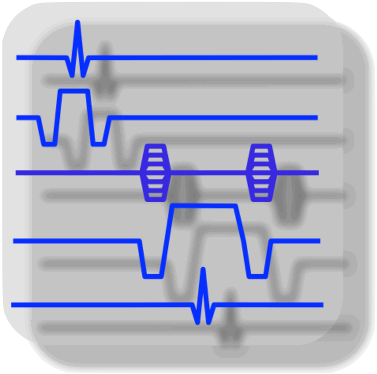


Principles of MRI

EE225E / BIO265

RF Excitation (Chap. 6)

- Energy is deposited into the system
- RF pulses used for:
  - Excitation
  - Contrast manipulation
  - Refocussing (...more later)
  - Saturation
  - Tagging
  - Transfer of magnetization

RF Excitation (Chap. 6)

- History:
  - –80’s - 90’s Lots of Research
  - –Mid 90’s most problems figured out
  - –Mid 2000 new burst of research with multiple xmitters

Excitation

- Excitation is short ~2-3ms. Can neglect relaxation
- Simplified Block eq

$$\vec{B}_{ROT} = \left(B_0 - \frac{\omega_0}{\gamma}\right)\hat{k}_r + \vec{G}\cdot\vec{x}\,\hat{k}_r + B_{1,x}\hat{i}_r + B_{1,y}\hat{j}_r$$

And $$\left(\frac{d\vec{M}}{dt}\right)_{ROT} = -\gamma \vec{B}_{ROT} \times \vec{M}_{ROT}$$

- Only defines rotations!

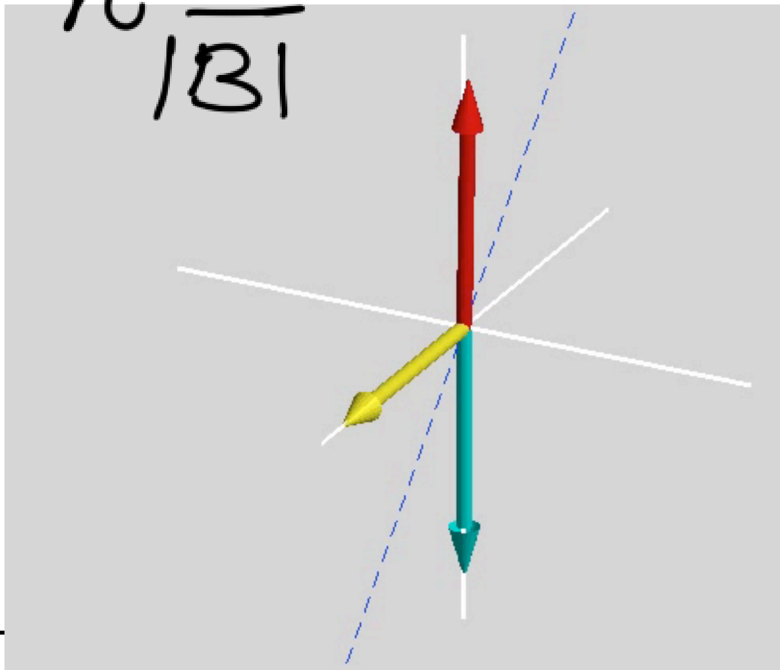
Excitation

- $\vec{M}$ Precesses around $\vec{B}$
- Frequency of rotation is

$$\omega = \gamma |\vec{B}|$$

normal

- Axis of rotation is $\vec{n} = \frac{\vec{B}}{|B|}$



Rotating Frame

- In rotating frame at ω_0, $B+\omega_0/\gamma = 0$

$$B_{\text{eff}} = [B_{1x}, B_{1y}, \gamma\vec{G}\cdot\vec{r}]^T$$

- Bloch equation for excitation:

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \begin{bmatrix} 0 & \gamma\vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma\vec{G}\cdot\vec{r} & 0 & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

RF Excitation

- Several Special cases:

  – Gradient is on: $\gamma \vec{G} \cdot \vec{r} \neq 0$
  excitation is spatially selective (next week)

  – Gradient is off: $\gamma \vec{G} \cdot \vec{r} = 0$
  non-selective excitation (Today)

Non Selective Excitation

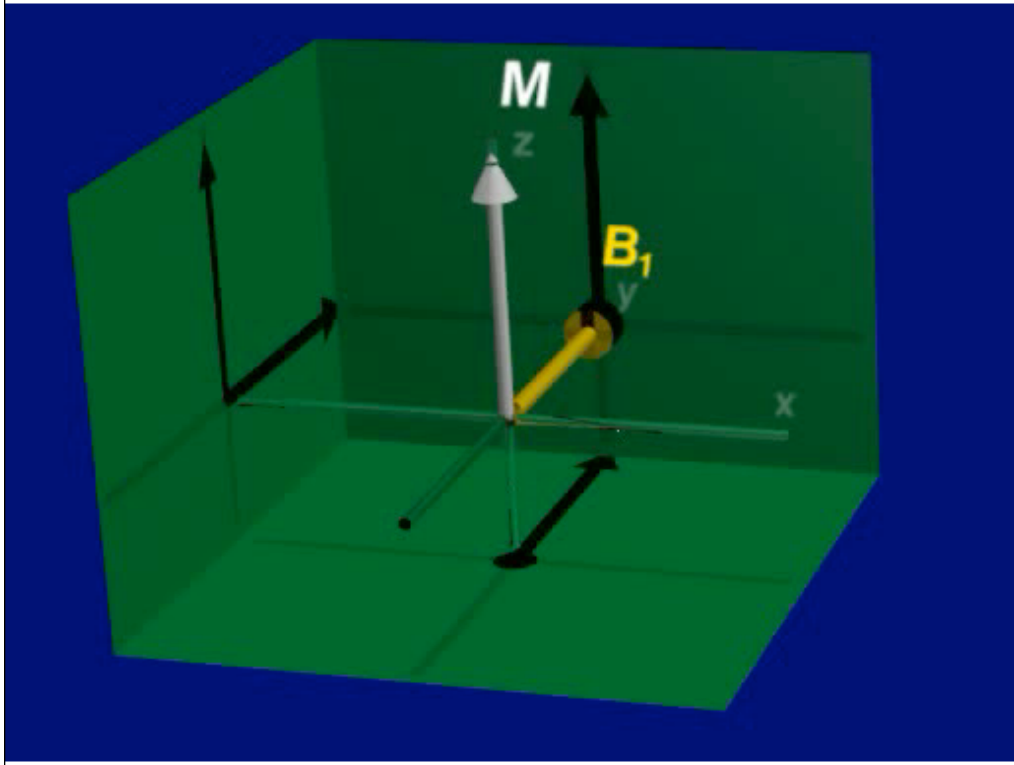
$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\gamma B_{1y} \\ 0 & 0 & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$B_{\text{eff}} = [B_{1x}, B_{1y}, 0]^T$$

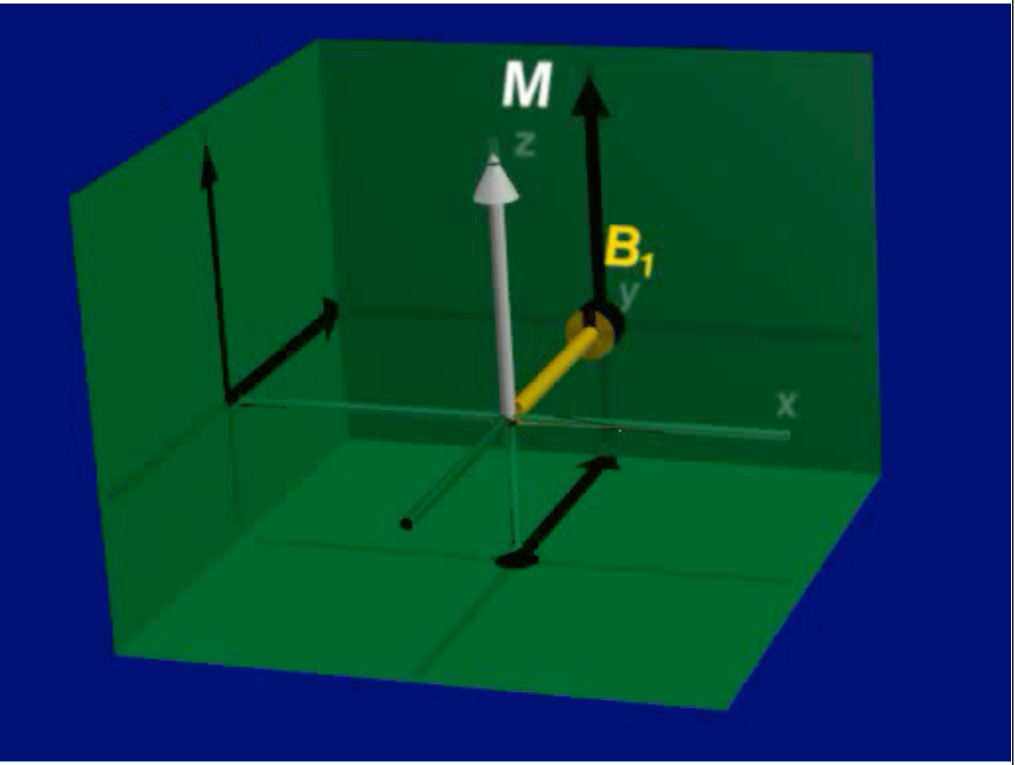
- Magnetization precesses around B_{eff}

RF Excitation

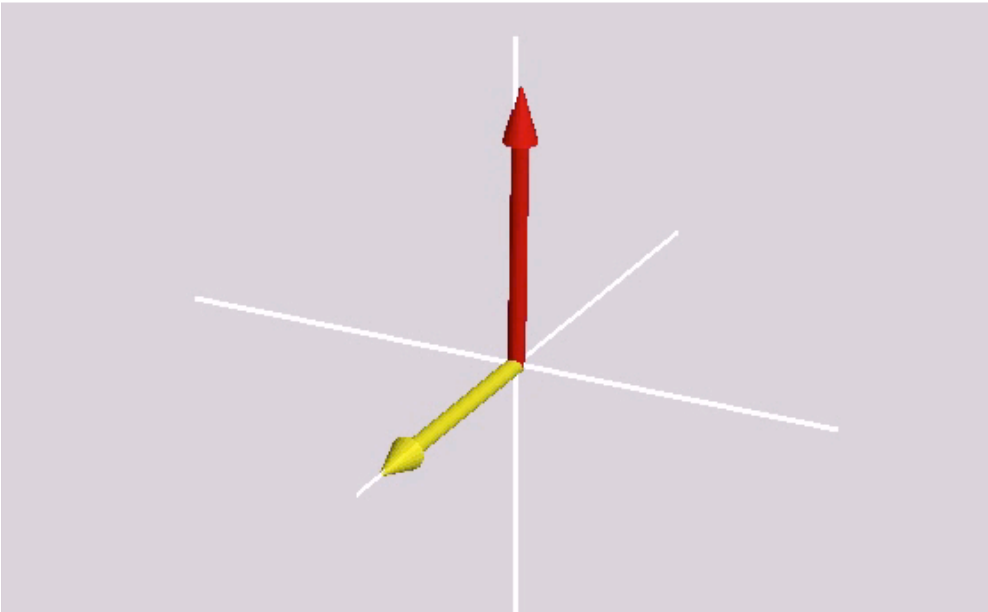
Lab Frame



Rotating Frame



RF Excitation



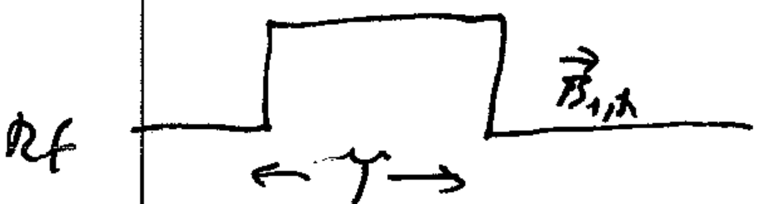
$B_{eff} = (B_{1,x}, 0, 0)$

$\vec{n} = (1, 0, 0)$

$\omega_1 = \gamma B_{1x}$

$\vec{M}$ ROTATES IN z-y PLANE.

If B_{1y} is constant



ANGLE OF ROTATION IS $\boxed{\Theta = \gamma B_{1,x} \tau}$

Time Varying B1

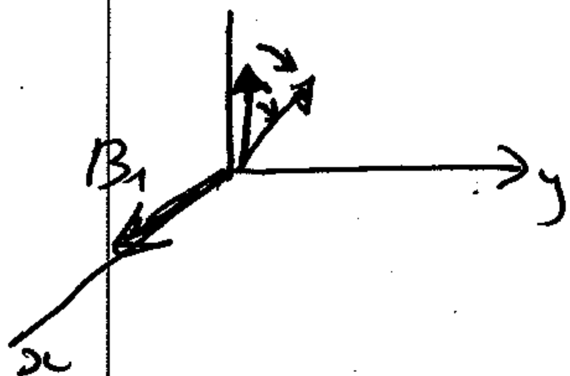
- For time varying B1:

$$\theta = \gamma \int_0^t B_{1x}(\tau)d\tau$$

- Rotations about a common axis add!
(not true in general)

Useful Rotations

$0 < \Theta < \frac{\pi}{2}$



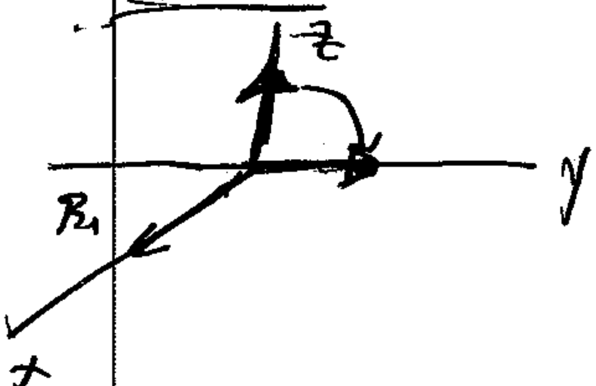
<u>SMALL TIP-ANGLE EXCITATION</u>

CREATES SOME M_{xy}

LEAVES SOME M_z

USEFUL FOR FAST IMAGING.

$\Theta = \pi/2$



<u>EXCITATION PULSE</u>

TAKES M_z INTO M_{xy}

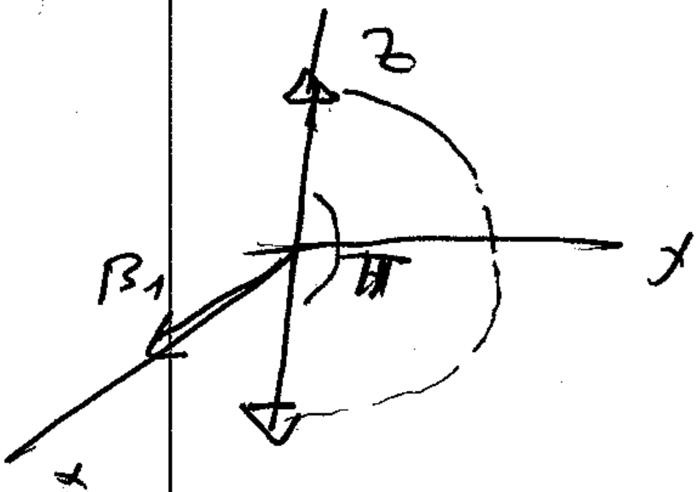
MAX SIGNAL

NO M_z LEFT!

Useful Rotations

$\Theta = \pi$

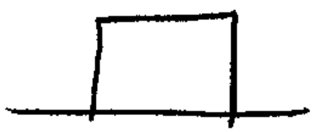
INVERSION PULSE



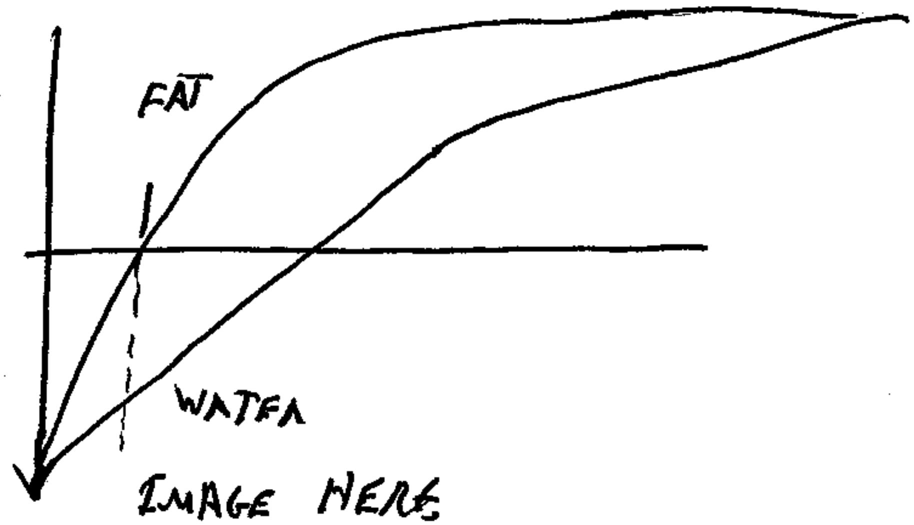
TAKES M_z INTO M_{xy}

USEFUL FOR T_1 CONTRAST

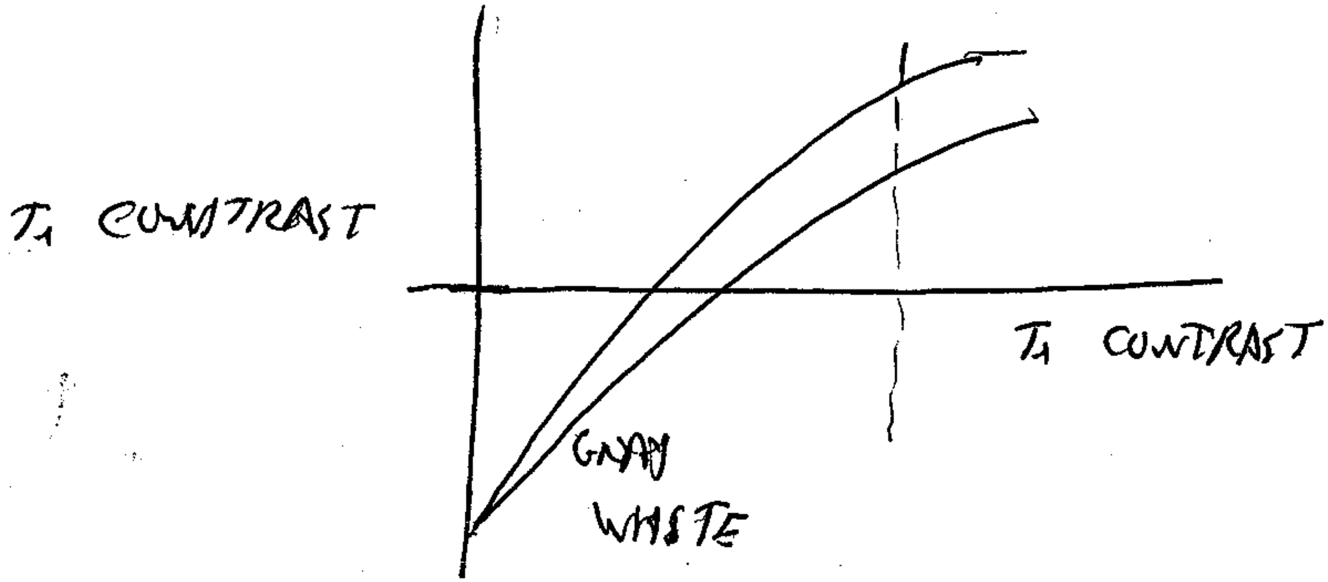
FAT SUPPRESSION.



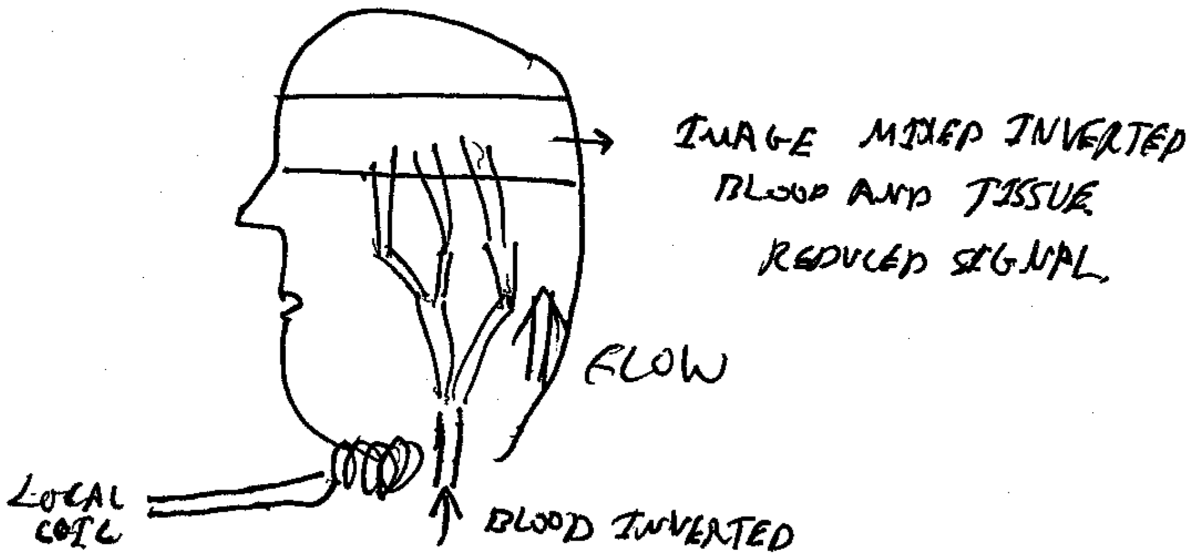
FAT-SUPPRESSION



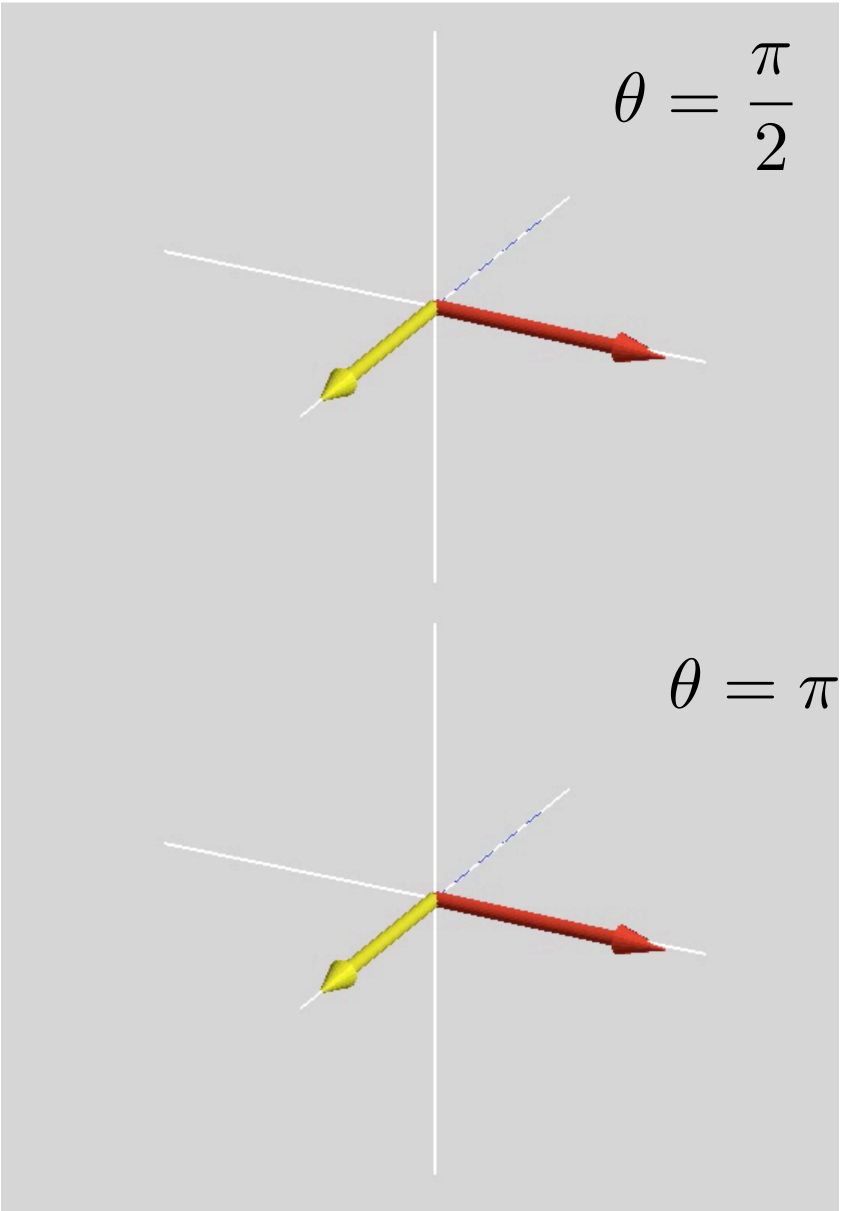
Useful Rotations

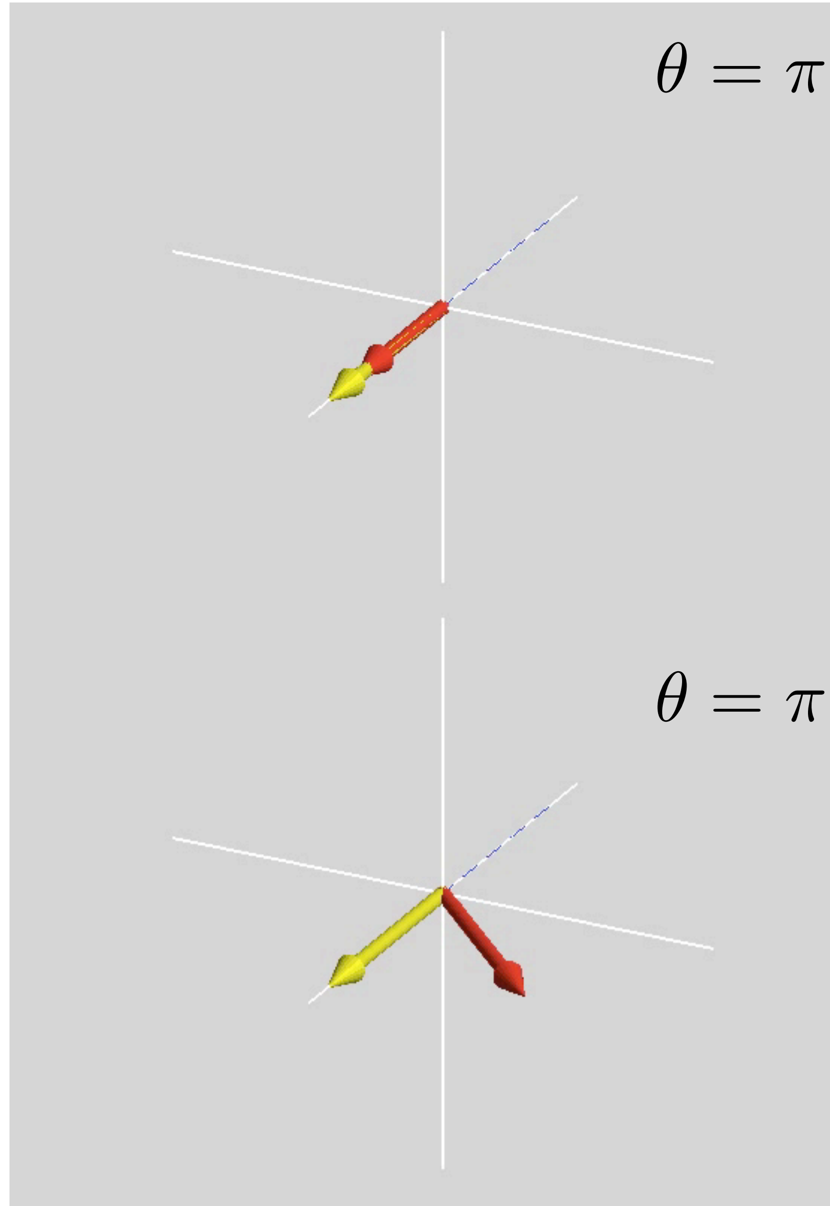


ASL



What do these pulses do?

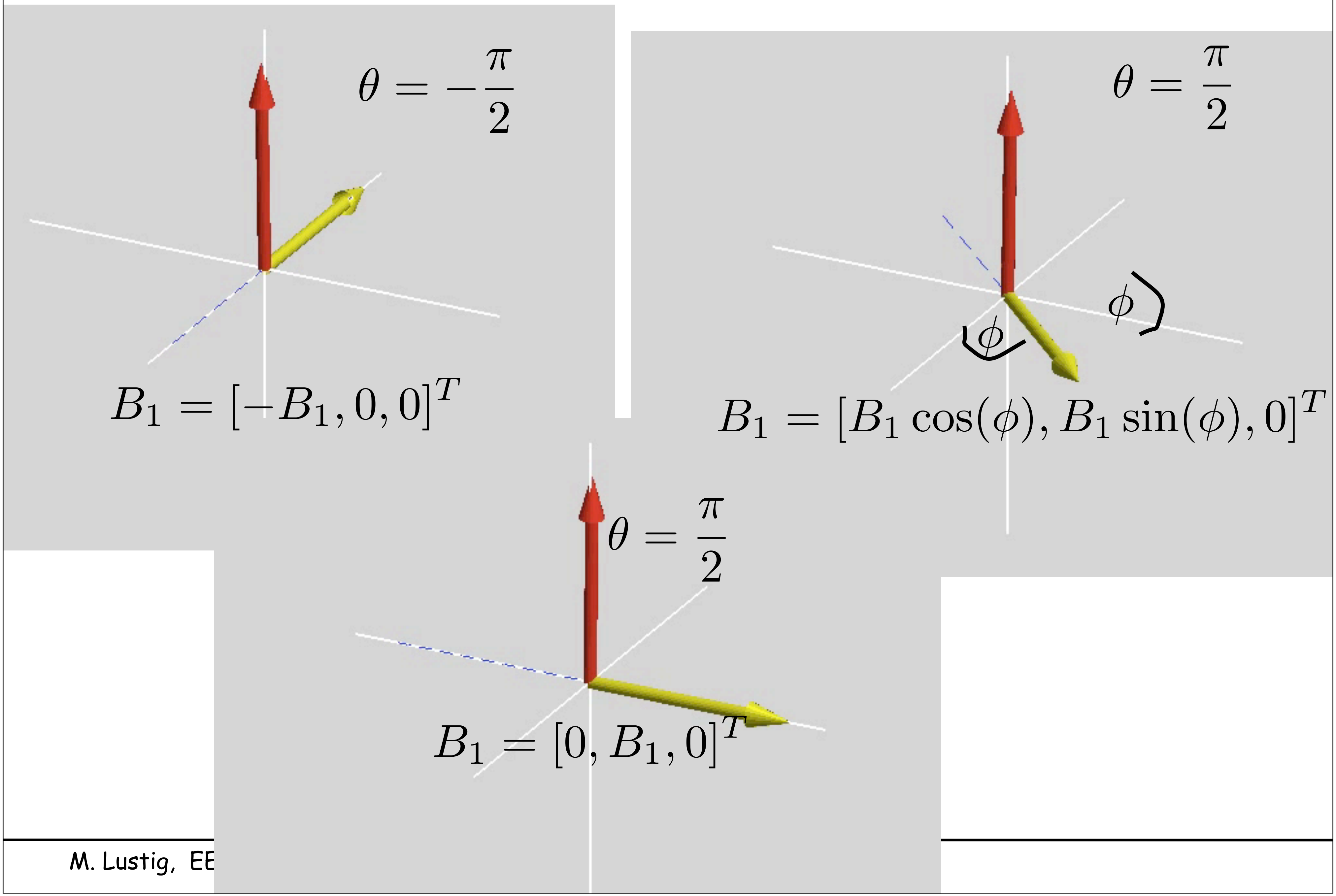




Example: Non selective RF

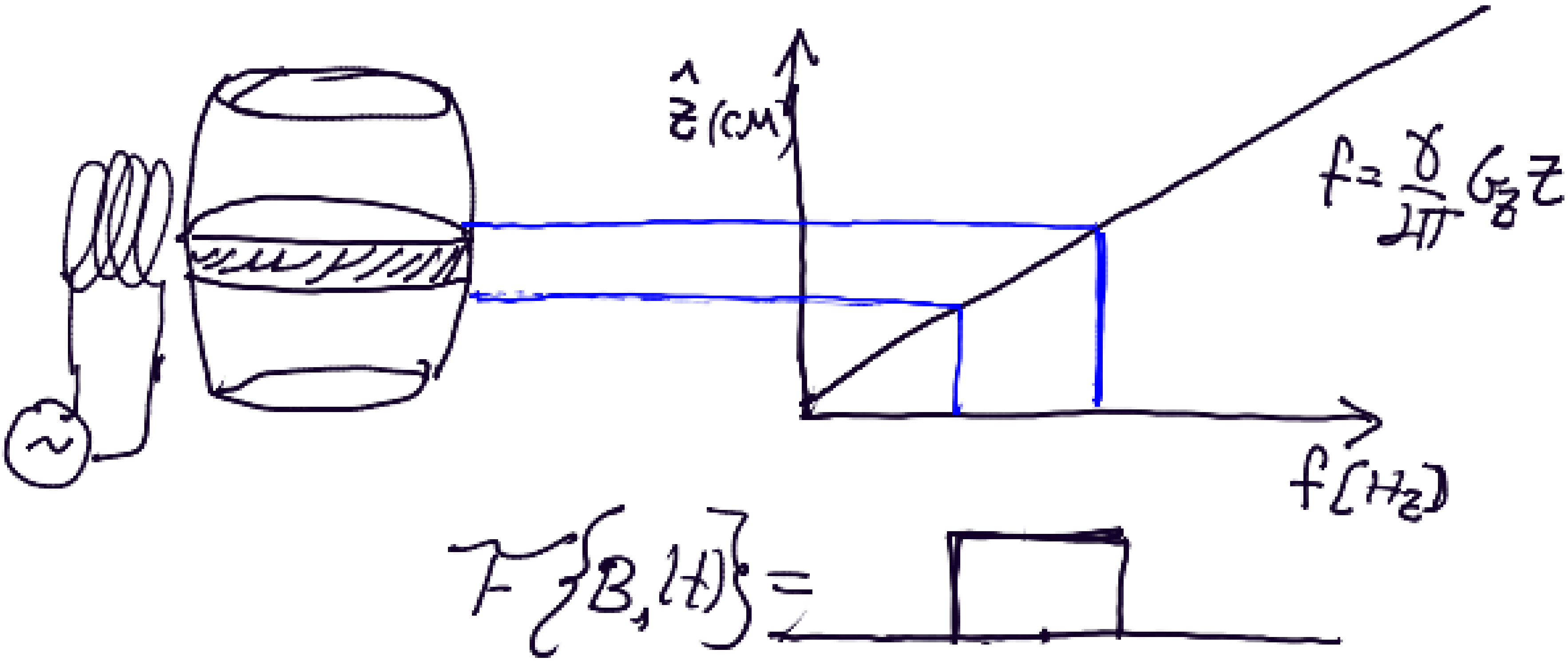
- Typical Numbers:
  - B1 = 0.1G
  - $\omega 1 = \gamma B1 = 2\pi$ 425.7 rad/sec
  - f1 $\approx$ 426Hz
- For a $\pi/2$ pulse: 1/4 rotation
  - $\tau \approx 1000/426 * 1/4 = 0.6$ ms

What do these pulses do?



Slice Selective Example

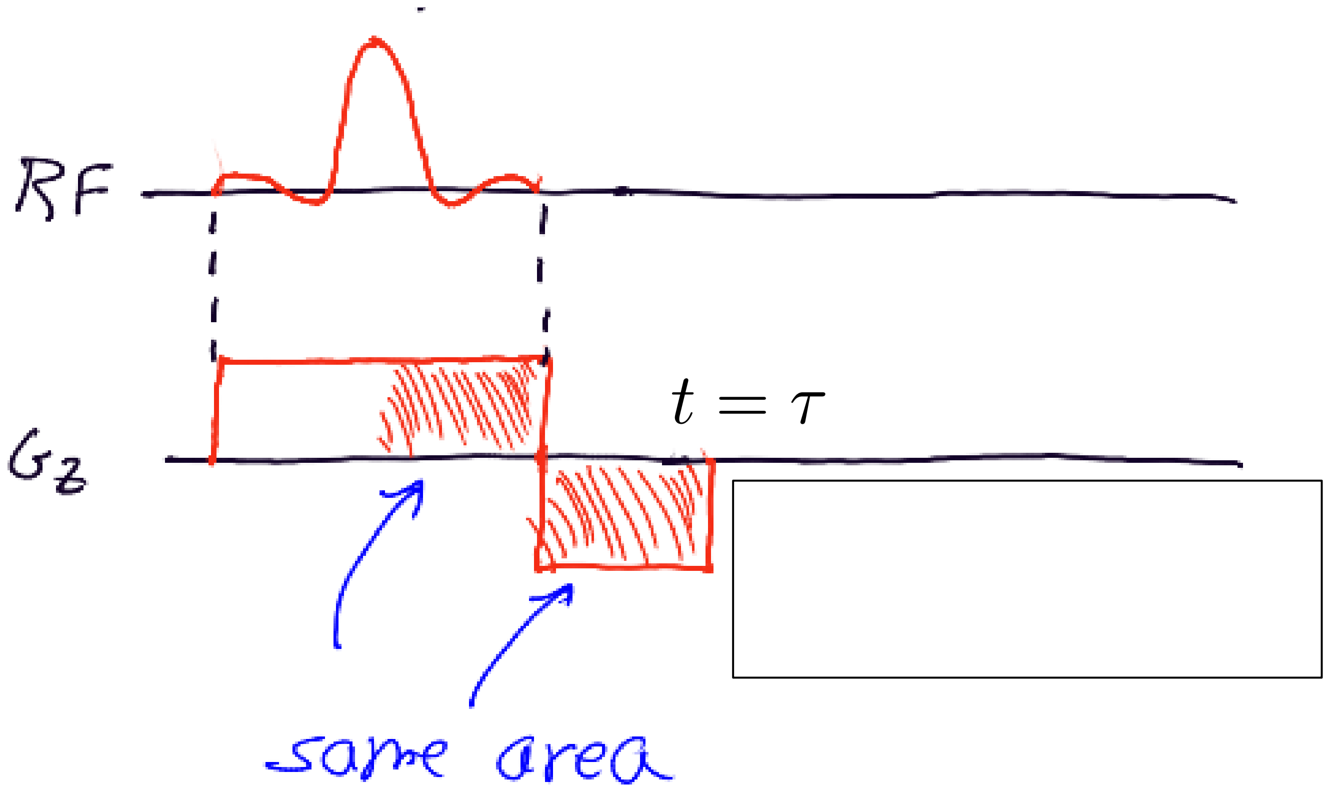
$$\vec{G} = [0,\ 0,\ \vec{G}_z]^T$$



Only spins near resonance frequency are excited.

Slice Selective Example

pulse sequence:



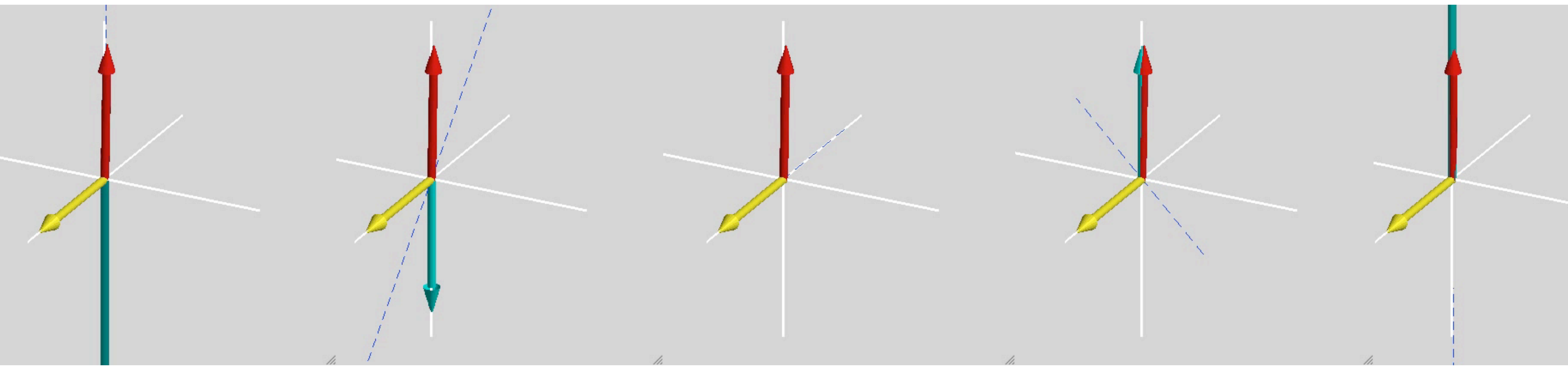
Q: What is: $M_{xy}(\vec{r}, \tau)$?

Slice Selectivity as Rotations

- B_{1x}, B_{1y} are the same EVERYWHERE
- $G_z z$ changes linearly with z

Example: Gz=1G/cm, B1x=0.16G:

$G_z z << -B_1$	$G_z z \approx -B_1$	$G_z z = 0$	$G_z z \approx B_1$	$G_z z >> B_1$



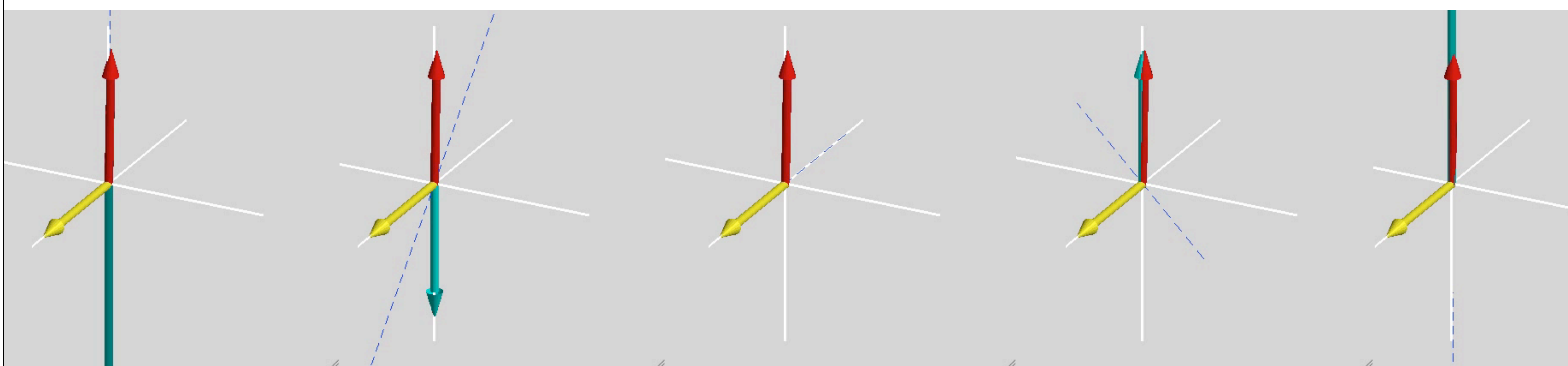
z=-10cm	z=-0.16cm	z=0 cm	z=+0.16cm	z=+10cm

Slice Selectivity as Rotations

- B_{1x}, B_{1y} are the same EVERYWHERE
- $G_z z$ changes linearly with z

Example: Gz=1G/cm, B1x=0.16G:

$G_z z << -B_1$ $\quad$ $G_z z \approx -B_1$ $\quad$ $G_z z = 0$ $\quad$ $G_z z \approx B_1$ $\quad$ $G_z z >> B_1$



z=-10cm $\quad$ z=-0.16cm $\quad$ z=0 cm $\quad$ z=+0.16cm $\quad$ z=+10cm

Slice Selectivity

- Simple Cases:
  - On-Resonance ⇒ same as Non-Selective

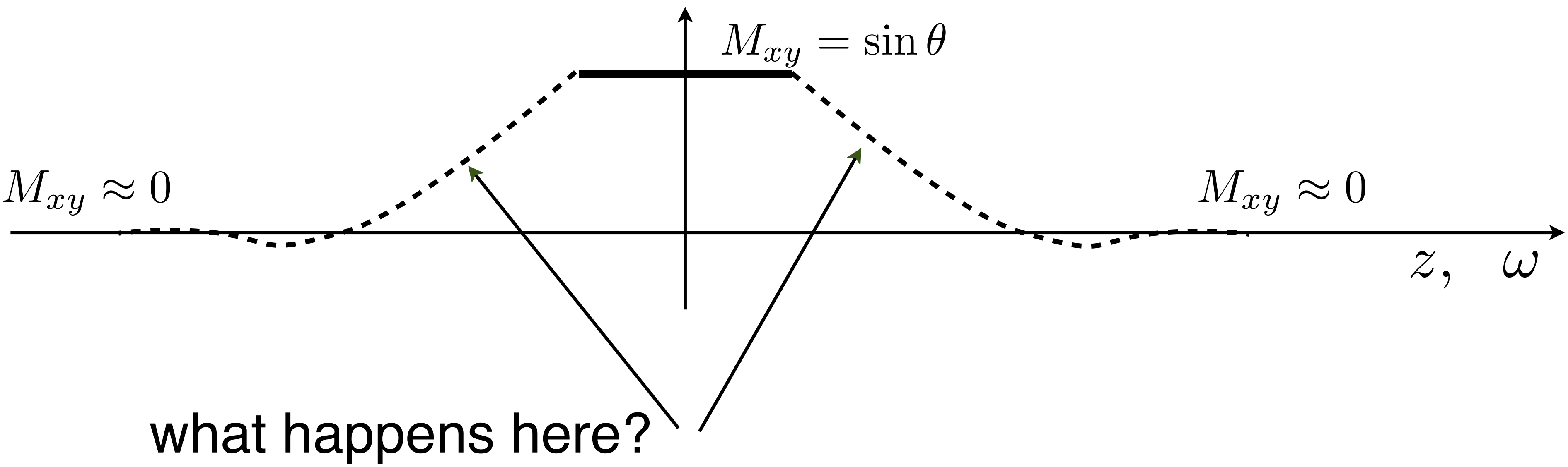
$$G_z z = 0 \Rightarrow \;\; \theta = \gamma \int_0^{\tau} B_1(t)dt$$

  - Far off resonance, $G_z z$ dominates

$$\gamma |G_z z| >> |B_1| \Rightarrow \;\; \vec{n} \approx [0, 0, 1]$$

no M_{xy} produced!

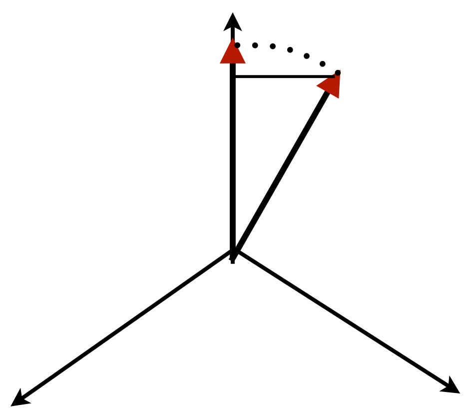
Slice Selectivity



- In general, a hard problem
  - Rotations ⇒ fundamentally non-linear
- Many Solutions for special cases
  - Including most interesting ones

Small Tip-Angle Excitation Pulses

- Basic idea:
  - Tip angle is small
  - $M_z \approx M_0$ throughout the excitation pulse



$$M_z = \cos(\theta) M_0 \approx M_0$$

$$M_{xy} = \sin(\theta) M_0 \approx \theta M_0$$

Bloch Equation - Selective RF

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \begin{bmatrix} 0 & \gamma \vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma \vec{G}\cdot\vec{r} & 0 & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

If $M_z \approx M_0$ the last equation decouples!

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \begin{bmatrix} 0 & \gamma \vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma \vec{G}\cdot\vec{r} & 0 & \gamma B_{1x} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

Bloch Equation - Small Tip Approximation

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \end{bmatrix} = \begin{bmatrix} 0 & \gamma\vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma\vec{G}\cdot\vec{r} & 0 & \gamma B_{1x} \end{bmatrix} \begin{array}{c} \begin{bmatrix} M_x \\ M_y \\ \end{bmatrix} \\ \mathrm{M0} \end{array}$$

Bloch Eq. Simplifies to:

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \gamma\vec{G}\vec{r} \\ -\gamma\vec{G}\vec{r} & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \end{bmatrix}}_{\text{Precession (like reception)}} + \underbrace{\begin{bmatrix} -\gamma B_{1y} \\ \gamma B_{1x} \end{bmatrix} M_0}_{\text{Excitation}}$$

Note: Mx, My, G, B1 are a function of time!

Small Tip-Angle Approximation

- Example: ISO-Center B1=B1x

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \end{bmatrix} = \begin{bmatrix} 0 \\ \gamma B_{1x} \end{bmatrix} M_0$$

- $M_{xy} = M_x + iM_y$ is linear with B_1 !

$$M_{xy} = i\gamma B_{1x} M_0 t$$

Derivation

$$
\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \end{bmatrix} = \begin{bmatrix} 0 & \gamma \vec{G}\vec{r} \\ -\gamma\vec{G}\vec{r} & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \end{bmatrix} + \begin{bmatrix} -\gamma B_{1y} \\ \gamma B_{1x} \end{bmatrix} M_0
$$

$$
\dot{M}_x + i\dot{M}_y = -i^2\gamma\vec{G}\cdot\vec{r}\,M_y - i\gamma\vec{G}\cdot\vec{r}\,M_x - \left(i^2\gamma B_{1y} + i\gamma B_{1x}\right)M_0
$$

$$
= \left(-i\gamma\vec{G}\cdot\vec{r}\right)\underbrace{\left(M_x + iM_y\right)}_{M_{xy}} + i\underbrace{\left(\gamma B_{1x} + i\gamma B_{1y}\right)}_{\gamma B_1}M_0
$$

Derivation

$$\dot{M}_{xy} = (-i\gamma \vec{G}\cdot\vec{r}) M_{xy} + i\gamma B_1 M_0$$

- Solve like in the reception case!
  - Integrating factor: $e^{i\int_{-\infty}^{t}\gamma\vec{G}\cdot\vec{r}d\tau}$

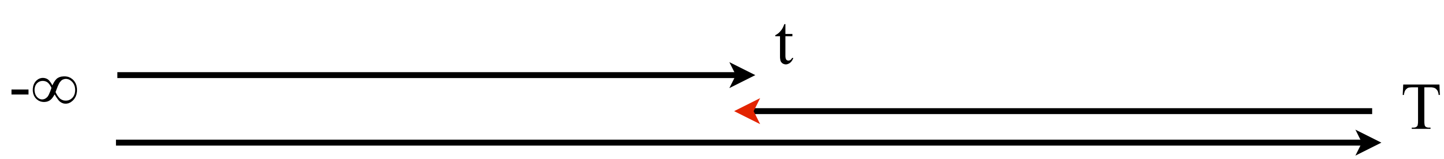
$$\frac{d}{dt}\left[M_{xy}(\vec{r},t)\, e^{i\int_{-\infty}^{t}\gamma\vec{G}\cdot\vec{r}\,d\tau}\right] = i\gamma B_1(t) M_0 e^{i\int_{-\infty}^{t}\gamma\vec{G}\cdot\vec{r}\,d\tau}$$

Derivation

- Integrate from -∞ to t=T to find $M_{xy}(r,T)$

$$M_{xy}(\vec{r},T)\cdot e^{i\int_{-\infty}^{T}\gamma \vec{G}(\tau)\vec{r}\,d\tau} = \int_{-\infty}^{T} iM_0\gamma B_1(t)\, e^{i\int_{-\infty}^{t}\gamma \vec{G}(\tau)\cdot\vec{r}\,d\tau}\, dt$$

$$M_{xy}(\vec{r},T) = \int_{-\infty}^{T} iM_0\gamma B_1(t)\, e^{i\int_{-\infty}^{t}\gamma \vec{G}(\tau)\cdot\vec{r}\,d\tau - i\int_{-\infty}^{T}\gamma \vec{G}(\tau)\vec{r}\,d\tau}\, dt$$



Derivation

$$M_{xy}(\vec{r},T) = iM_0 \int_{-\infty}^{T} \gamma B_1(t)\, e^{-i\int_t^T \gamma \vec{G}(\tau)\cdot\vec{r}\, d\tau}\, dt$$

Small Tip-Angle Approximation

- Solution for general Eq. at time t=T

$$M_{xy}(\vec{r},T) = iM_0 \int_{-\infty}^{T} \gamma B_1(t) e^{-i2\pi \vec{k}(t)\cdot\vec{r}} dt$$

,where

$$\vec{k}(t) = \frac{\gamma}{2\pi} \int_{t}^{T} \vec{G}(\tau) d\tau$$

- k(t) is area of the <u>remaining</u> gradient

Example: Slice Selection

$$M_{xy}(z,T) = iM_0 \int_0^T \gamma B_1(t) e^{-i2\pi k_z(t)z} dt$$

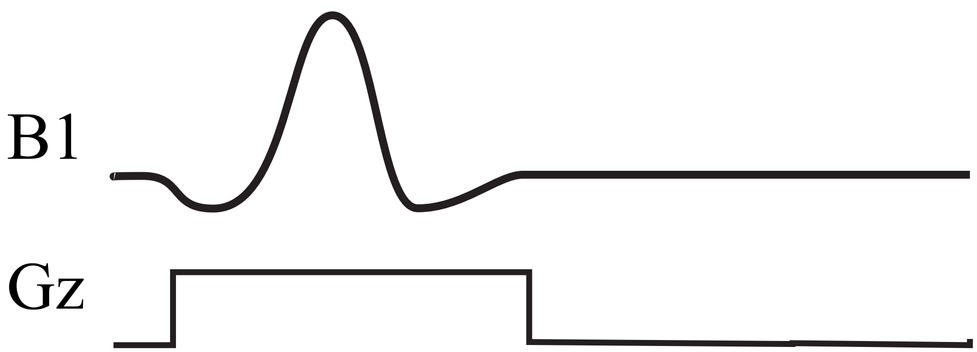
- This is not exactly a Fourier transform
- Would like:

$$M_{xy}(z,T) = iM_0 \int_K W(k) e^{-i2\pi k_z z} dk_z$$

$$W(k) = \frac{2\pi B_1(k)}{|G(k)|}$$

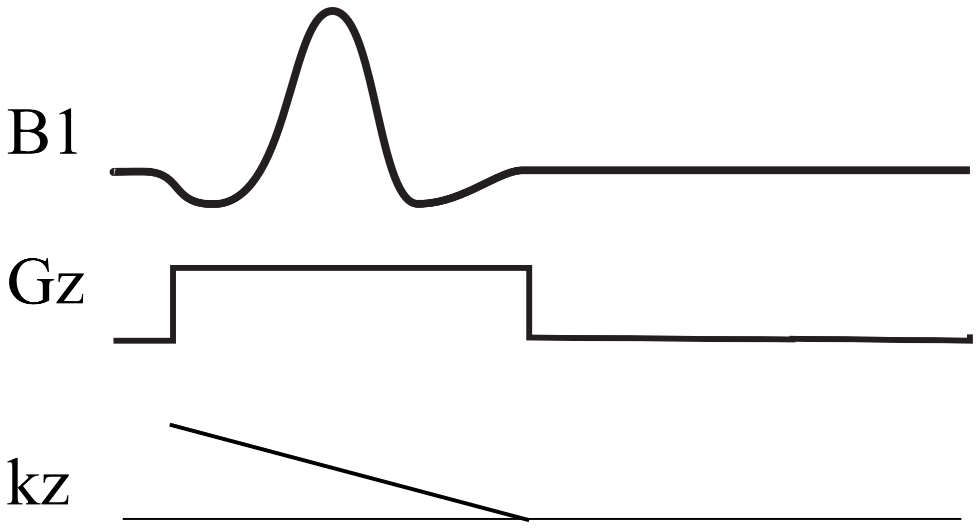
Example: Slice Selection

$$M_{xy}(z,T) = iM_0 \int_0^T \gamma B_1(t) e^{-i2\pi k_z(t)z} dt$$

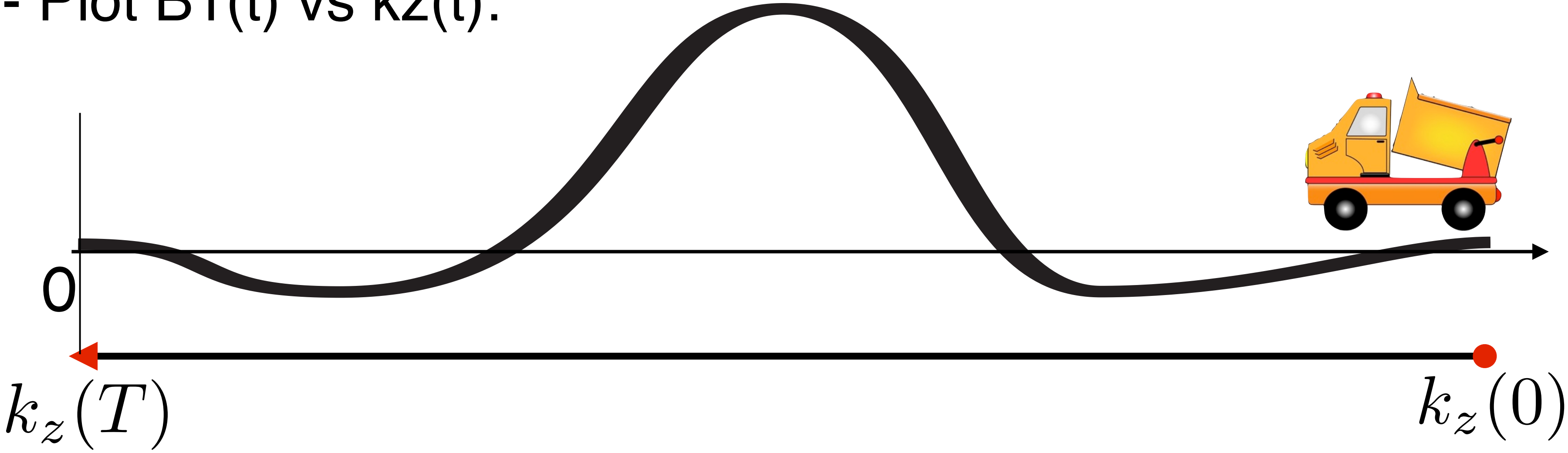


- First find $k_z(t)$
- Map B1(t) to $k_z(t)$
- Compute the integral (Fourier transform)

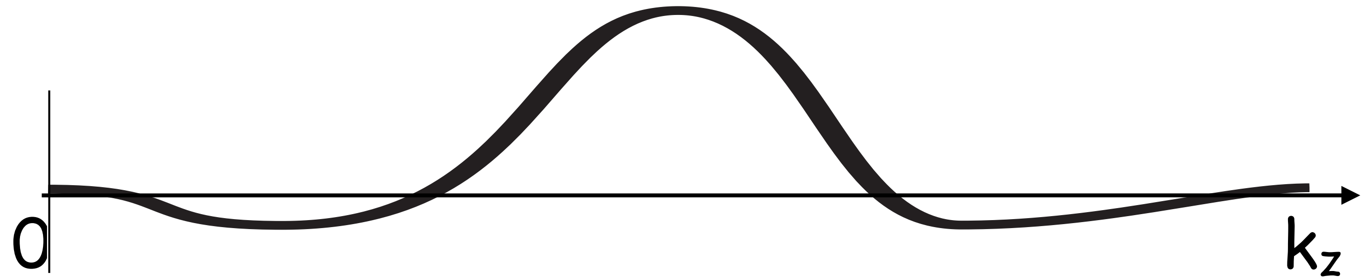
Example: Slice Selection



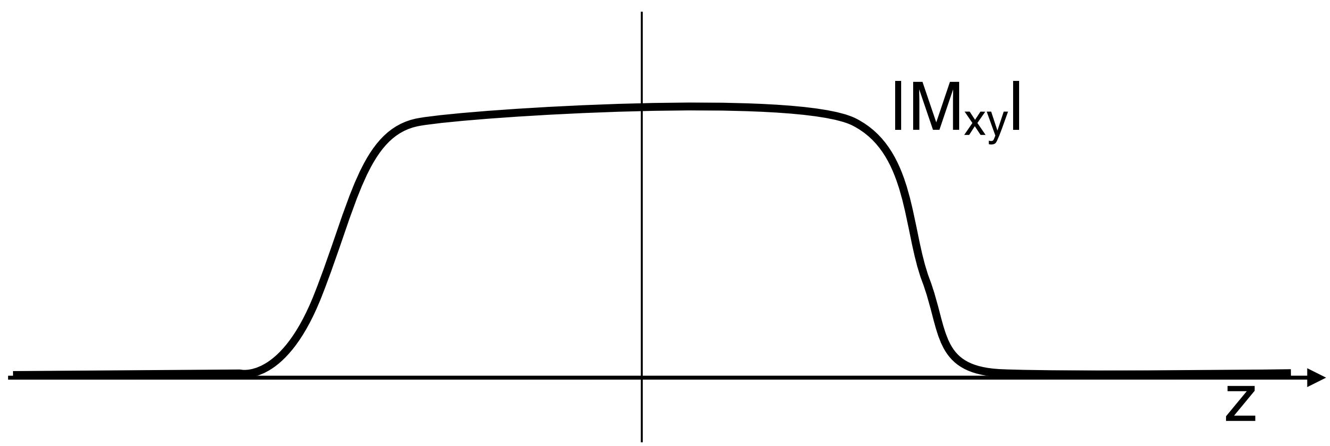
- Plot B1(t) vs kz(t):



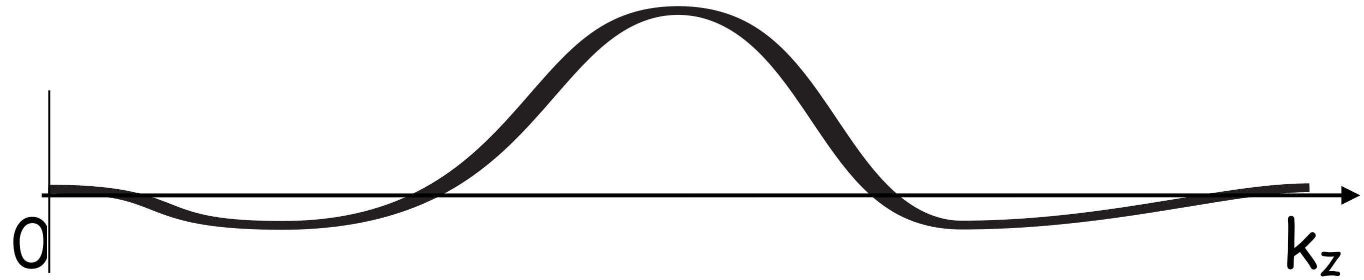
Example: Slice Selection



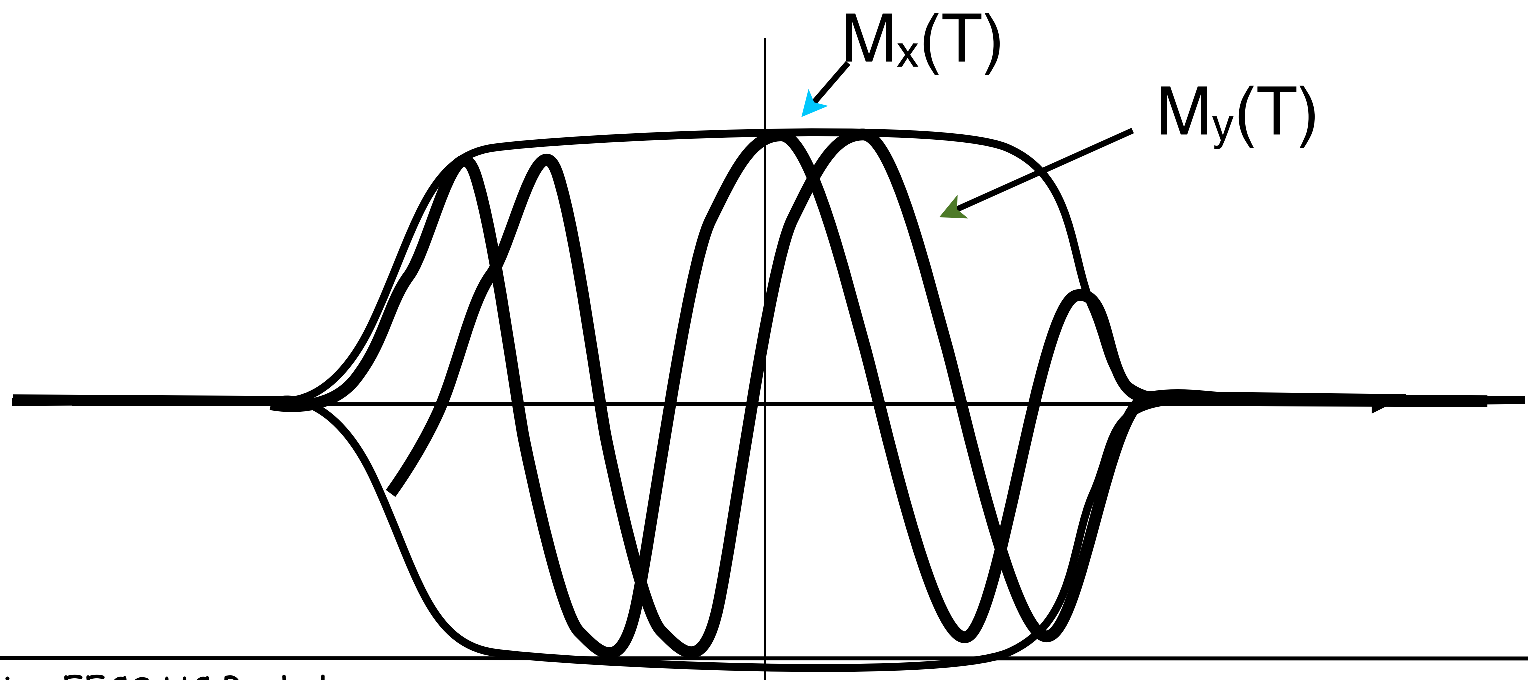
- B1 is not centered in k-space
- We get the right magnitude Mxy



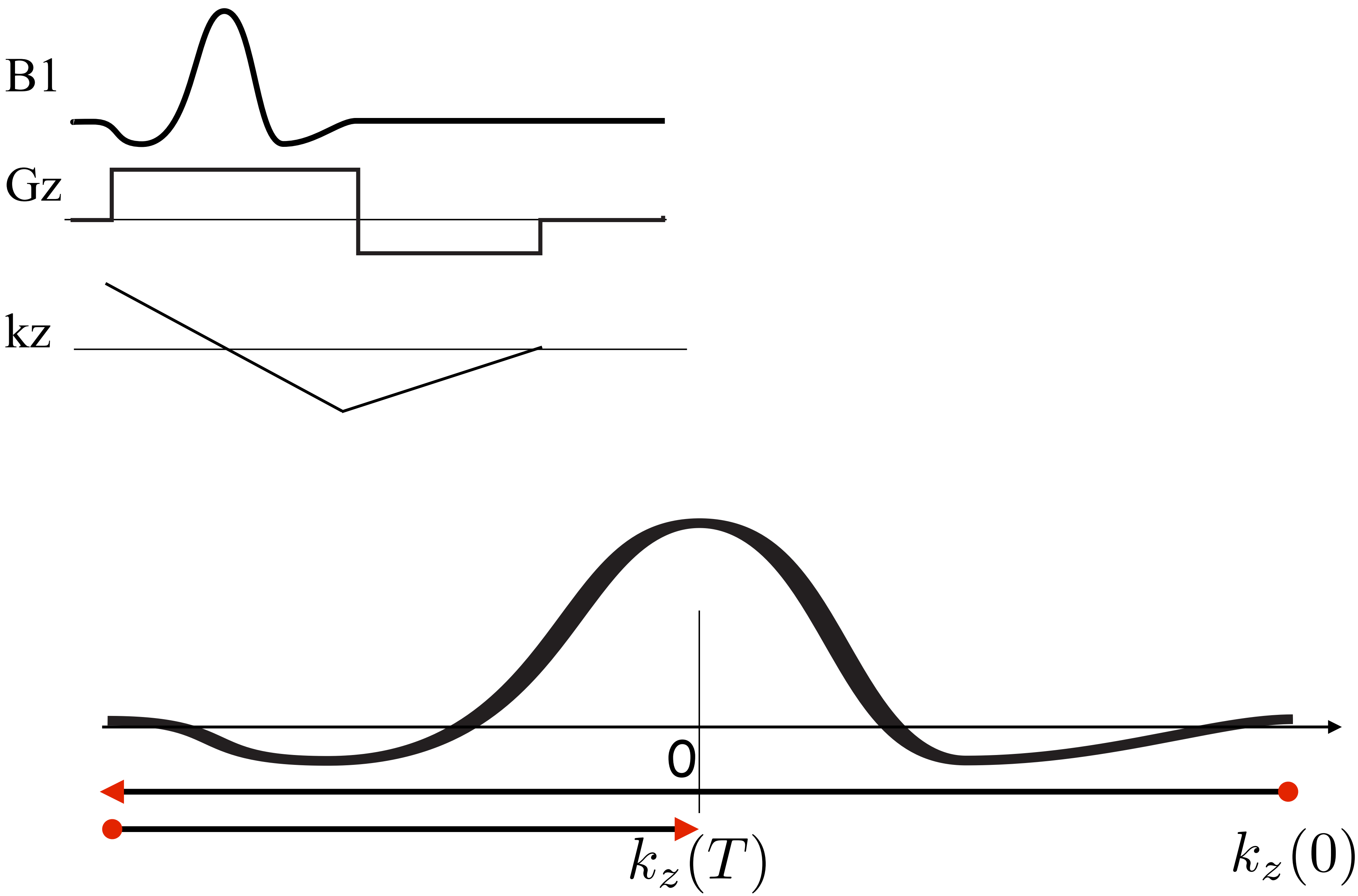
Example: Slice Selection



- But we get phase across the slice.... signal cancels!

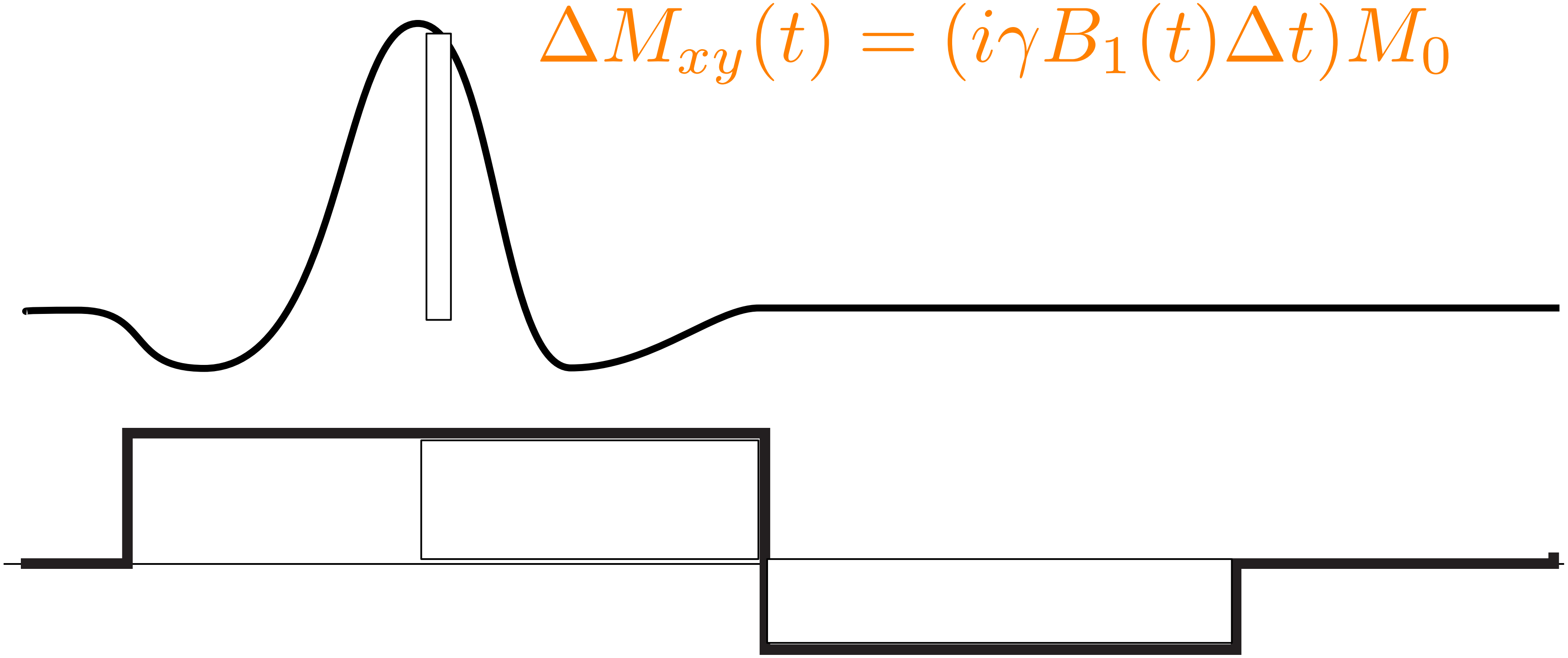


Slice Refocussing



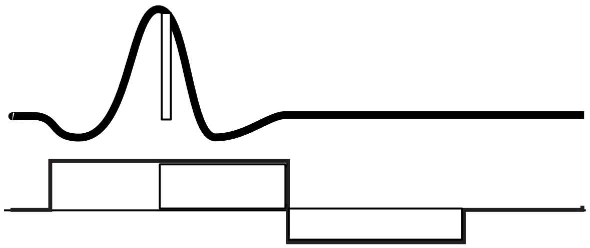
Graphical Interpretation

$$\Delta M_{xy}(t) = (i\gamma B_1(t)\Delta t)M_0$$



- At each time-point new x-verse magnetization is created
- The new magnetization exhibits precession

Graphical Interpretation



$$\Delta M_{xy}(t) = (i\gamma B_1(t)\Delta t)M_0$$

$$k_z(t) = \frac{\gamma}{2\pi}\int_t^T G_z(\tau)d\tau$$

area of remaining gradient

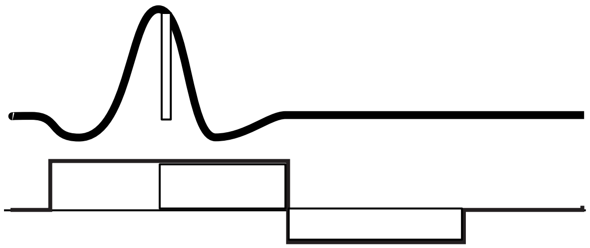
- Magnetization at position z precesses through angle:

$$\theta = -2\pi k_z(t)$$

- Magnetization excited at t, at position z will end up:

$$(iM_0\gamma B_1(t)dt)e^{-i2\pi k_z(t)z}$$

Graphical Interpretation



$$\Delta M_{xy}(t) = (i\gamma B_1(t)\Delta t)M_0$$

- Each magnetization increment is created/precesses independently! Result is the sum of all.

- Sum up magnetizations from t=0 to t=T

$$M_{xy}(z,T) = iM_0 \int_0^T \gamma B_1(t) e^{-i2\pi k_z(t)z} dt$$

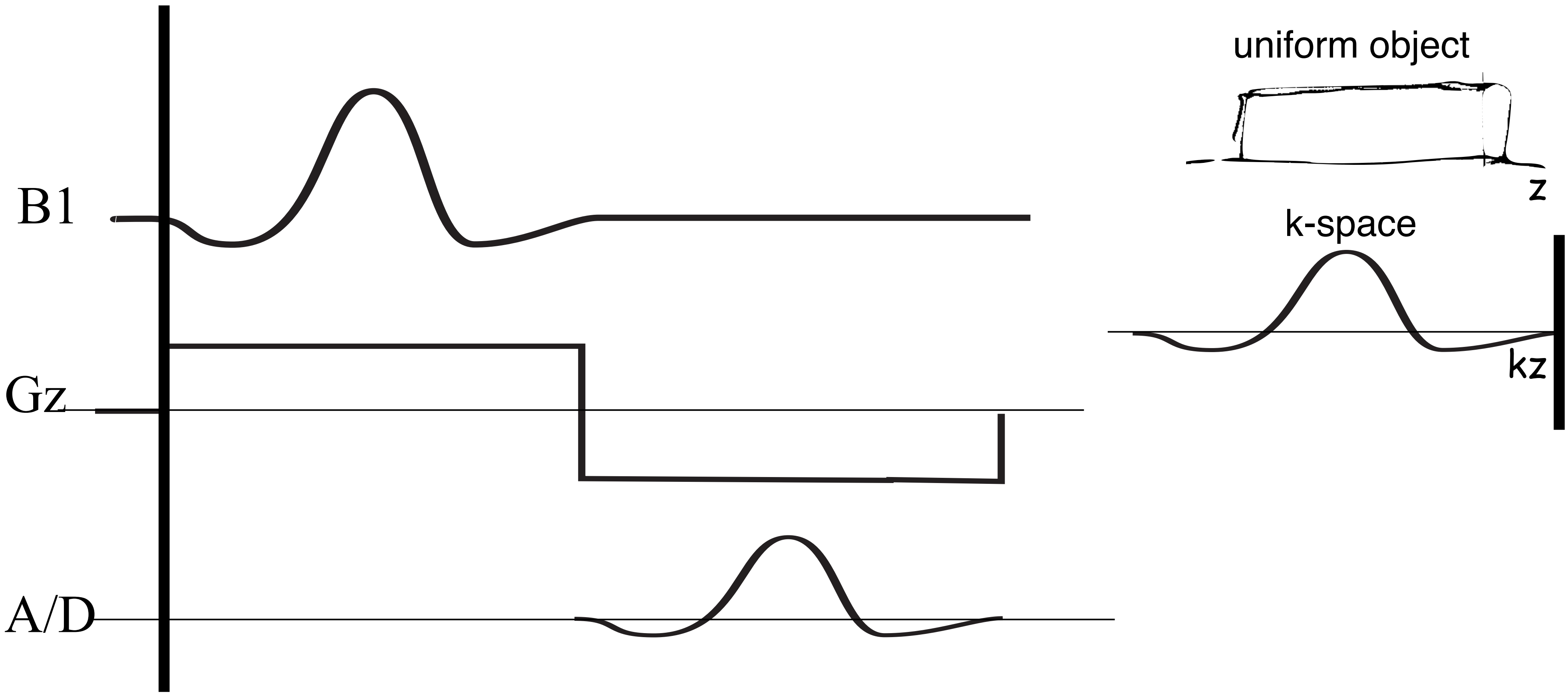
Excitation k-space

J. Pauly, D. Nishimura and A. Makovski "A k-space analysis of small-tip-angle excitation" JMR, 1989;81:43-56

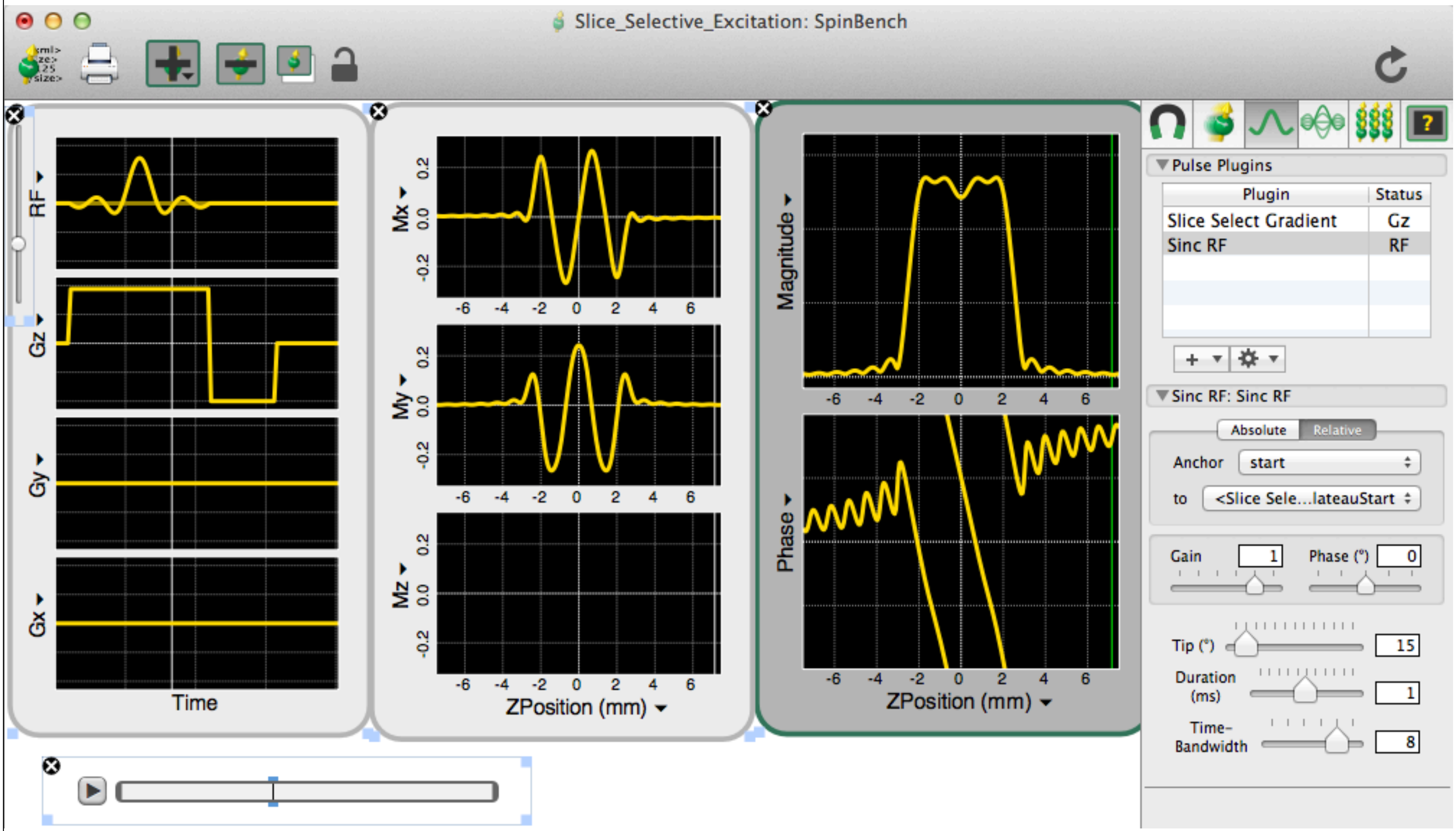
* Slightly different conventions than we used.

RF and readout

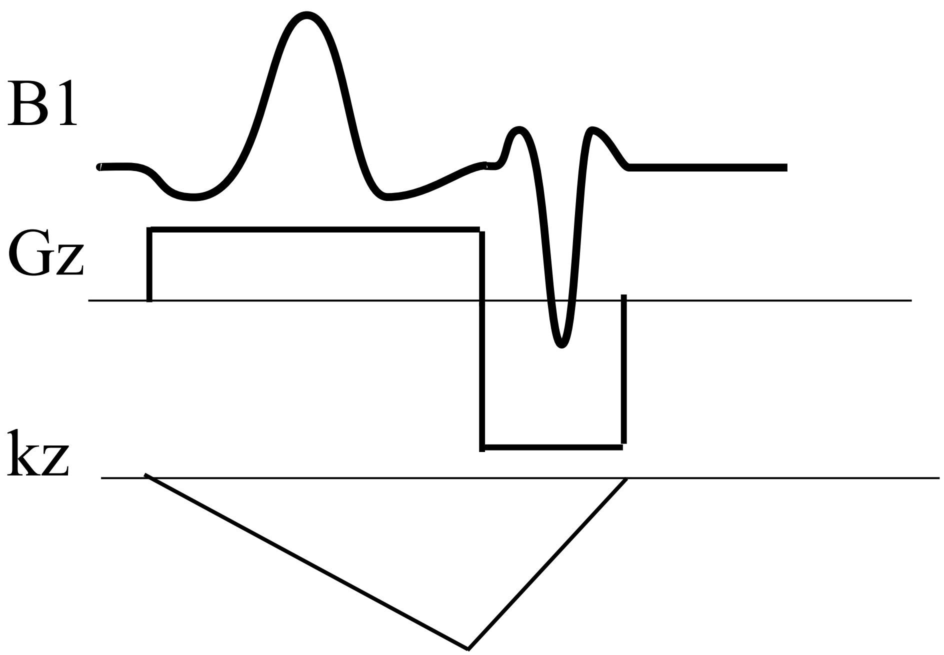
- RF burns magnetization in k-space
- Receive reads the burned magnetization (weighted by object)

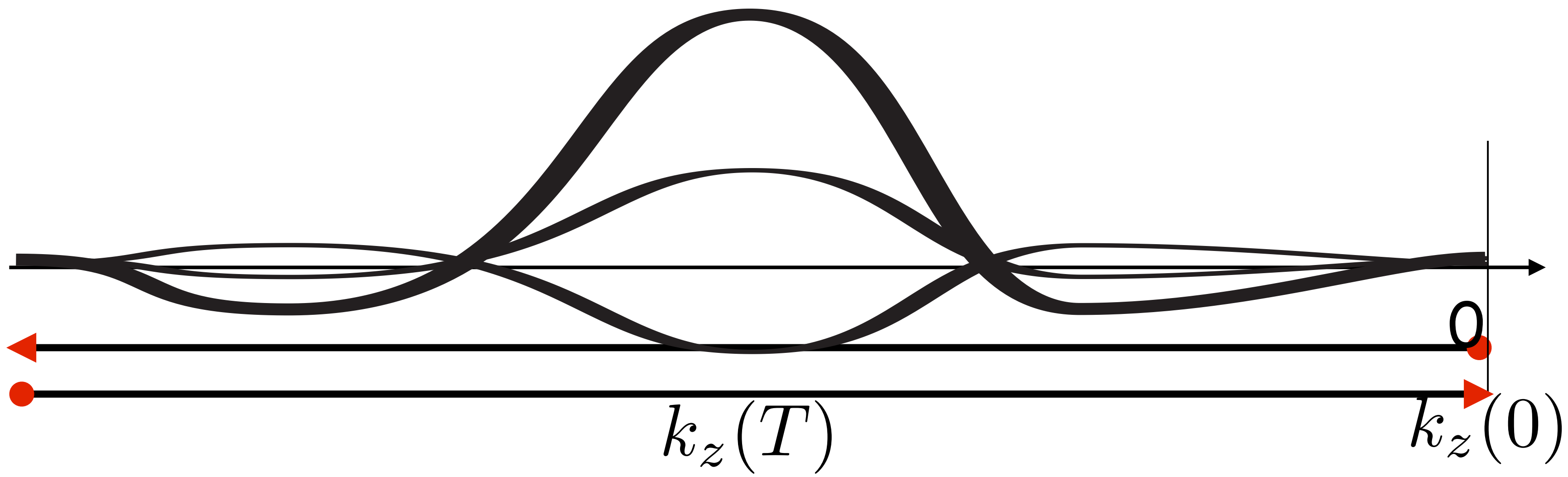


Spin Bench simulation



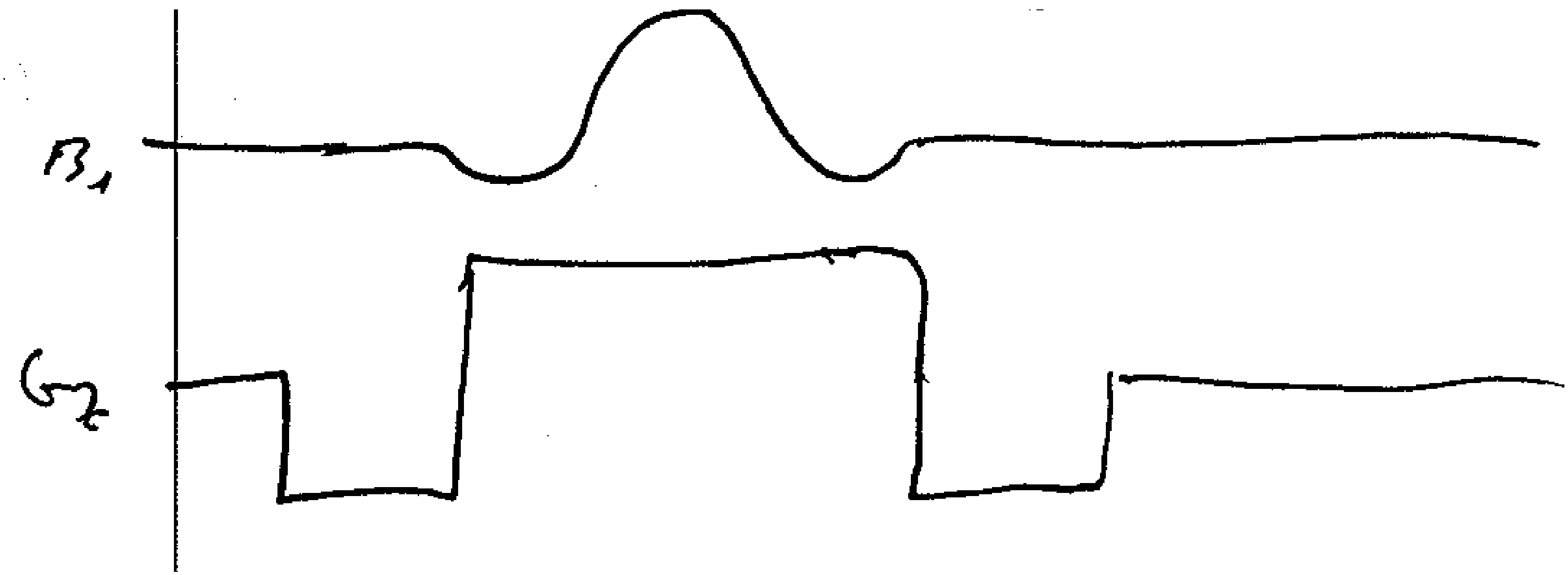
k-Space Weighting Example



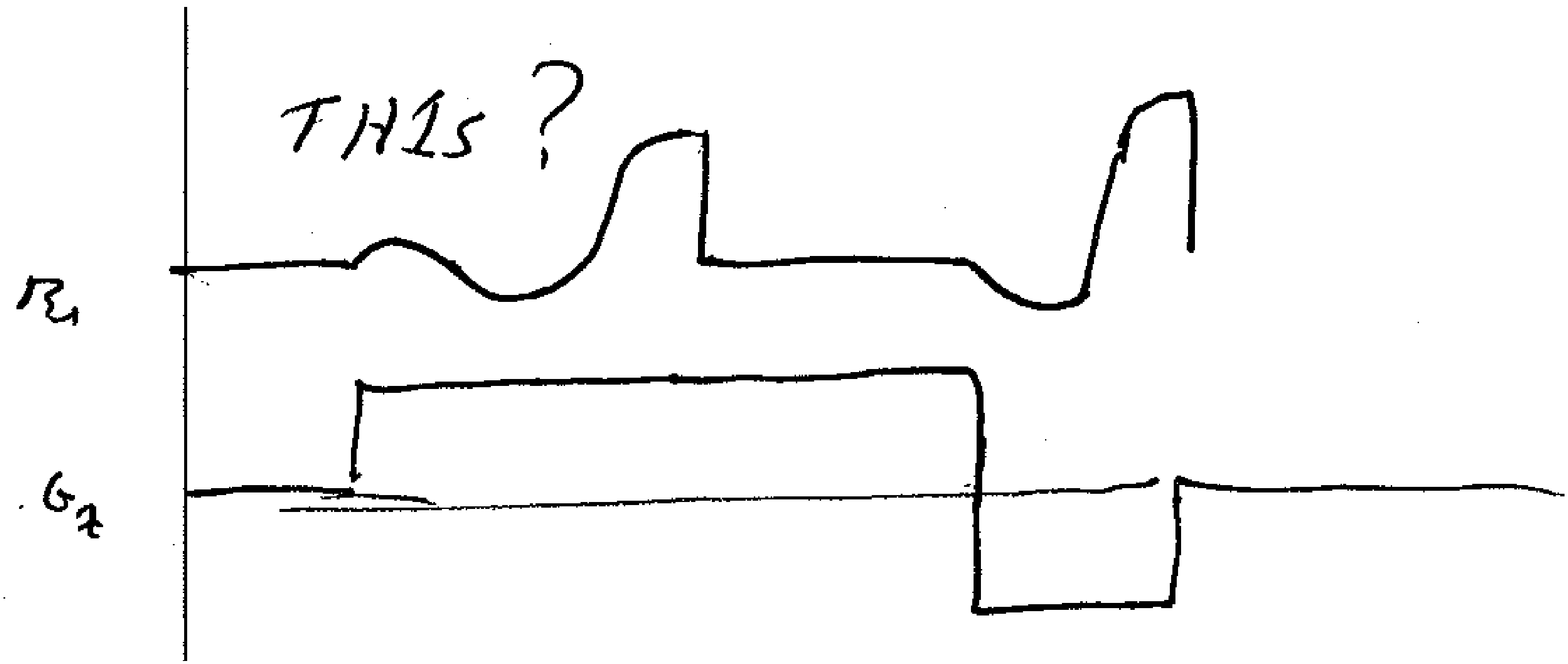


Examples

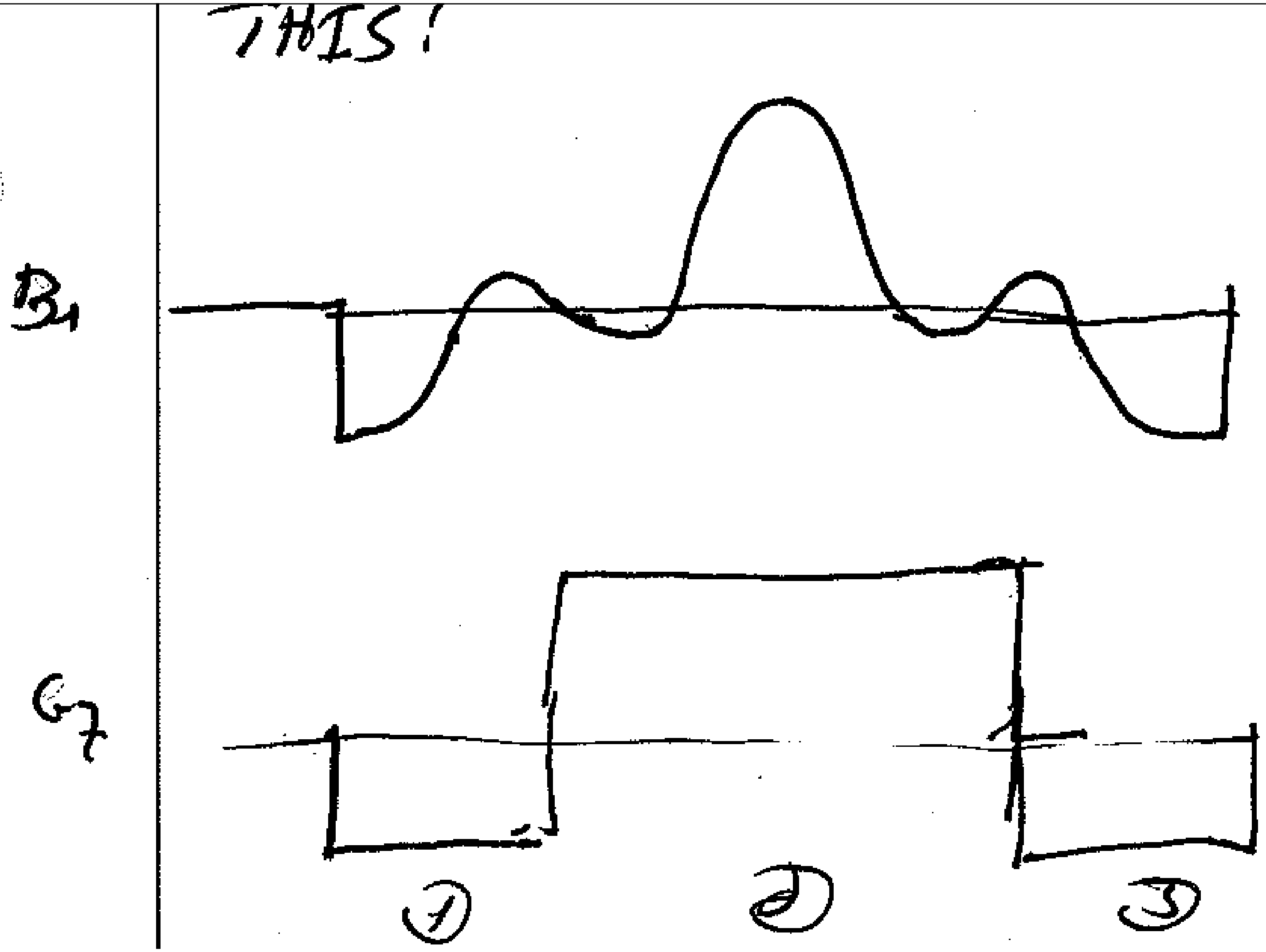
What does this do?



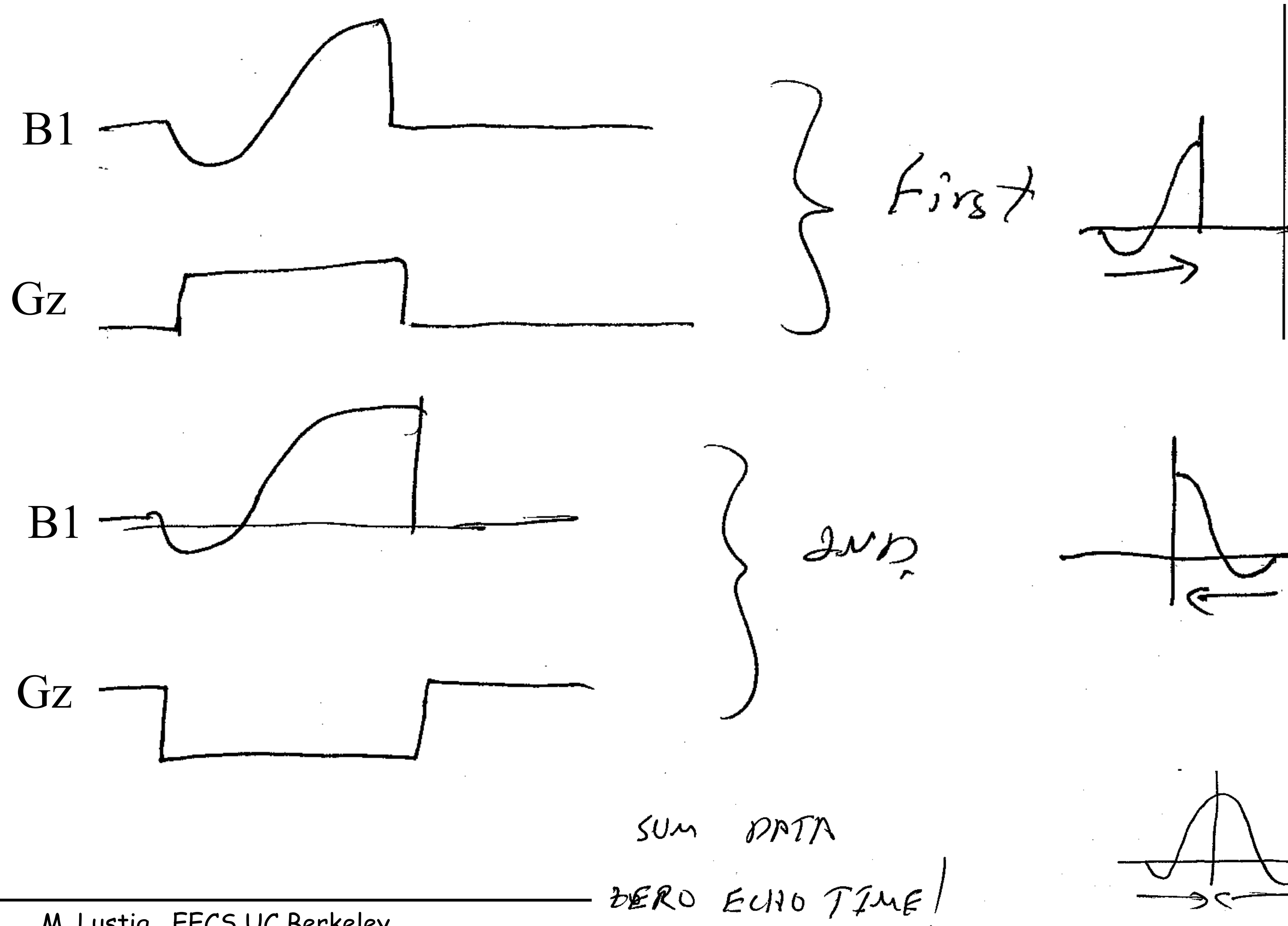
Examples



Exam

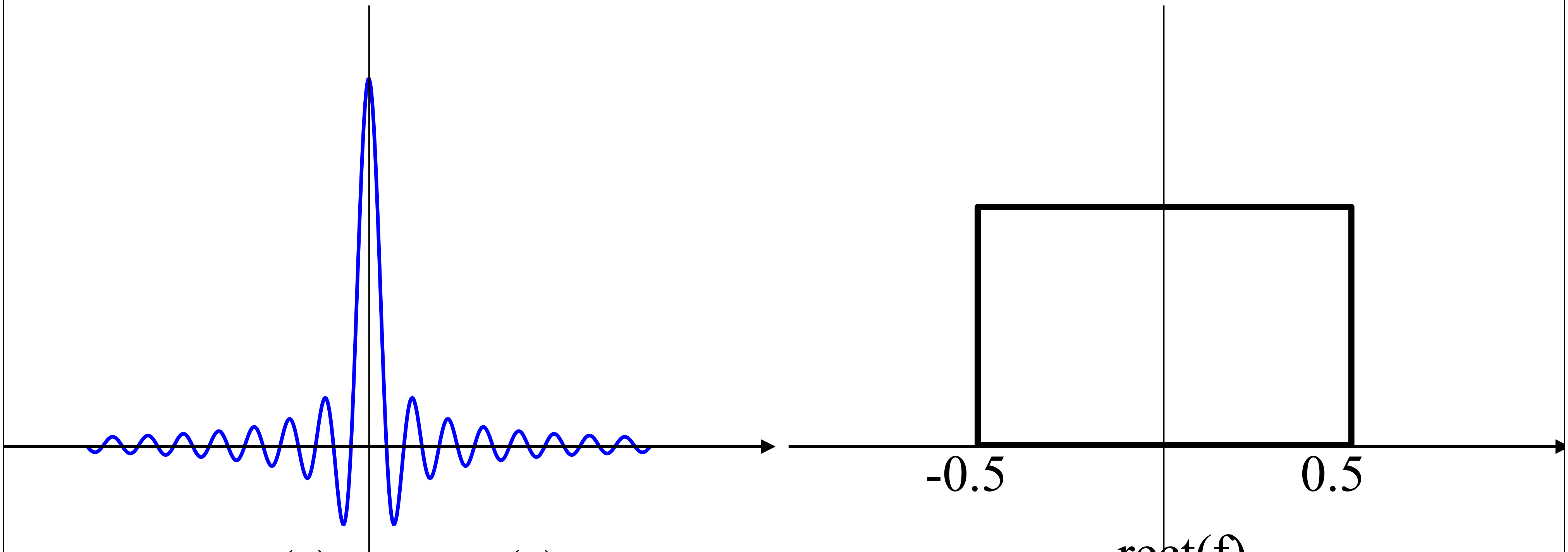


Multiple Excitations



RF Pulse Design

- Choose, B1(t) with a nice transform



$B_1(t) \propto \text{sinc}(t)$

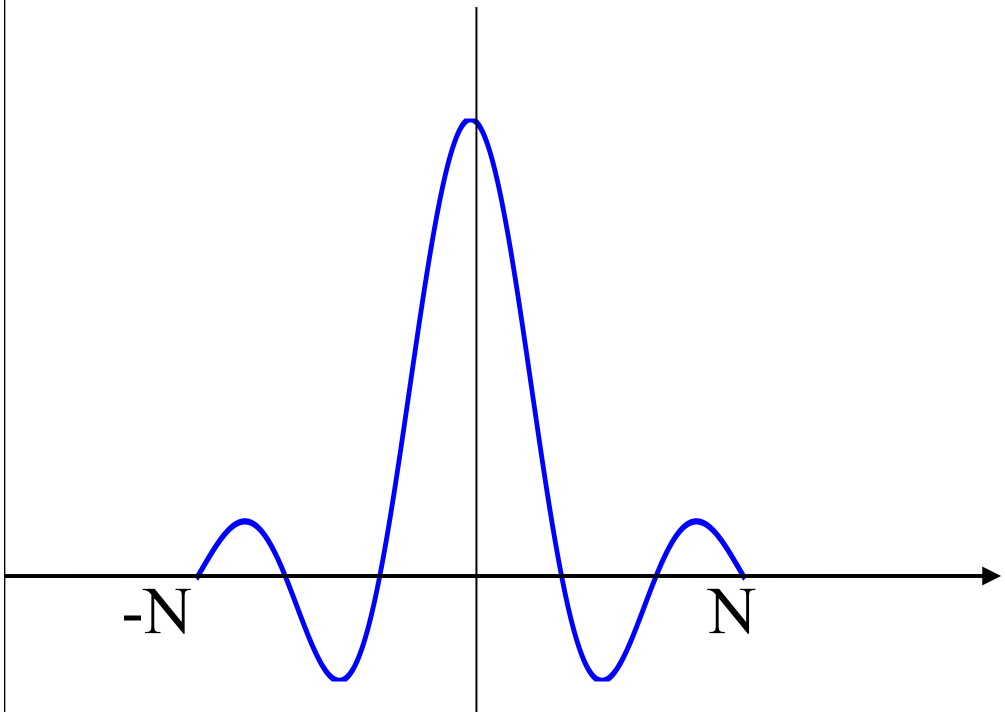
t[ms]

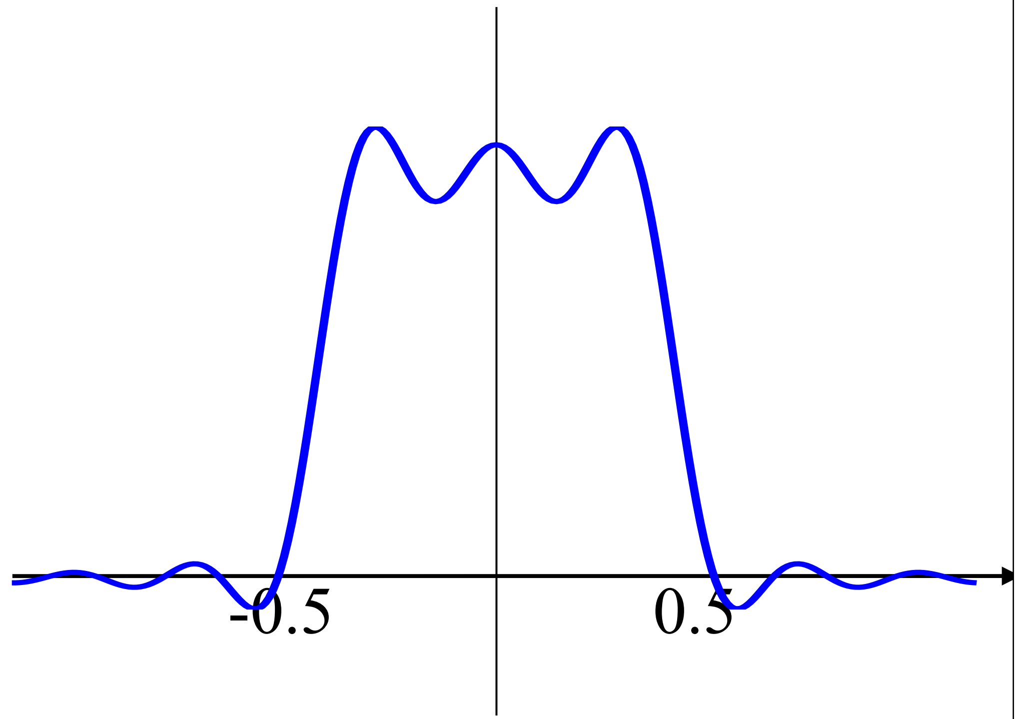
rect(f)

f[KHz]

Not prectical, since sinc is continuous indefinitely

Truncated sinc



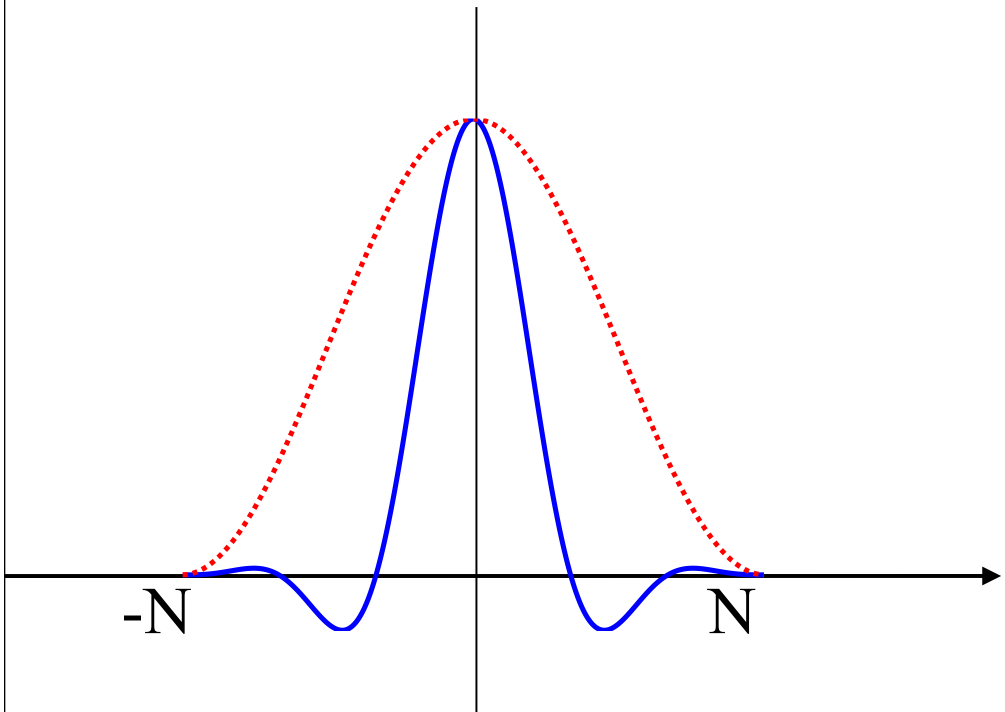


$$B_1(t) \propto \mathrm{sinc}(t)\mathrm{rect}(t/2N)$$

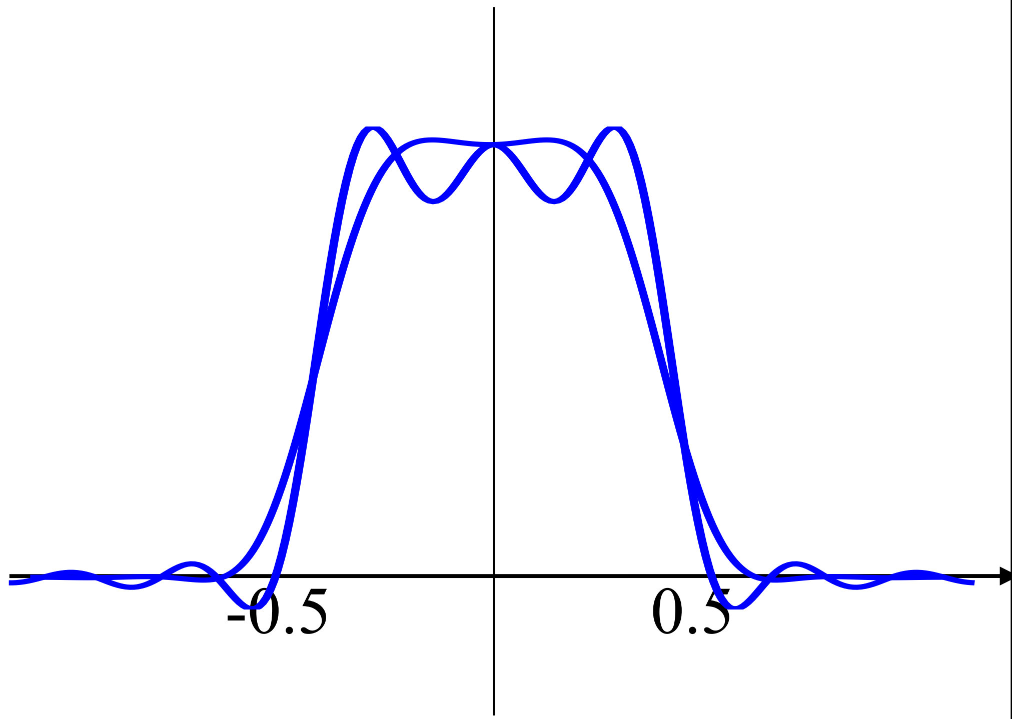
rect(f)*2N sinc(2Nf)

Too much Ripple!

Windowed sinc



$$B_1(t) \propto \mathrm{sinc}(t) w(t/2N)$$



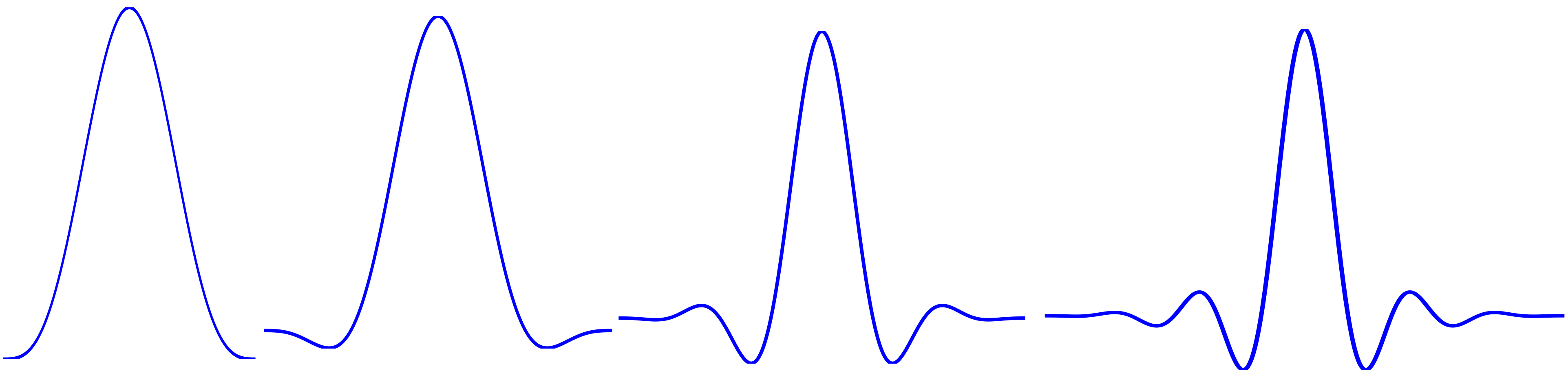
rect(f)*2N W(2Nf)

Hanning Window

Characterization of Pulse Shape

Time-Bandwidth Product

$T(BW) = (2N)1 = 2N \quad \Rightarrow$ Total # of zero crossings



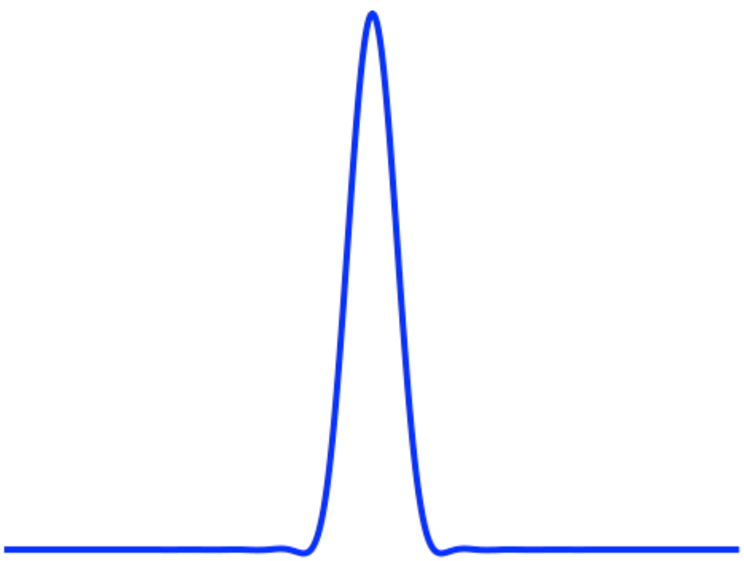
TBW=2 — rapid imaging

TBW=4 — 180

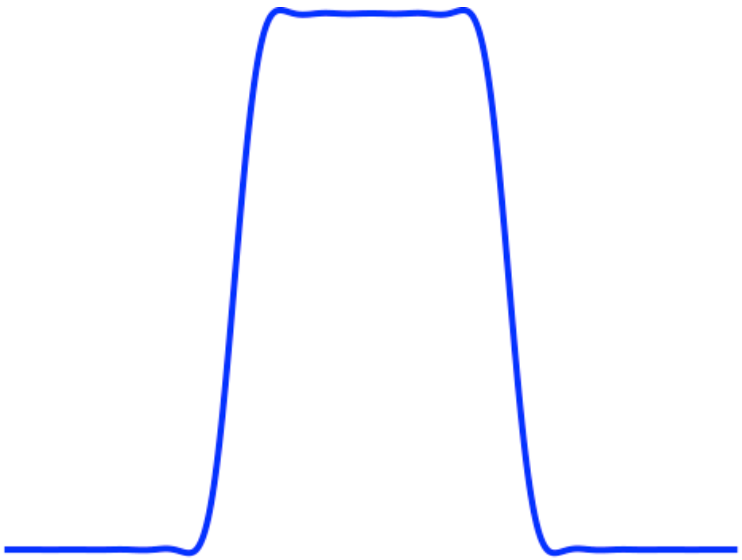
TBW=8 — 90

TBW=12 — slab and saturation

Slice Profile

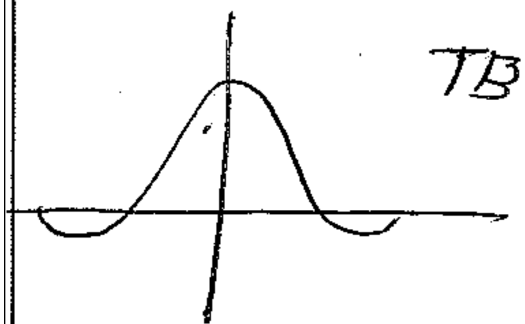


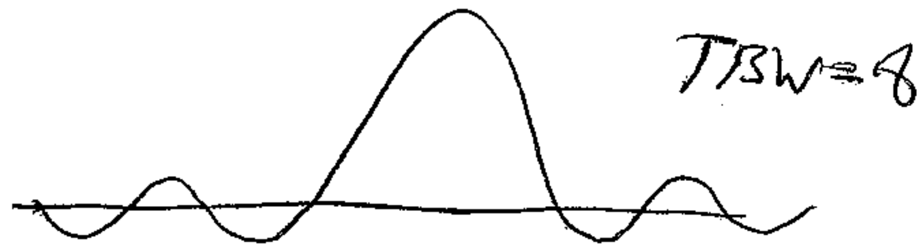
TBW=2

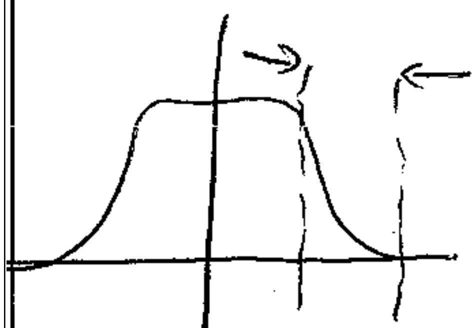


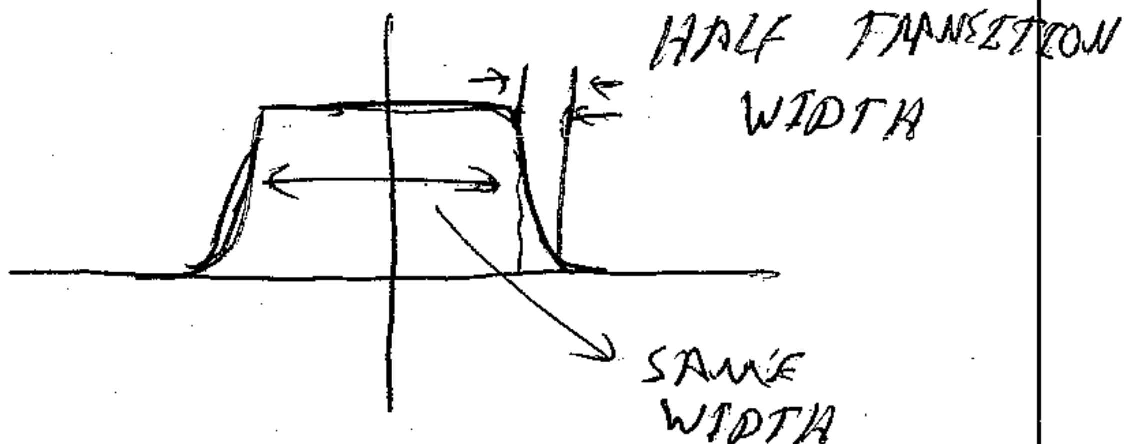
TBW=12

IF WE FIX BANDWIDTH AND MAKE T LONGER



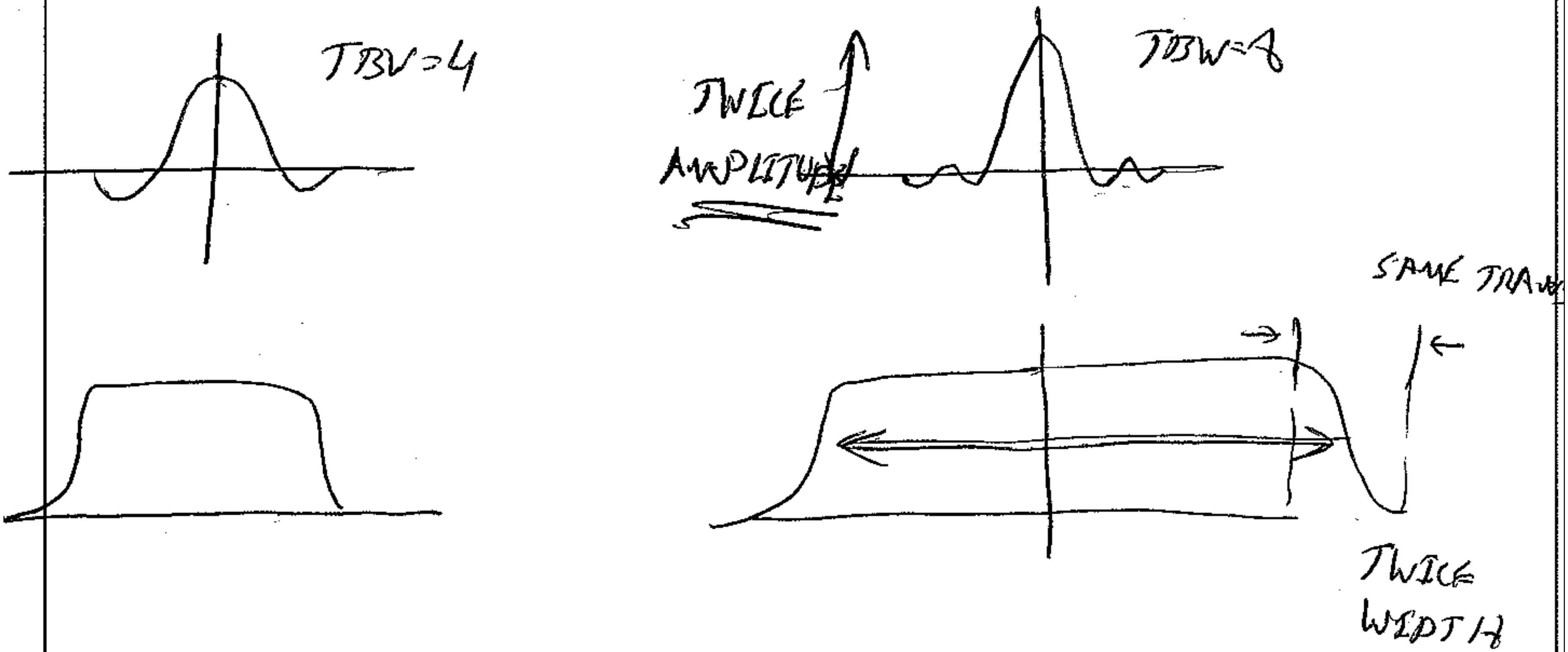






MORE SELECTIVE PROFILE

IF WE FIX DURATION INCREASE BW

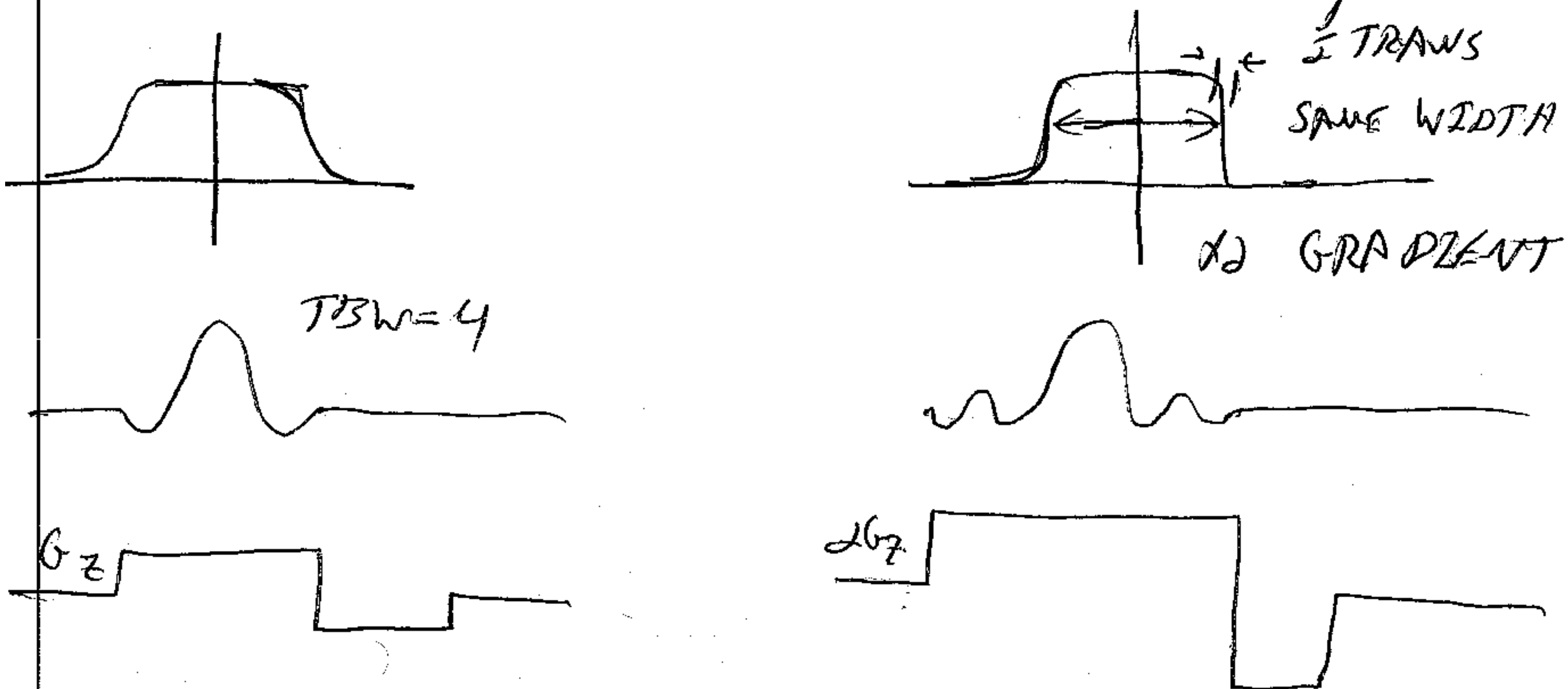


WIDER EXCITATION

TYPICALLY IN MRI WE FIX DURATION, AND

ADJUST THE GRADIENT AMP. TO COMPENSATE FOR

FOR THE INCREASE IN BW



DOUBLE BW (kHz)

DOUBLE GRADIENT (kHz/cm)

$\Rightarrow$ SAME WIDTH!

EXAMPLE

WE WANT A TBW=8 PULSE WITH 2[ms] DURATION

SLICE THICKNESS IS 1[cm]. WHAT IS THE GRADIENT AMP?

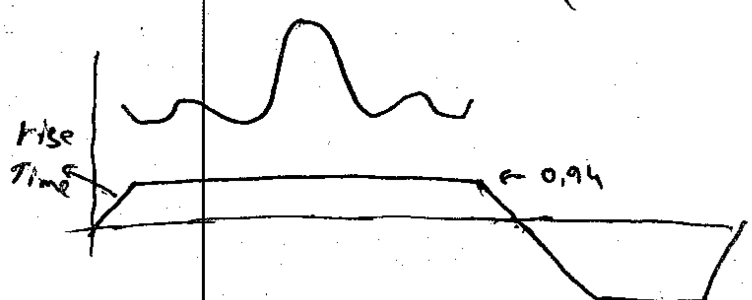
$T(BW)=8$

$2ms \cdot BW = 8 \Rightarrow BW = 4\,KHz$

$\Delta Z = 1\,cm$

$$\underbrace{\frac{\gamma}{2\pi} G \Delta Z}_{BW} = 4\,kHz$$

$$\left(4.257\ \frac{kHz}{G}\right) G \cdot (1\,cm) = 4\,KHz \Rightarrow G = 0.94\ \frac{G}{cm}$$



Q. WHAT IS G IF SLICE = 0.5 cm?

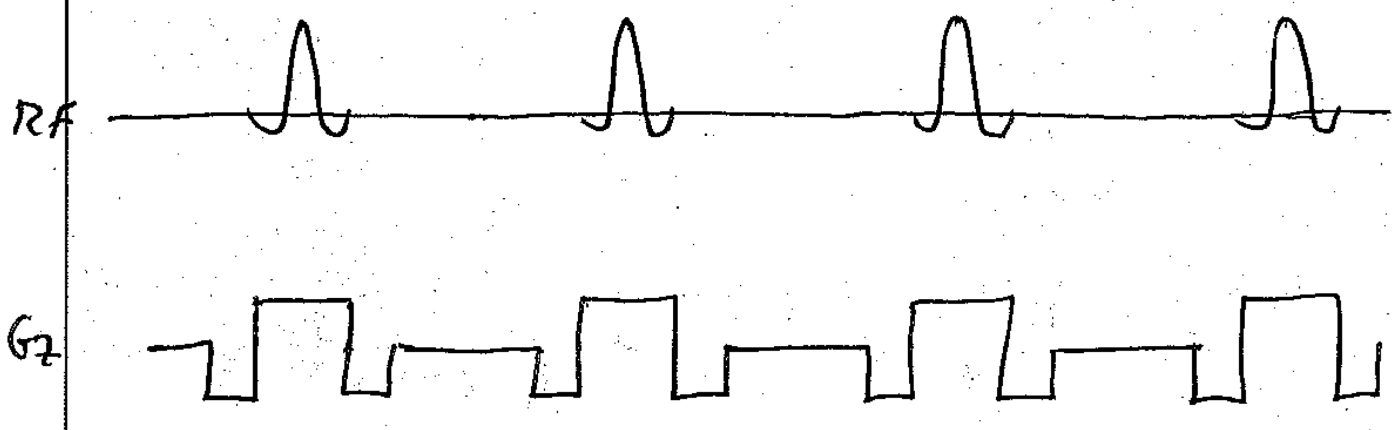
A. $G = 1.88\ \frac{G}{cm}$

Q WHAT IF $G_{max} = 0.94\ \frac{G}{cm}$?

$TBW = 4 \Rightarrow$ LESS SELECTIVE

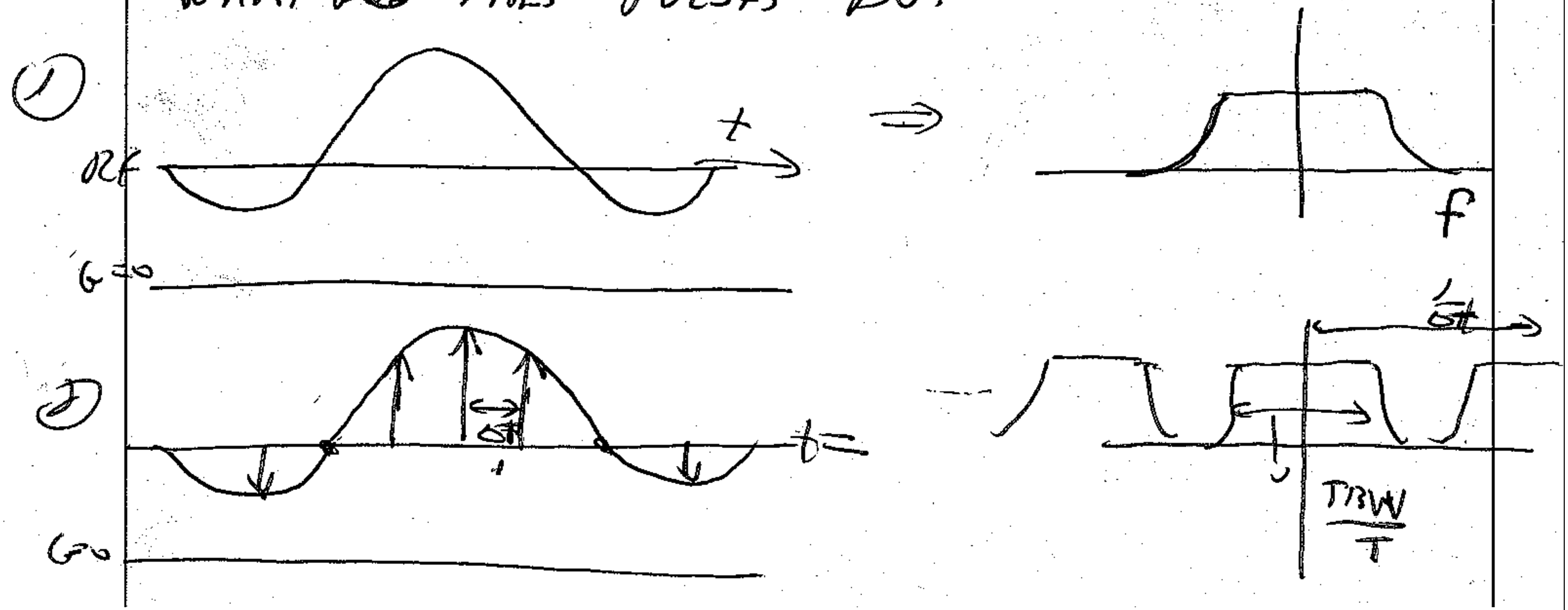
$T = 4ms \Rightarrow$ LONGER DURATION

WHAT DOES THIS PULSE DO?

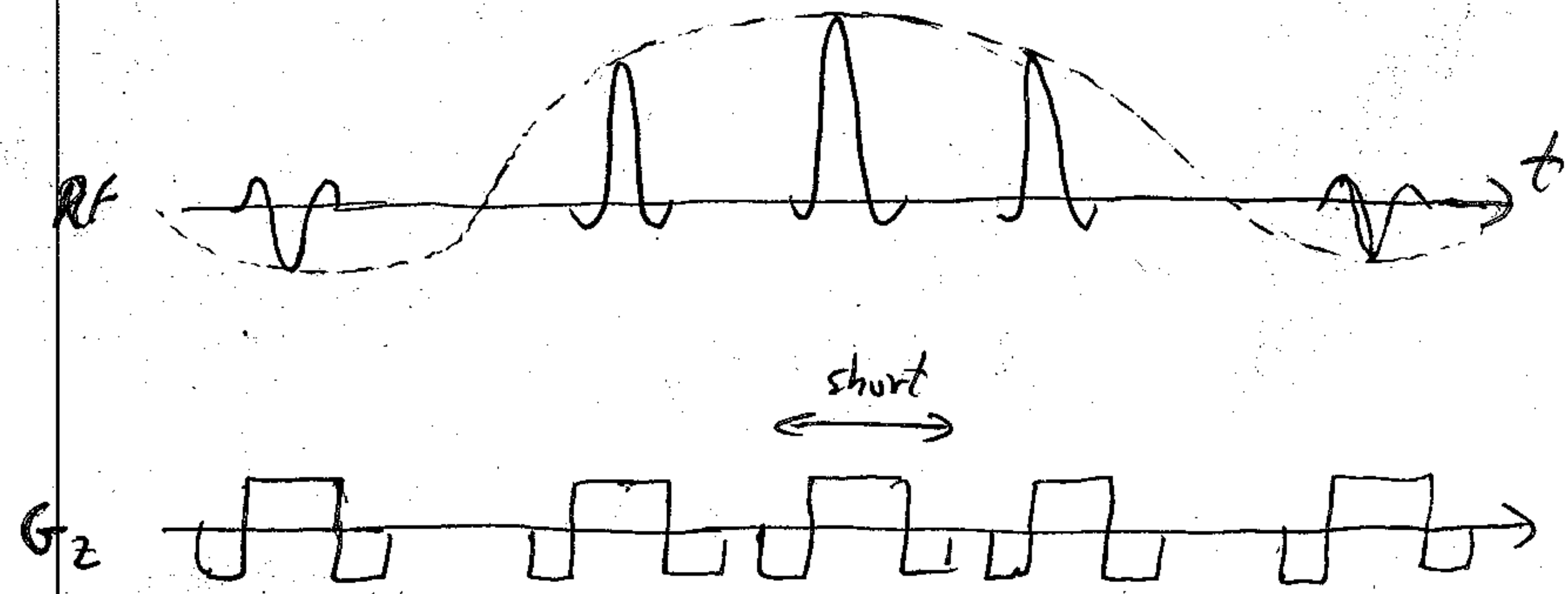


SPECTRAL PULSE

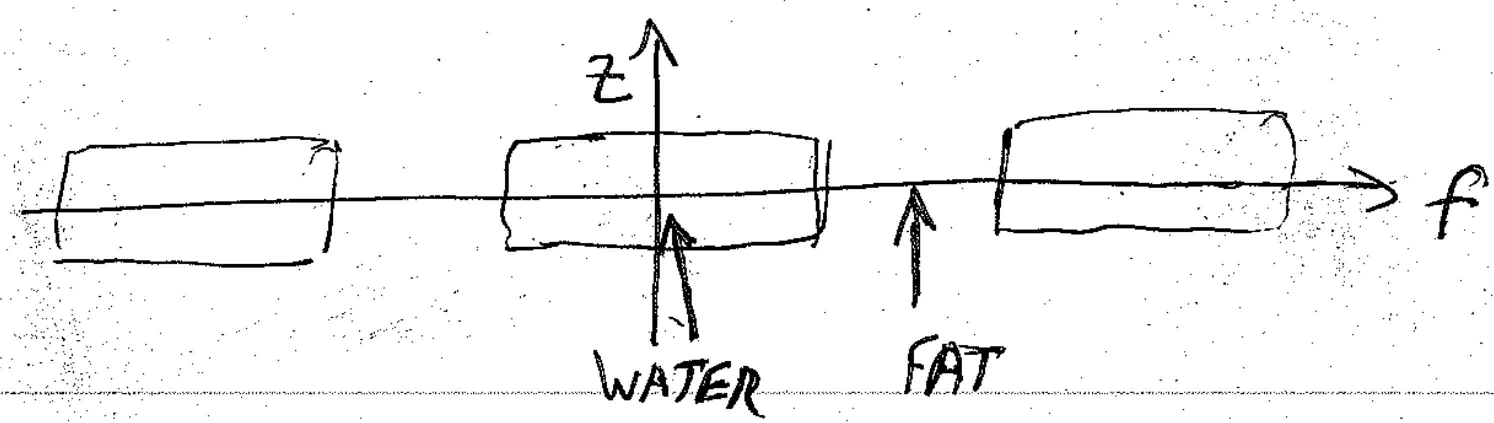
WHAT DO THES PULSES DO?



REPLACE HARD RF WITH SLICE SELECT SUB-PULSES



SPATIAL AND SPECTRAL SELECTIVITY !



WHAT HAPPENS AT LARGE FLIPS?

MANY METHODS:

- NUMERICAL OPTIMIZATION
- OPTIMAL CONTROL (CONOLLY)
- SLR (PAULY)
- PERTUBATION THEORY

OPTIMAL CONTROL!

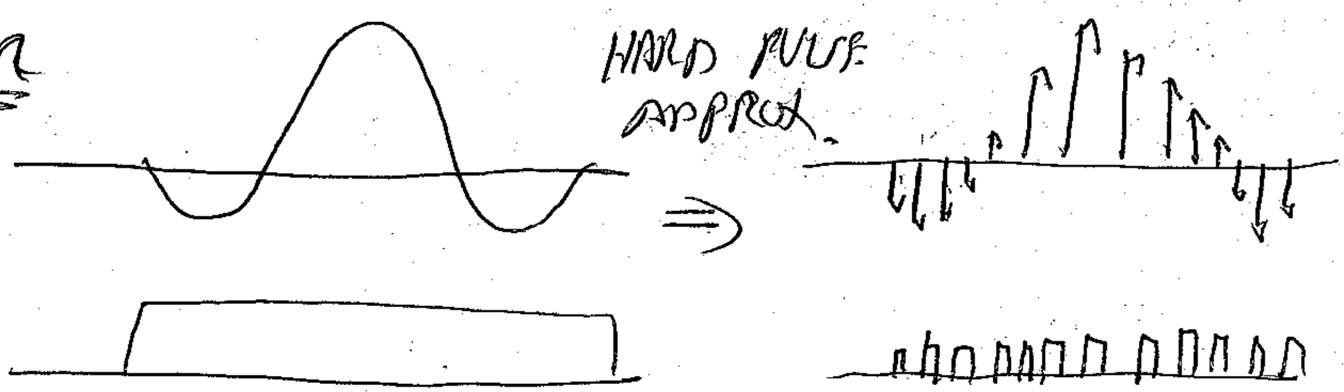
handles handles

$$\begin{pmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{pmatrix} = \begin{pmatrix} 0 & \gamma \vec{G}\cdot\vec{r} & -\gamma B_{1y} \\ -\gamma \vec{G}\cdot\vec{r} & 0 & \gamma B_{1x} \\ \gamma B_{1y} & -\gamma B_{1x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

handles

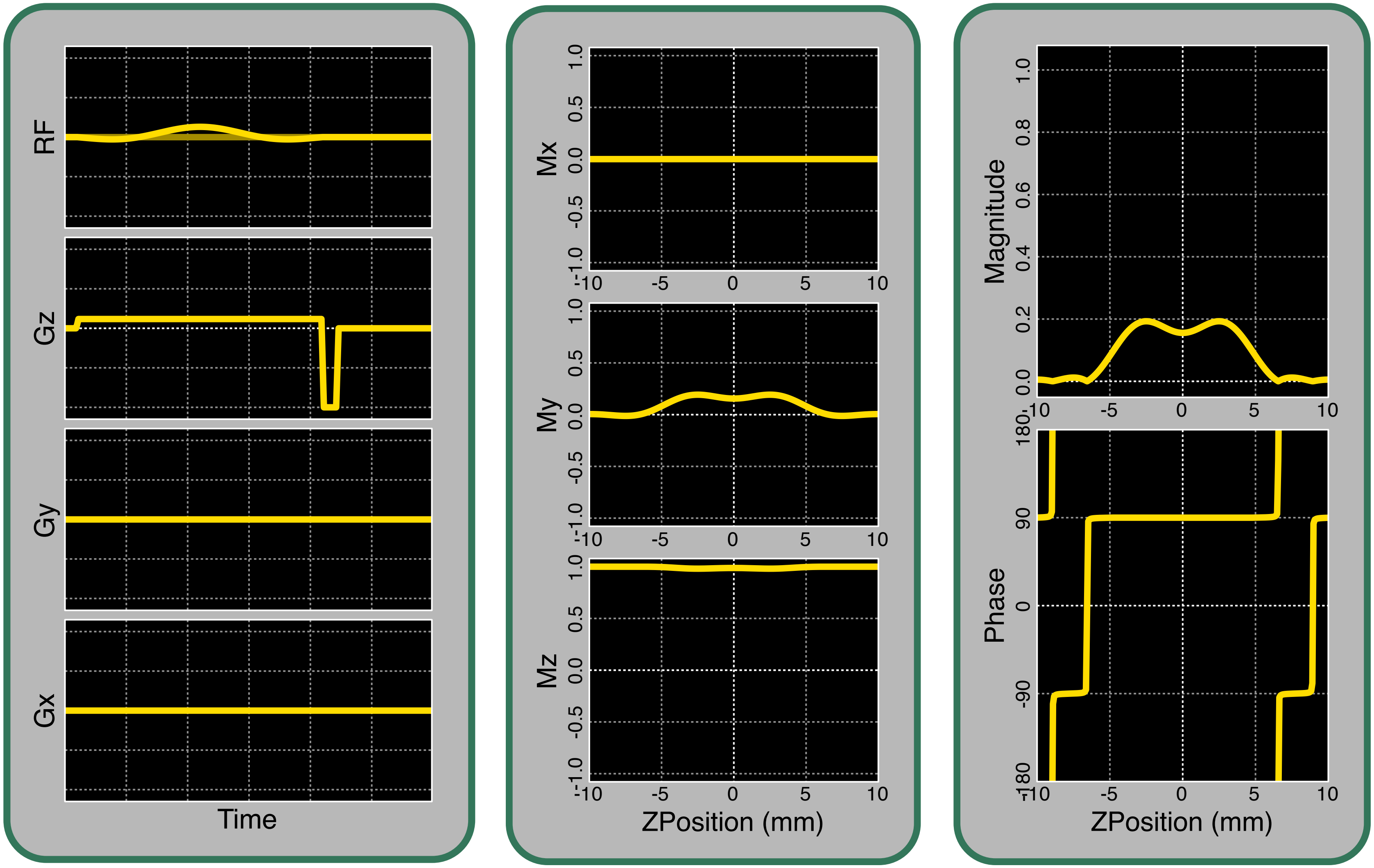
Design $G(t), B_1(t)$ to move spins at positions $\vec{r}$ to desired position

SLR

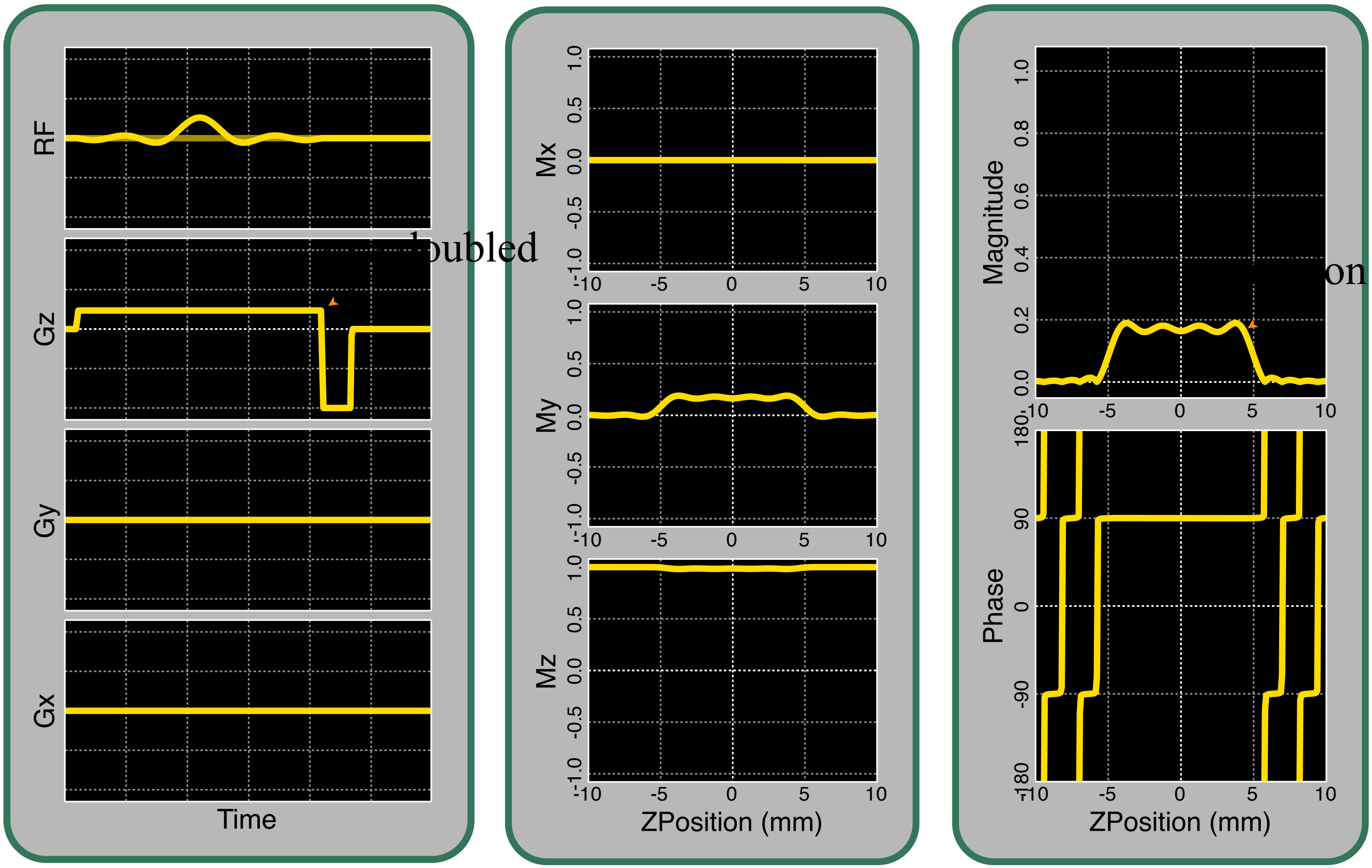


GOOD THINGS HAPPEN!

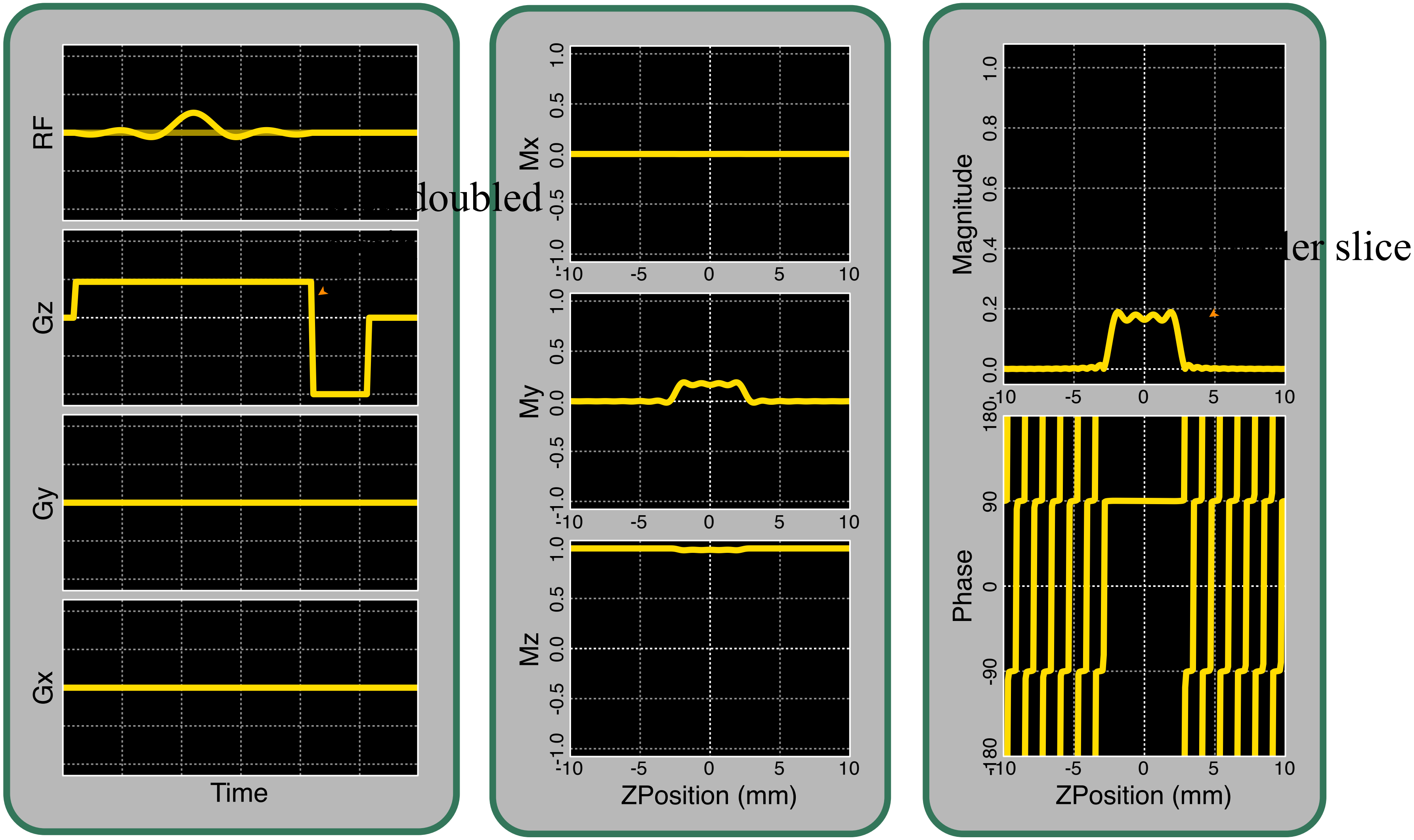
TBW=4, flip = 10, slice = 10mm, duration =2ms



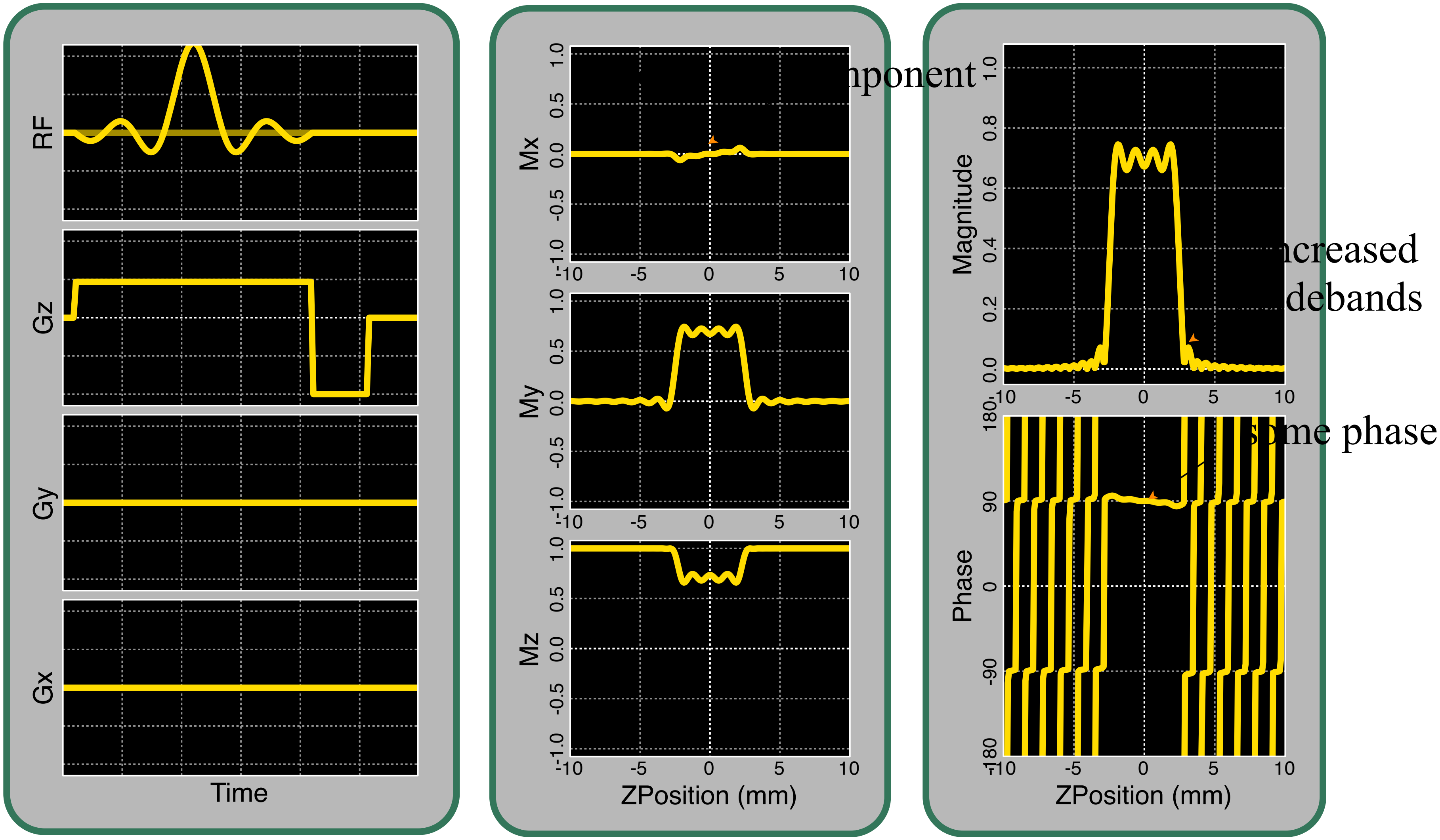
TBW=8, flip = 10, slice = 10mm, duration =2ms



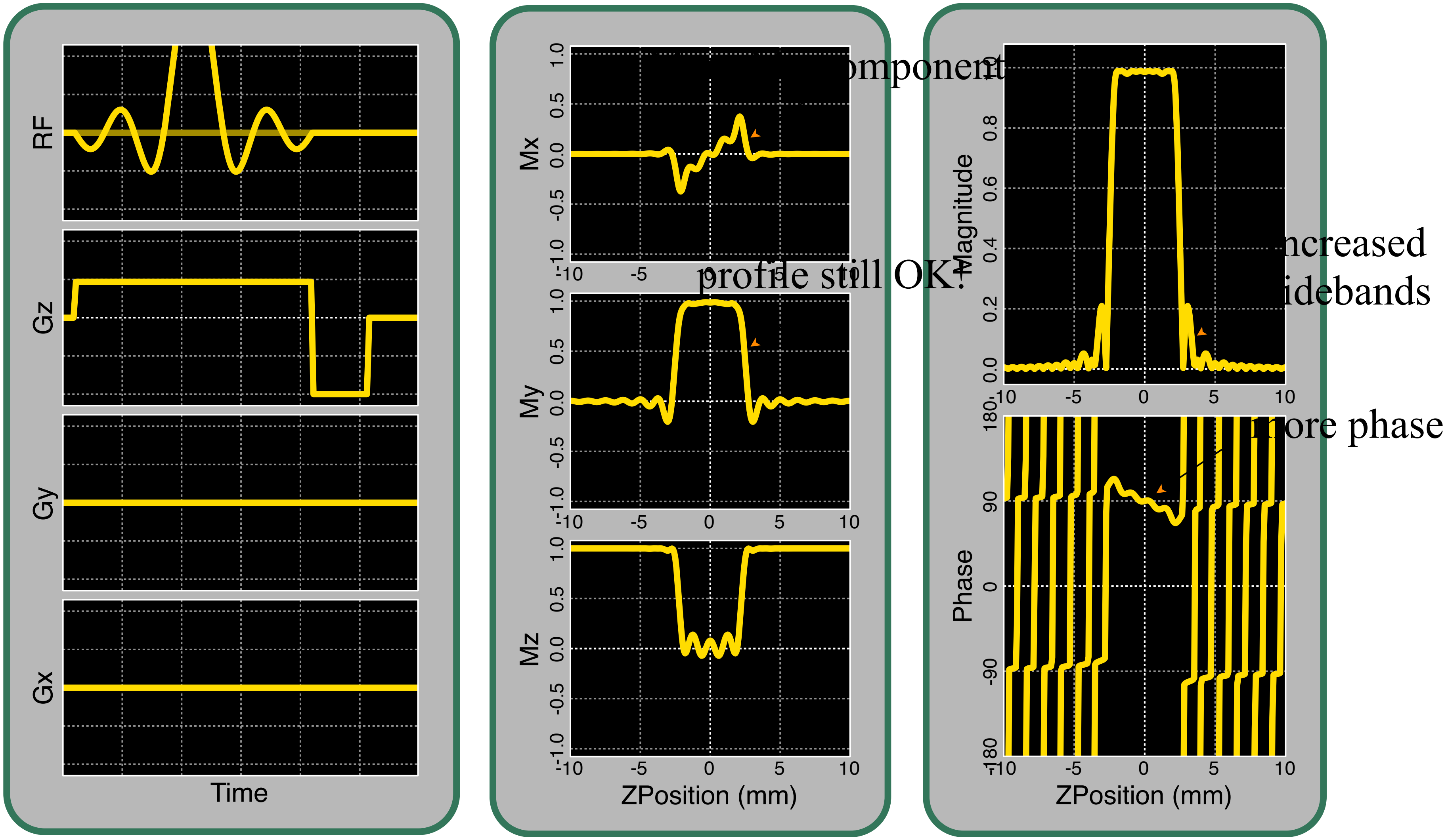
TBW=8, flip = 10, slice = 5mm, duration =2ms



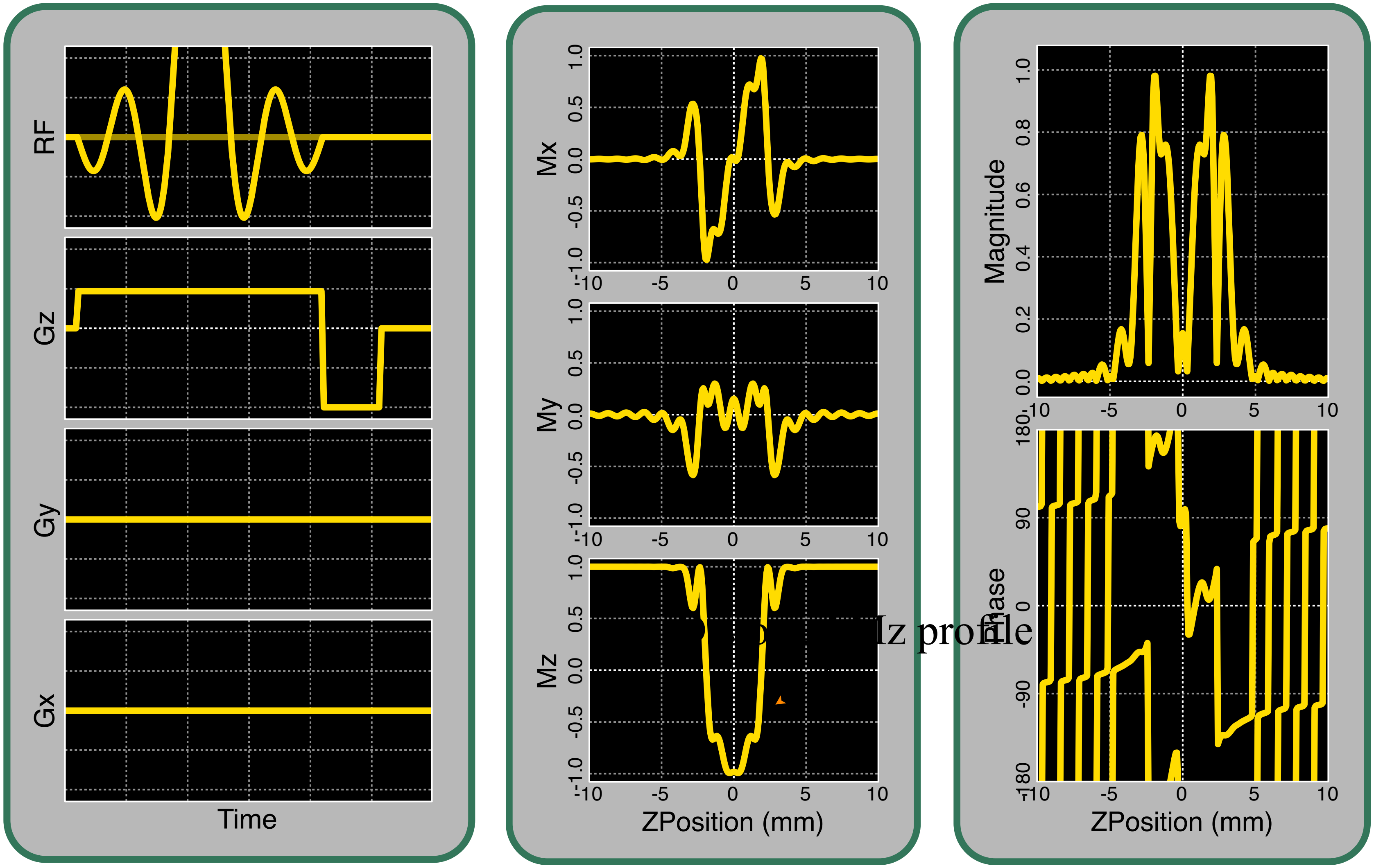
TBW=8, flip = 45, slice = 5mm, duration =2ms



TBW=8, flip = 90, slice = 5mm, duration =2ms

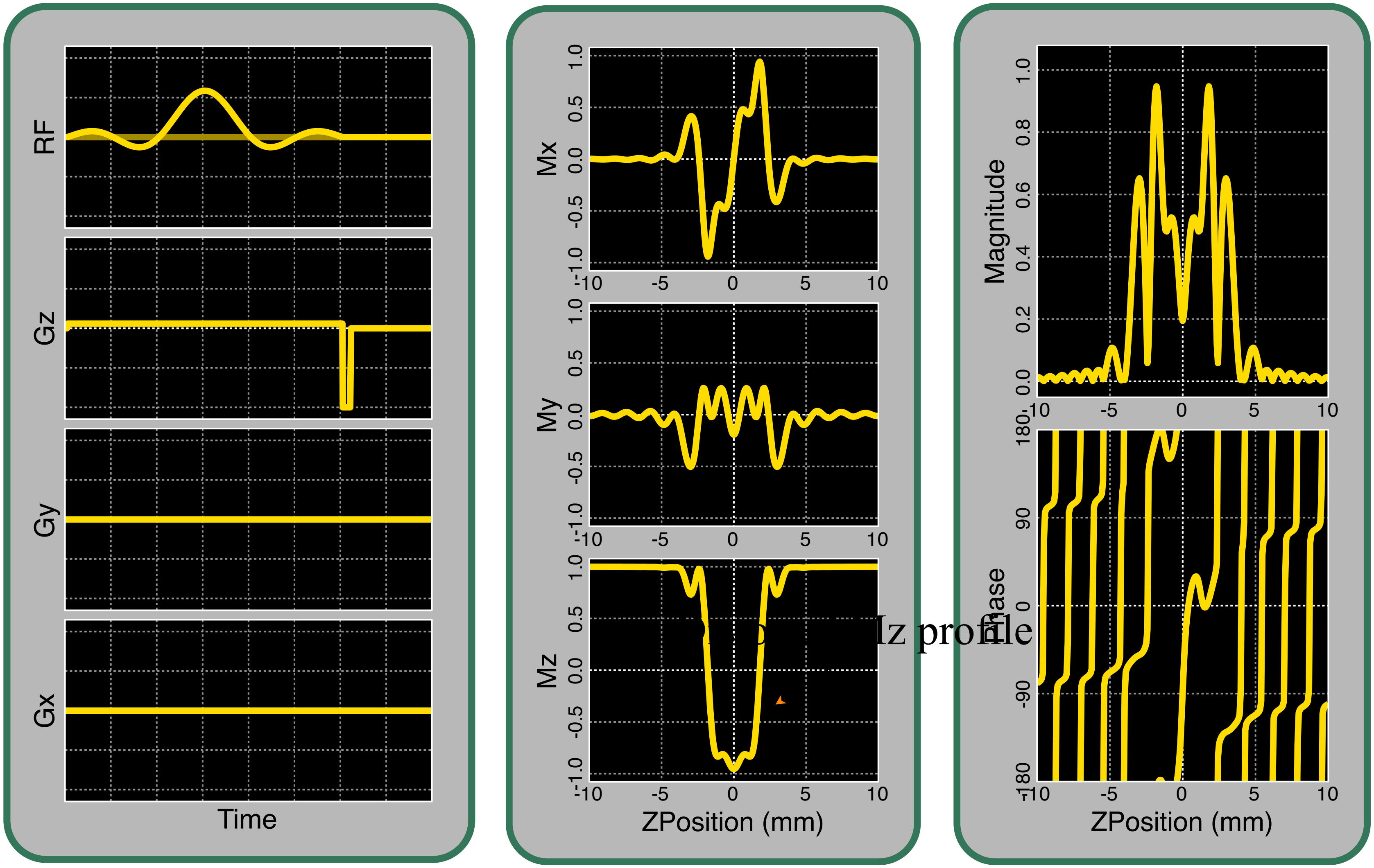


TBW=8, flip = 180, slice = 5mm, duration =2ms



180

small-tip Pulse

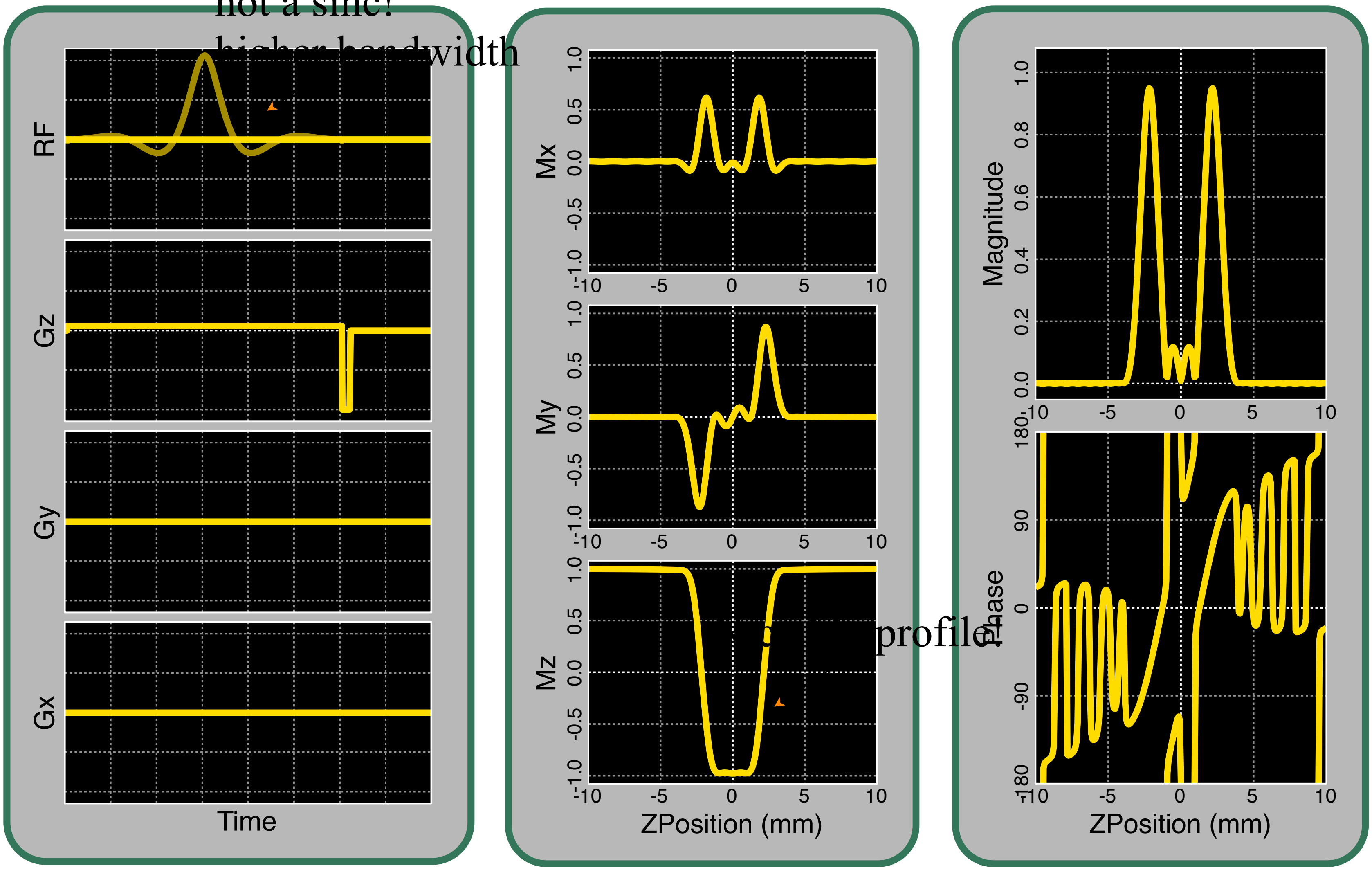


180

SLR Pulse

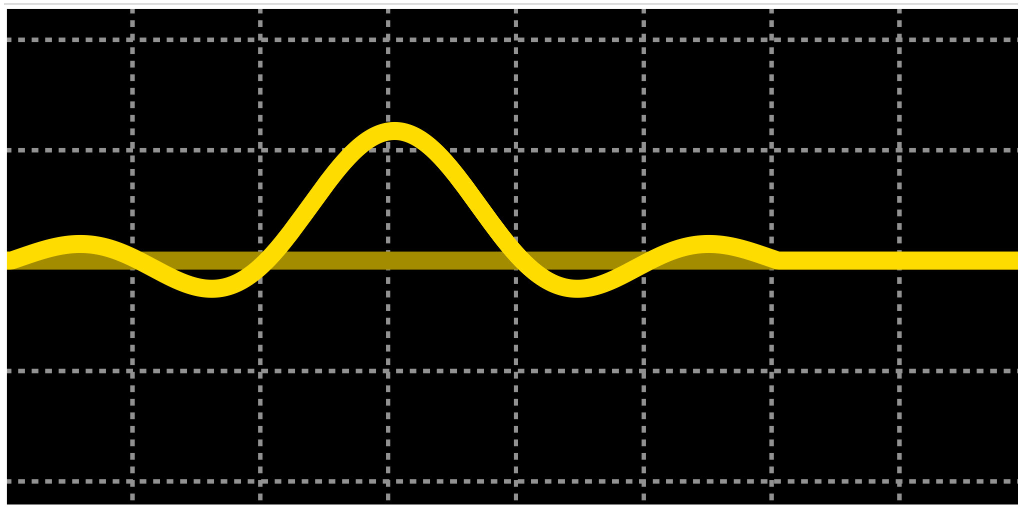
not a sinc!

higher bandwidth

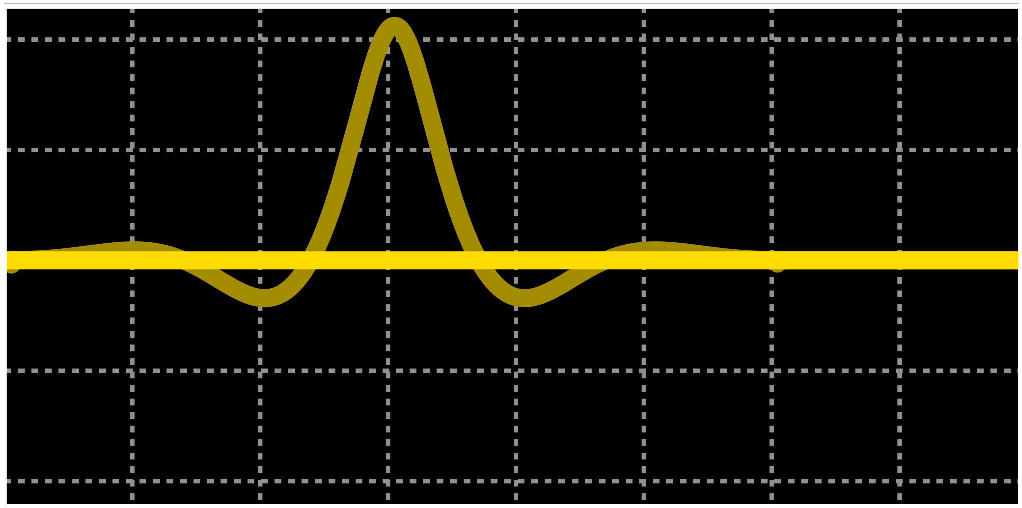


profile!

“small-tip” 180



SLR 180



Spectral-Spatial Pulse