Next:

- Practical Issues in MRI
- $T_2$ decay
  - Map decay to k-space
  - result in artifacts
    - Image weighting
    - blurring in the readout
- Off-resonance
Effect of T2 Decay on Imaging

\[ s(t) = \int_{\tilde{R}} M_{xy}(\tilde{r}, 0) e^{-\frac{t}{T_2}} e^{-i2\pi \tilde{k}(t) \cdot \tilde{r}} d\tilde{r} \]

- Signal decays along k-space trajectory

Approximation: Mxy created mid-RF Decays with T2 after
Effect of T2 Decay on Imaging

\[ k_x(t) = \frac{\gamma}{2\pi} G_x(t - TE) \Rightarrow t = \frac{k_x}{\frac{\gamma}{2\pi} G_x} + TE \]

\[ e^{-\frac{t}{T_2}} = e^{-\frac{TE}{T_2}} e^{-\frac{k_x}{\frac{\gamma}{2\pi} G_x T_2}} \]
Effect of T2 Decay on Imaging

- Two Effects:
  - Signal Loss by $e^{-\frac{TE}{T_2}}$ (T2 Weighting)
  - Apodization by $e^{-\frac{k_x}{2\pi G_x T_2}}$
    - Blurring in Image Domain (readout direction)
    - Usually minor effect
    - Reduced by increasing $G_x$

\[
e^{-\frac{t}{T_2}} = e^{-\frac{TE}{T_2}} e^{-\frac{k_x}{2\pi G_x T_2}}
\]
Point Spread Function

Decay

$T_2 = 20\text{ms}$

$T_2 = 5\text{ms}$

$T = 10\text{ms}$

PSF

$T_2 = 50\text{ms}$

$T_2 = 20\text{ms}$

$T_2 = 5\text{ms}$

$TE = 10\text{ms}$, $T_{\text{readout}} = 20\text{ms}$

$T_2 = 5\text{ms}$

$T_2 = 20\text{ms}$

$T_2 = 100\text{ms}$
Effect of T2 Decay on Imaging

- Two effects:
  - Signal loss by \( \exp(-TE/T_2) \) (T2 weighting)
  - Apodization - causes blurring in readout

\[ T_2 = 5\text{ms}, \quad T_2 = 20\text{ms}, \quad T_2 = 100\text{ms} \]

\[ \text{TE}=10\text{ms}, \quad T_{\text{readout}}=20\text{ms} \]
Off - Resonance

• So far, assumed $B_0$ constant. But $B_0$ varies due to:
  – Main field inhomogeneity ($\sim 1$ppm)
  – Object magnetic susceptibility
  – Chemical shift
Main Field Inhomogeneity

• Magnet is designed to be homogeneous over a (spherical) volume

• Typical numbers:
  – Bare magnet ~10-100 ppm
  – Shimmed magnet ~1 ppm
    @3T 1ppm is 127Hz

• Generally, main magnet inhomogeneity is not a limitation
Object Susceptibility

- Most biological objects perturb the field
  \[ \Delta B_{z,\text{tissue}} \approx \chi B_{0,\text{freespace}} \]

- \( \chi \) is the magnetic susceptibility

- Larmor frequency lower in tissue than air
  \( \chi_{\text{water}} = -9.05 \text{ppm w.r.t free-space} \)
  \( \chi_{\text{air}} = 0.36 \text{ ppm w.r.t free-space} \)

for a sphere:

\[ \Delta B_z = \frac{\Delta \chi B_0}{3} \left( \frac{a}{r} \right)^3 (3 \cos^2 \theta - 1) \]
Object Susceptibility

• Complex behavior at boundaries.
  – Depends on $\Delta\chi$ and geometry
  – Typical $\Delta B_0 \pm 3$ ppm ($-12 < \chi < -6$)
Macroscopic Effect

- Problem areas:
  - Brain above sinuses, auditory canals
  - Heart surrounded by lungs
  - Abdomen

Field inhomogeneity brain@1.5T
Macroscopic Effects - Metal Artifacts

(a) Depicts a simulated dipole pattern of metal-induced field inhomogeneities from a cylindrical object, where the sample field inhomogeneities along the $x$-axis (solid line) and along the $z$-axis (dashed line) are plotted in (b). When the field inhomogeneities superimpose upon the frequency induced by the slice-select gradient, the resulting excitation profiles correspond to the frequency bands shown in (c). The bottom three sample distorted excitation profiles in (d, e, f) show that the spins are excited when their precession frequencies fall into the frequency bands (highlighted in gray in (c)) corresponding to the excited slices. As the excited slices contain spins from different slice locations, the distorted excitation profiles lead to through-plane distortions.
Macroscopic Effects - Metal Artifacts

metal artifact

corrected (SEMAC)

- $\chi_{\text{titanium}} = 182$
- $\chi_{\text{stainless steal (nonmagnetic)}} = 3520-6700$


M. Lustig, EECS UC Berkeley
Microscopic Effects

• Lungs:
  – Approx: 1/6 tissue, 5/6 air

• Result:
  – Distribution of field/frequencies

  – Tells a lot about microstructure of tissue
Blood

- Water $\chi = -9.05$
- Hemoglobin molecule (deoxy) $\chi = 0.15$
- Red Blood cells (deoxy) $\chi = -6.52$
- Deoxy blood $\chi = -8.77$
- Oxy blood $\chi = -9.05$


+ John F. Schenck: Review article: Role of magnetic susceptibility in MRI
Chemical Shift

- Protons in complex molecules are “shimmed” by adjacent spins and electrons

\[ B_{cs} = B_0 (1 - \sigma) \]

- \( \sigma \) is a shielding constant - depends on molecular structure

- Example: Lipids
Chemical Shift

- Example: Lipids

\[
\begin{align*}
\text{H} & \quad \text{H} & \quad \text{H} & \quad \text{H} & \quad \text{H} & \quad \text{H} & \quad \text{H} \\
\text{C} & \quad \text{C} & \quad \text{C} & \quad \text{C} & \quad \text{C} & \quad \text{C} & \quad \text{H} \\
\text{H} & \quad \text{H} & \quad \text{H} & \quad \text{H} & \quad \text{H} & \quad \text{H} & \quad \text{H}
\end{align*}
\]

Lipids from CH2 is 3.4ppm below water @3T ≈ 440 Hz

ppm

\begin{align*}
\text{CH} & \quad \text{H}_2\text{O} & \quad \text{CH}_2 & \quad \text{CH}_3 & \quad \text{TMS} \\
5.4 & \quad 4.7 & \quad 1.3 & \quad 0.9 & \quad 0
\end{align*}
Heterogeneous Tissue

• Tissue is a combination of
  – Chemical shift
  – Susceptibility
  – Geometry

• Results are complex
  – Interesting cases:
    • Blood (fMRI)
    • Lungs
    • Trabecular bone
    • Iron in brain / liver
Effect on Imaging

- Magnitude
- Phase
- Geometric
- Blurring

- All depend on spatial scale we look at, and acquisition strategy
Off Resonance Effect on FID

- Simple spectroscopy experiment
  - On resonance

\[ e^{-t/T_2} \]

Signal decays with \( T_2 \)
Off Resonance Effect on FID

• Simple spectroscopy experiment
  – With Susceptibility variation, FID is:

Why?
Off Resonance Effect on FID

- The field is:

\[ B(\vec{r}) = B_0 + E(\vec{r}) \]

uniform error-field

- Error is spatial and spectral
- In the rotating frame \( w_0 \)

\[ \vec{B} = E(\vec{r})\hat{k} \]

\[ \omega_E(\vec{r}) = \gamma E(\vec{r}) \]

\[ f_E(\vec{r}) = \frac{\gamma}{2\pi} E(\vec{r}) \]
Off Resonance Effect on FID

\[ m_{x,y}(\vec{r}, t) = m_{x,y}(\vec{r}, 0) e^{-i\omega E(\vec{r})t} e^{-\frac{t}{T_2(\vec{r})}} \]

- Received signal is:

\[ s(t) = \int_{\vec{R}} m_{x,y}(\vec{r}, 0) e^{-i\omega E(\vec{r})t} e^{-\frac{t}{T_2(\vec{r})}} d\vec{r} \]

- with time, phase dispersion causes signal cancellation and loss.

magnetization \hspace{1cm} \text{dephasing} \hspace{1cm} \text{relaxation}
Off - Resonance Effects on FID
Off Resonance Effects on FID

\[ e^{-\frac{t}{T_2}} \]

\[ e^{-\frac{t}{\gamma B_0}} \]

INHOMOGENEOUS \( B_0 \)

GRADIENTS

"Signal from entire volume"
Off Resonance Effects on FID

\[ \text{Gradients are very strong inhomogeneity!} \]

\[ \text{Inhomogeneity } \Rightarrow \text{SIGNAL LOSS KNOWN AS } T_2^* \]

\[ \text{Approximate as exponential.} \]

\[ s(t) = \int_{0}^{t} \text{m}_y(t) e^{-\frac{t}{T_2^*}} e^{-\frac{t}{T_2}} \text{d}t \]

\[ \int_{\text{voxel}} \text{m}_y(t) \text{d}v \]

\[ \frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'} \]

\[ \text{RELAX} \quad \downarrow \quad \text{DEPHASE} \]

\[ R_* = R_2 + R_2' \]
Off Resonance Effects on FID

- T2*: Not a good model for Large-Scale variations (dephasing near sinuses)
- T2*: Is a good model for small scale distributed variations (dephasing near capillaries)
Twinkle, Twinkle, T$_2^*$

T$_2^*$ ("tee-two-star"): time constant for the decay of signals measured by NMR

T$_2^*$ depends on . . .
- random spin-spin interactions (which govern T$_2$)
- inhomogeneities in the magnetic field (which make T$_2^*$ shorter than T$_2$)
Example fMRI
Off-Resonance Effects on Imaging

The $x$-verse magnetization is:

$$m_{xy}(\vec{r}, t) = m_{xy}(\vec{r}, 0) e^{-i \omega_E(\vec{r}) t} e^{-\frac{t}{T_2(\vec{r})}} e^{-i 2 \pi k_x(t) x} d\vec{r}$$
Off-Resonance Effects on Imaging

neglecting $T_2$ and substituting for $t$:

$$m_{xy}(\vec{r}, t) = m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r}) \left( \frac{k_x(t)}{2\pi G_x} + TE \right)} e^{-i2\pi k_x(t)x} d\vec{r}$$

Or,

$$m_{xy}(\vec{r}, t) = m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r}) TE} e^{-i2\pi k_x(t) \left( x + \frac{\omega_E(\vec{r})}{\gamma G_x} \right)} d\vec{r}$$

phase/dephasing  displacement
Off-Resonance Effects on Imaging

\[ m_{xy}(\vec{r}', t) = m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r}) T E} e^{-i 2\pi k_x(t) \left(x + \frac{\omega_E(\vec{r})}{\gamma G_x}\right)} d\vec{r} \]

phase/dephasing  
displacement

An on-resonance spin at position:

\[ x' = x + \frac{\omega_E(\vec{r})}{\gamma G_x} \]

Produces the same signal as a spin at x, with off-resonance!
Examples:

- shape
- signal loss
- readout
- direction?
Examples: Phase vs Echo Time

- water and lipids in phase
- water and lipids out-of-phase
Example: Chemical Shift

\[ x' = x + \frac{\omega_E(\vec{r})}{\gamma G_x} \]

Increasing \( G_x \) reduces chemical shift artifact
Example:

Let,

\[ \frac{\gamma}{2\pi} G_x = 1 \text{ KHz/cm} \]

\[ E(\vec{r}) = 3 \text{ ppm} \]

\[ \omega_E(\vec{r}) = 2\pi \cdot 200 \text{ Hz} \]

\[ \Delta x = x' - x = \frac{\omega_E(\vec{r})}{\gamma G_x} = \frac{2\pi \cdot 200 \text{ Hz}}{2\pi \cdot 1 \text{ KHz/cm}} = 0.2 \text{cm} \]

At:

\[ \delta_x = 1 \text{ mm} \quad \text{this is a shift of two pixels!} \]
Effects in Spin-Warp

- **Off-Resonance**
  - Modest spatial distortions (few pixels)
  - Relatively benign artifacts
  - Reduce artifacts with large Gx

- **Chemical-Shift**
  - Fat shift of -220Hz @ 1.5T
  - Fat image is displaced from Water
  - In practice F/W shift limited to ~2pixels

2 pixels shift are two cycles of linear phase across k-space

\[
\frac{2 \text{ cyc}}{220 \text{ Hz}} \approx 9.1 \text{ ms}
\]
Off-Resonance in EPI
Example: Lipids

Phase accumulation

0.44cyc ⇔ 1ms

1ms ⇒ 0.44cyc

0.44cyc ⇔ 1ms

1ms ⇒ 0.44cyc

96ms ⇒ 42.24cyc

Point-spread function
Off-Resonance in EPI

- Point-spread function
- Simulation
- Leg
Off-Resonance in Spiral

phase accumulation

point-spread function

2 ms $\Rightarrow$ 0.88cyc
Off-Resonance in Spiral

Readout time @3T:

- 2ms
- 5ms
- 13ms

Spiral scan with linear off-resonance
Echoes

• Early in NMR people noticed the following odd behavior (Hann, 1950)

Two 90° excitations, separated by T cause large signal to form at 2T, Why?
Spin Echo Pulse Sequence

$\mathbf{RF}$

$0^-$  $0^+$  $T_2^*$  $2T$

$\tau^-$  $\tau^+$

$90$  $180$

$?$
Spin Echo Pulse Sequence

Any signal loss due to dephasing is recovered at the spin-echo.

If $wE(r,t)$ changes over time, refocusing is not perfect.

Provides probe to measure molecular motion.
Multi-Spin Echo - CPMG
Non uniform dephasing

- Any signal due to dephasing $\omega_E(r)t$ is refocussed.
- If $\omega_E(r,t)$ refocussing not perfect.

$T_2^\ast$ Huge!

but, constant dephasing shows as $T_2$
T2(T)

Large vessels

small vessels
Low Flip Angle Spin Echoes

- Any pulse can produce a spin-echo
  - 180 maximally refocuses magnetization
  - Lower refocusing power with lower flip angle
  - 90 refocusses 1/2 (see homework)
Low Flip Angle Spin Echoes

• Refocusing RF result can be decomposed to several components:
  – Pass through
  – Refocused
  – Parasitic excitation
  – Magnetization stored in Mz
Low Flip Angle Spin Echoes

\[ M = R_z(\omega_E \tau) R_x(\alpha) R_z(\omega_E \tau) R_x(\alpha) M_0 \]

For 90 degrees:

\[ M_{xy} = \frac{1}{2} \sin(\omega_E 2\tau) M_0 - i \sin^2(\omega_E \tau) M_0 \]

In general.... I think

\[ M_{xy} = A_1 \cos(2\omega_E \tau) + A_2 \sin(2\omega_E \tau) + B_1 \cos^2(\omega_E \tau) + B_2 \sin^2(\omega_E \tau) + C_1 \cos(\omega_E \tau) + C_2 \sin(\omega_E \tau) \]

\[ \text{passthrough} \quad \text{refocused} \quad \text{parasitic} \]
Phase Graphs

- Useful to find echo formations

\[
\alpha \quad \alpha \\
\text{passthrough} \quad \text{parasitic excitation} \\
\text{refocused} \quad \text{echo} \\
\text{longitudinal magn.}
\]
Phase Graphs

Figure 1: Phase graph showing the splitting of transverse magnetization into 3 parts by an RF-pulse. A part of the magnetization is converted to z-magnetization, which is modulated along ω and which can be retrieved by a later α-pulse to form a spin echo ('180°-pulse').

3. A part of the magnetization is converted to z-magnetization, which is modulated along ω and which can be retrieved by a later α-pulse to form a spin echo ('180°-pulse').

Such a phase graph is very practical to predict the time points of echo formation after a series of non-equidistant pulses. For a CPMG-sequence (or other pulse sequences with regular pulse spacing [7]) it has also been used to calculate the signal amplitudes by superposition of all different refocusing pathways leading to echo formation. This approach to calculate the echo amplitude by a sum over all pathways has been called the partition method [7]. The number of refocusing pathways increases extremely rapidly with the echo number: After the Nth pulse 3N − 1 echo formation pathways occur [9], as demonstrated in Fig. 2.

Analytical solutions based on the partition method have been published only up to the fourth echo [8]. The calculation of echo amplitudes for typical echo train lengths used in RARE sequences is feasible by computers, however, rather unhandy.

1.3 Extended Phase Graphs

Figure 2: Repetitive splitting of the phase graph as shown in Fig. 1 and described in the text. In total 3N − 1 echo formation pathways occur.

The extended phase graph (EPG) - algorithm [9,10] was devised to create a mathematical framework, by which the ease of qualitative insight into the refocusing pathways offered by the phase graph is transformed into a straightforward algorithm for 3.* from document by Matthias Weigel
Extended Phase Graphs

• Each component has different magnitude and phase
• Echo consist of contribution from many pathways
• Can track to calculate magnitude & Phase of echo (Beyond scope... see document by Matthias Weigel)
Spin Echo Imaging

- Spin echo negates phase -- Conjugation
- How to incorporate in pulse sequence?

\[ M_{xy}(\vec{r}, t) = M_{xy}(\vec{r}, 0) e^{-i2\pi(k_x(t)x + k_y(t)y)} \]
Spin Echo Imaging

- Spin echo negates phase -- Conjugation
- How to incorporate in pulse sequence?
REGULAR HOFT GRADIENT ECHOS
NEGATE GRADIENT LEFT OF 180

TRIVIAL ADAPTATION.

MINIMUM MOTION ARTIFACTS.

WANT GRADIENT ECHO TO HAPPEN WITH SPIN-ECHO.
imperfect 180 causes parasitic which is not phase encoded (HW)
MULTI-SLICE

SIMPLE B-WEIGHTS SPIN-ECHU.

\[ \text{May} (\text{no}) e^{-\frac{t}{T}} \]

FNS MILLISECONDS

SECONDS
INEFFICIENT!  ADD MULTISLICE MAKE NO SELECTIVE

WHEN IMAGING ONE SLICE, OTHER RECOVER!

VERY EFFICIENT ALWAYS ACQUIRING DATA.
what's the phase graph for all pathways?
Fast/Turbo Spin-Echo (RARE)

Hennig ‘86
figure from Bernstein, King and Zhou
Other non-idealities

• RF Inhomogeneity
  – Affects Excitation
    • Spatially varying flip-angle
    • Spatially varying phase
    • Can cause problem for fat saturation
  – Worse @ high field (wave effects)

– Affects receive
RF excitation inhomogeneity
AFFECTS RECEIVE:

\[ \mathcal{S}(\hat{\mathcal{R}}) = \int c(\hat{\mathcal{R}}, \hat{\mathcal{H}}) m(\hat{\mathcal{R}}, \hat{\mathcal{H}}) e^{-i\mathcal{H}(\hat{\mathcal{R}} \cdot \hat{\mathcal{H}})} d\mathcal{R} \]
CAN BE A GOOD THING: SENSITIVITY ENCODING.

\[
\psi_1(\mathbf{b}) = \int \frac{m(\mathbf{r}_0)}{\mathbf{r}} C_1(\mathbf{r}) e^{-i 2 \pi (\mathbf{b} \cdot \mathbf{r})} d\mathbf{r}
\]

\[
\psi_2(\mathbf{b}) = \int \frac{m(\mathbf{r}_0)}{\mathbf{r}} C_2(\mathbf{r}) e^{-i 2 \pi (\mathbf{b} \cdot \mathbf{r})} d\mathbf{r}
\]

To reconstruct \( m_1 \) & \( m_2 \) need \( n \) encodings

but to reconstruct \( m \) need (maybe) \( \frac{n}{2} \) encodings.

More later!
Gradient non-idealities

GRADIENTS

(-) Non-Linearity: Gradient Roles Off

\[ z \uparrow [\] \Rightarrow (\) \]

Easy to correct by interpolation

For \( z \uparrow \) results
Concomitant Gradient (Maxwell Terms)

- **Concomitant Gradients**

  Ideal gradients violate Maxwell Eqn.

  For cylindrical coils used in MRI

\[
\mathbf{B} = B_0 + \mathbf{G}_x x + \mathbf{G}_y y + \mathbf{G}_z z + \frac{1}{4B_0} \left[ \frac{G_x^2}{2} (x^2 + y^2) + \left(G_x^2 + G_y^2\right) z^2 - G_x G_z x - G_y G_z y \right]
\]

The main gradients

**Example:** At \( z = 20 \text{ cm} \), \( G_x = 40 \text{ mT/m} \); \( B_0 = 1.5 \text{ T} \)
Concomitant Gradient (Maxwell Terms)

\( \text{c) Concomitant Gradients} \)

Ideal Gradients Violate Maxwell Eqn.

For cylindrical coils used in MRI

\[
B = B_0 + 6_x x + 6_y y + 6_z z + \frac{1}{4\pi} \left[ \frac{G_x^2 (x^2 + y^2)^2 + (6_x + 6_y)^2 z^2}{4 B_0} \right]
\]

\[ \text{main gradients} \]

Example: At \( B = 20 \text{cm} \), \( 6_x = 40 \text{ mT/m} \), \( B_0 = 1.5 \text{T} \)

\[
\frac{G_x^2 z^2}{4 B_0} = \frac{(40 \times 10^{-3} \times 0.2)^2}{2 \times 1.5} = 2.13 \times 10^{-5} \text{T} = 14.2 \text{ ppm}
\]

\( G = 10 \text{ mT} \Rightarrow 0.969 \text{ ppm} \)

\( B_0 = 0.7 \text{T} \Rightarrow 65.2 \text{ ppm} \)
Concomitant Gradients

\[ \frac{G_x^2 z^2}{2 B_0} \]

*FIG. 3. Sagittal spiral scans acquired at isocenter (z/H_11005 = 0) with (a) \( g_m = 2.1 \text{ G/cm} \), and (b) \( g_m = 1.11 \text{ G/cm} \), using field strength \( B_0 = 1.5 \text{ T} \). Scan prescriptions are (a) 8 interleaves, 4096 readout points, 125 kHz bandwidth, 4 NEX; and (b) 16 interleaves, 2048 readout points, 64 kHz bandwidth, 2 NEX. Both (a) and (b) use spin echo, 27-cm field of view, TE = 15, TR = 2000, 1-cm thick slice. Spatial resolution, SNR, and off-resonance blurring are approximately the same for both cases. Arrows show areas of blurring differences.*

*FIG. 2. Axial spiral scans acquired at slice locations \( z/H_11005 \) and \( z/H_11005 + 10 \text{ cm} \) with (a) \( g_m = 2.1 \text{ G/cm} \), and (b) \( g_m = 0.524 \text{ G/cm} \), using field strength \( B_0 = 1.5 \text{ T} \). Scan prescriptions are (a) 8 interleaves, 4096 readout points, 62.5 kHz bandwidth, 4 NEX; and (b) 32 interleaves, 1024 readout points, 16 kHz bandwidth, 1 NEX. Both (a) and (b) use spin echo, 14-cm field of view, TE = 15, TR = 2000, 1-cm slice. Spatial resolution, SNR, and off-resonance blurring are approximately the same for all cases.*

king et. al, MRM 41:103-112(1999)
@z=10cm

king et. al, MRM 41:103-112(1999)

\[ g_m = 2.1 \text{ G/cm} \quad \text{and} \quad g_m = 0.524 \text{ G/cm} \]
Even finite-rise-time rate! (often ignored)
Eddy Currents

Generated by electric field from changing magnetic flux, build up in time varying gradients and decay in constant & SLEW rate.
\[ B_e(\mathbf{r}, t) = b_0(\mathbf{r}) + \mathbf{r} \cdot \mathbf{q}(\mathbf{r}) + \ldots \]
Eddy Currents

g(t) = -\frac{\partial b}{\partial t} \otimes e(t)

e(t) = H(t) \geq \alpha n e^{-\frac{t}{\tau}}

\uparrow \quad \uparrow \quad \uparrow

step function \quad \text{intensity} \quad \text{time constant}
Pre-emphasis

\[ \beta_0 \rightarrow \text{chunq raster phase} \]

\[ \text{gradient} \]

For short time constants

\[ G_{net}(t) = G_{applied} (1 - e^{-\lambda t}) \]

\[ \text{Benstein} \quad \text{ch. 10.3} \]