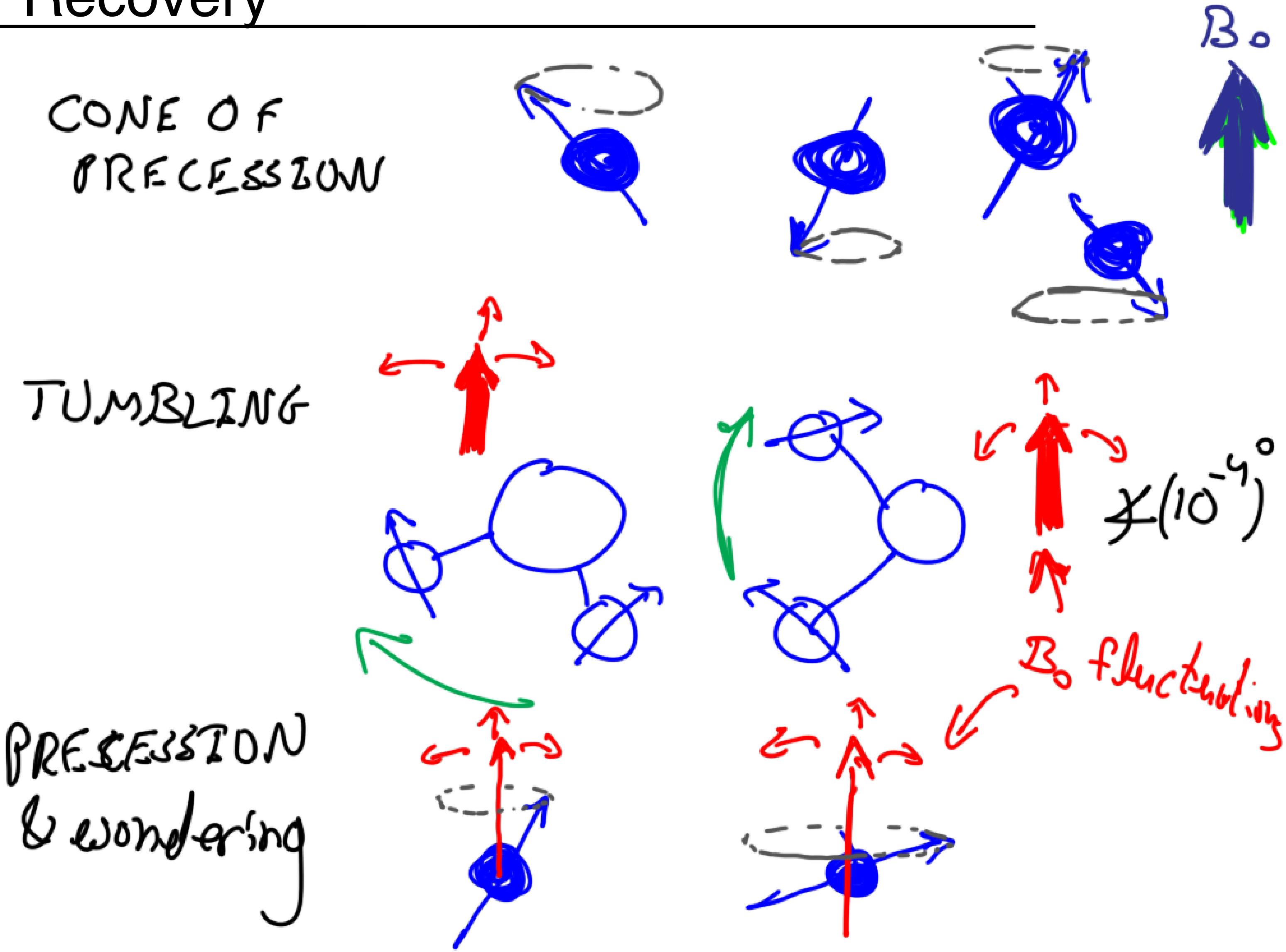




Relaxation

- T1 Recovery
  - –Longitudinal relaxation
  - –Due to Spin-Lattice interaction
  - –Thermal bouncing of molecules - lose cone of precession - align with field
  - –Strong dependency on B_0 , since energy level depends on B_0
  - –B_0 stong - hard to transition - T1 long

T1 Recovery



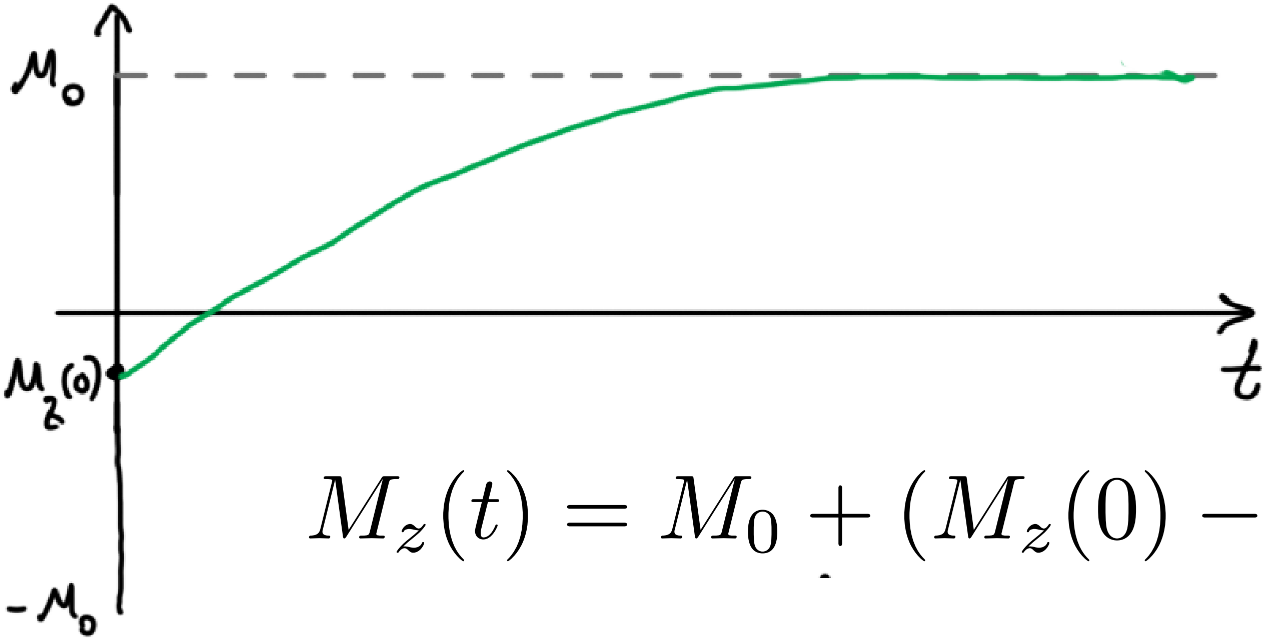
- Bias towards up - stable anisotropic dist.

T1 Recovery

- Magnetization recovers to equilibrium M_0

$$\frac{dM_z}{dt} = -\frac{M_z - M0}{T_1}$$

- Solution:



$$M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T1}$$

T1 Recovery

After 90° pulse, $M_z = 0$

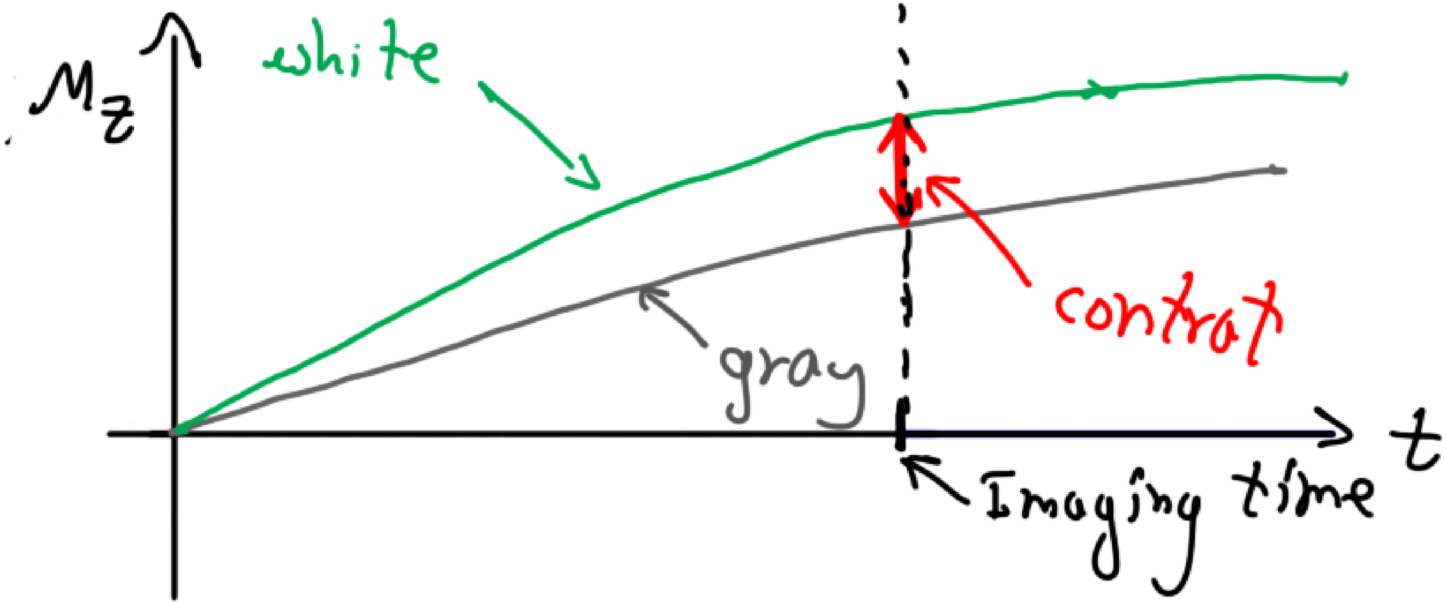
$$M_z(t) = M_0 - M_0 e^{-\frac{t}{T_1}} = M_0(1-e^{-\frac{t}{T_1}})$$

Major source of contrast as well!

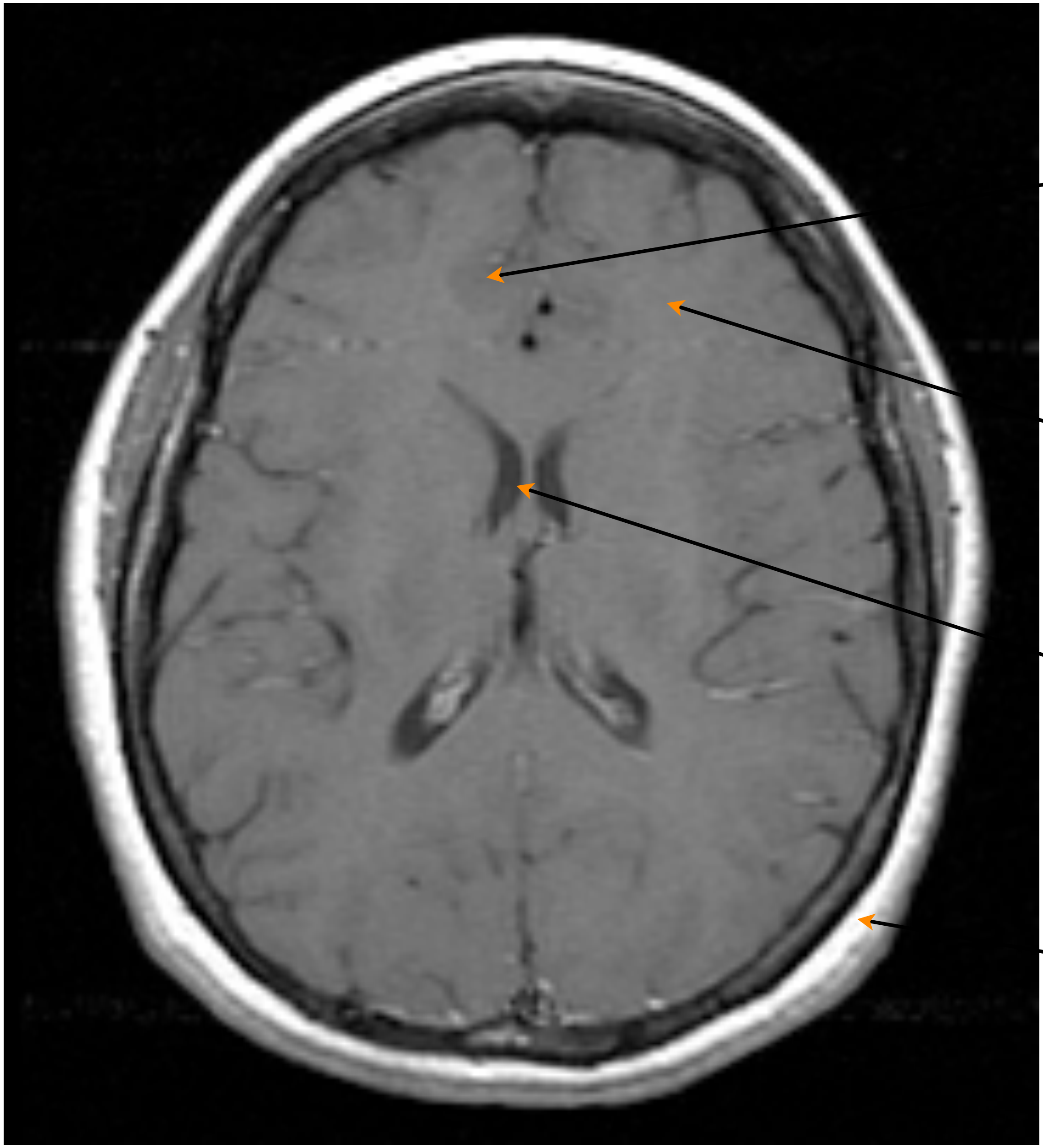
T1 Contrast

- Brain at 1.5T
  - –Gray Matter T1 = 900ms
  - –White Matter T1 = 800ms

Excite 90, wait, excite again, image....



T1 Contrast Example



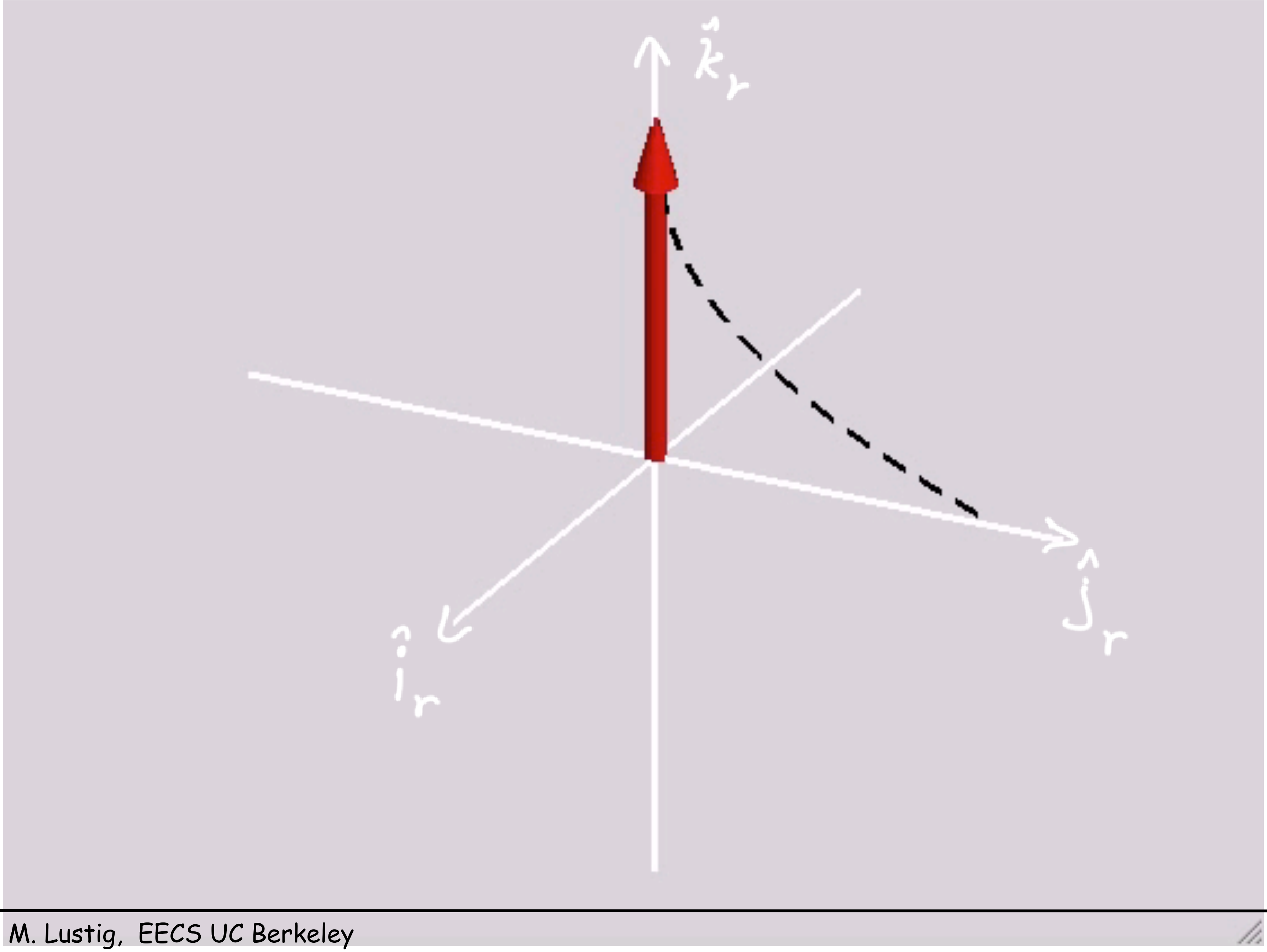
Gray matter darker

white matter lighter

CSF Dark!

Fat Bright!

Relaxation



The Bloch Equation

- Combine Precession and relaxation

$$\frac{d\vec{M}}{dt} = \underbrace{-\gamma\vec{B}\times\vec{M}}_{\text{precession}} - \underbrace{\frac{M_x\hat{i}+M_y\hat{j}}{T_2}}_{\text{transverse decay}} - \underbrace{\frac{M_z-M_0}{T_1}\hat{k}}_{\text{long. recovery}}$$

- Phenomenological: Fits observations
  - –Describes most of MRI
  - –Sometimes Fails.... J-coupling, Magn, transfer