

Principles of MRI

EE225E / BIO265

Chapter 07 Imaging Considerations

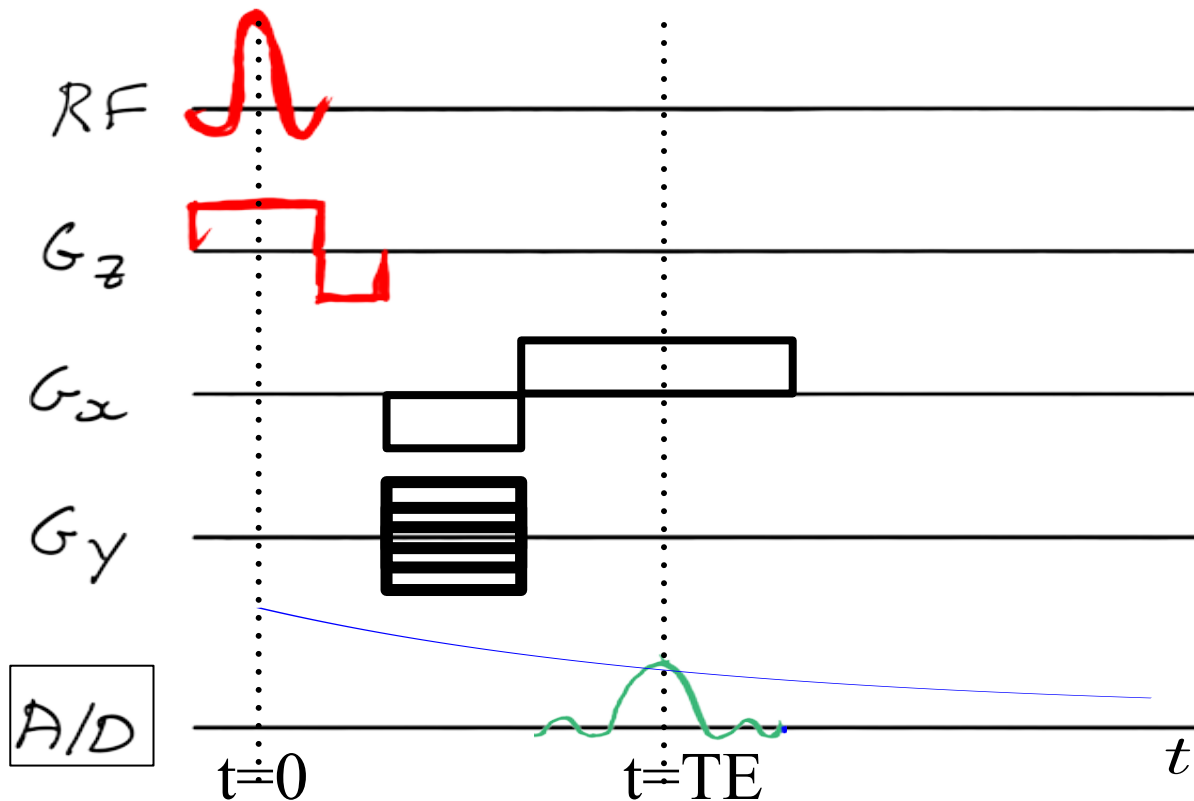
Next:

- Practical Issues in MRI
- T_2 decay
 - Map decay to k-space
 - result in artifacts
 - Image weighting
 - blurring in the readout
- Off-resonance

Effect of T2 Decay on Imaging

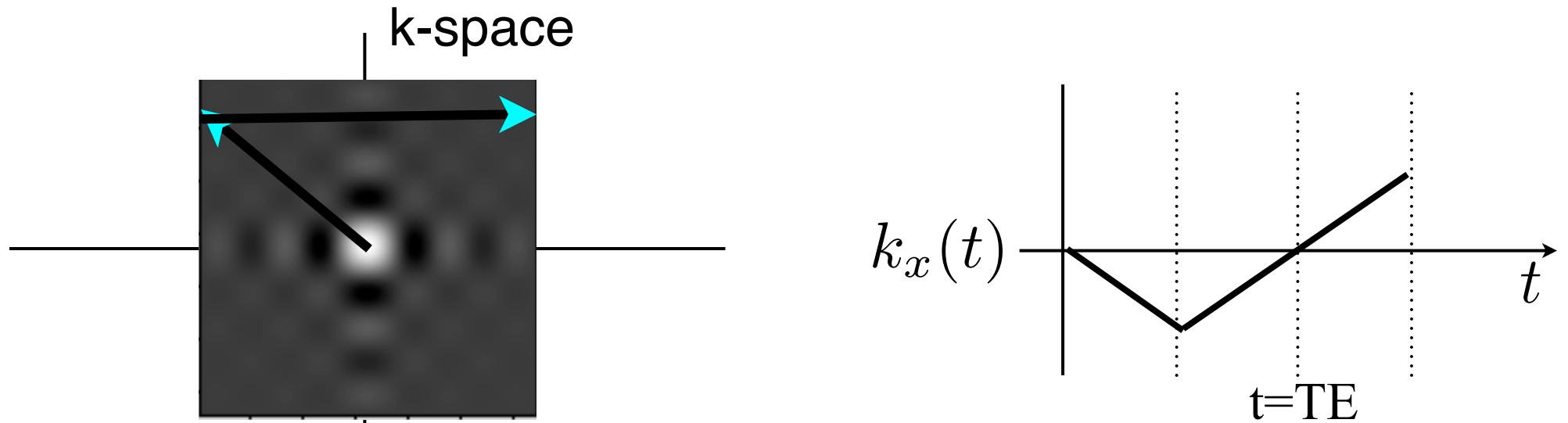
$$s(t) = \int_{\vec{R}} M_{xy}(\vec{r}, 0) e^{-\frac{t}{T_2}} e^{-i2\pi\vec{k}(t)\cdot\vec{r}} d\vec{r}$$

- Signal decays along k-space trajectory



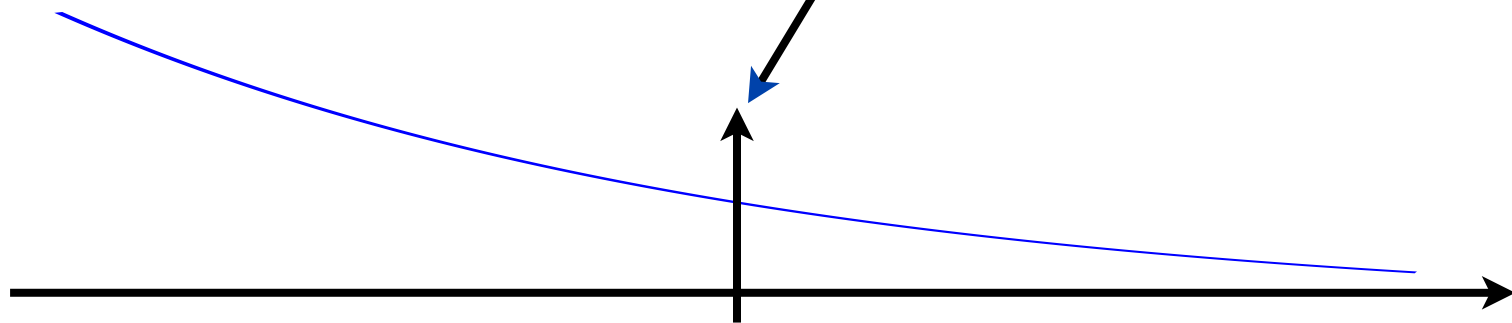
Approximation:
Mxy created mid-RF
Decays with T_2 after

Effect of T2 Decay on Imaging



$$k_x = \frac{\gamma}{2\pi} G_x (t - TE) \Rightarrow t = \frac{k_x}{\frac{\gamma}{2\pi} G_x} + TE$$

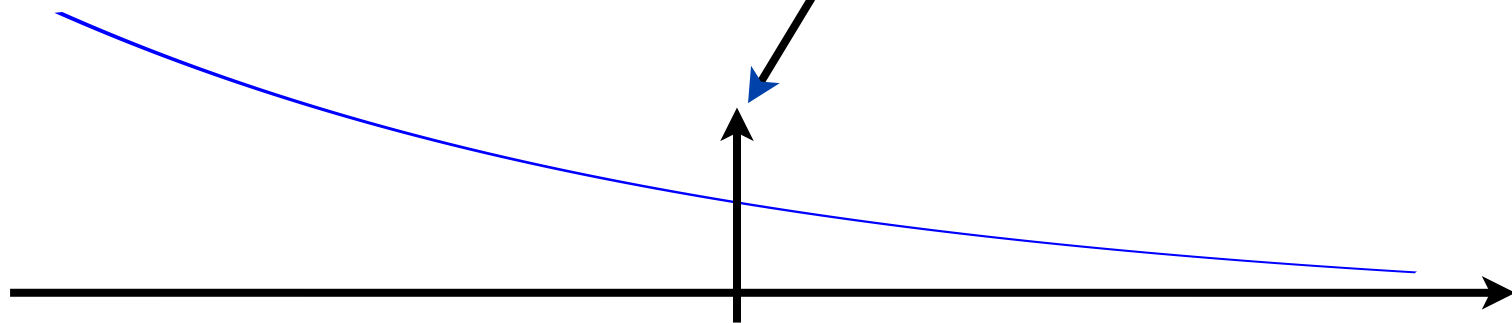
$$e^{-\frac{t}{T_2}} = e^{-\frac{TE}{T_2}} e^{-\frac{\frac{k_x}{\frac{\gamma}{2\pi} G_x} T_2}{T_2}}$$



Effect of T2 Decay on Imaging

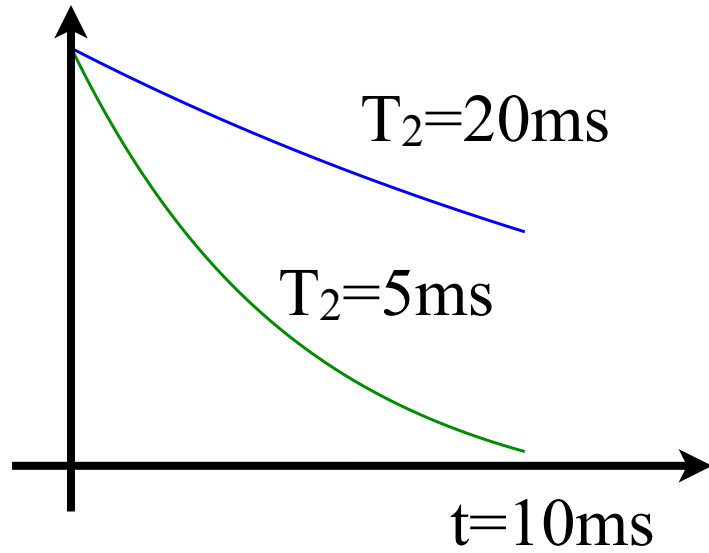
- Two Effects:
 - Signal Loss by e^{-TE/T_2} (T_2 Weighting)
 - Apodization by $e^{-\frac{k_x}{2\pi} G_x T_2}$
 - Blurring in Image Domain (readout direction)
 - Usually minor effect
 - Reduced by increasing G_x

$$e^{-\frac{t}{T_2}} = e^{-\frac{TE}{T_2}} e^{-\frac{k_x}{2\pi} G_x T_2}$$

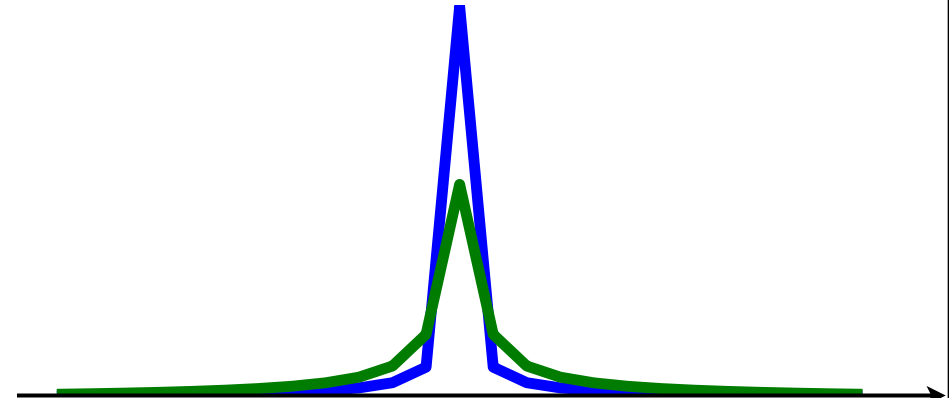


Point Spread Function

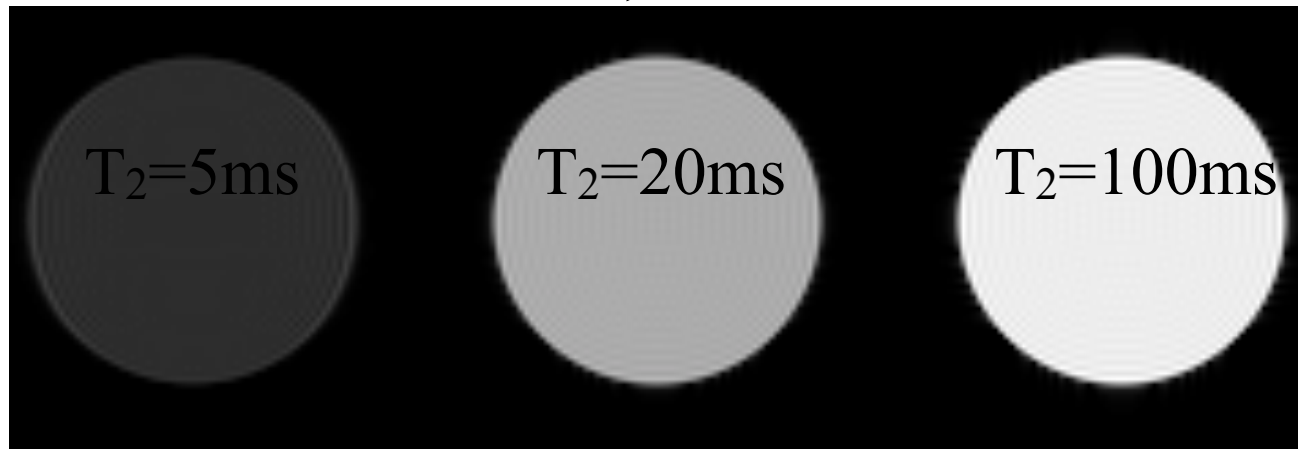
Decay



PSF



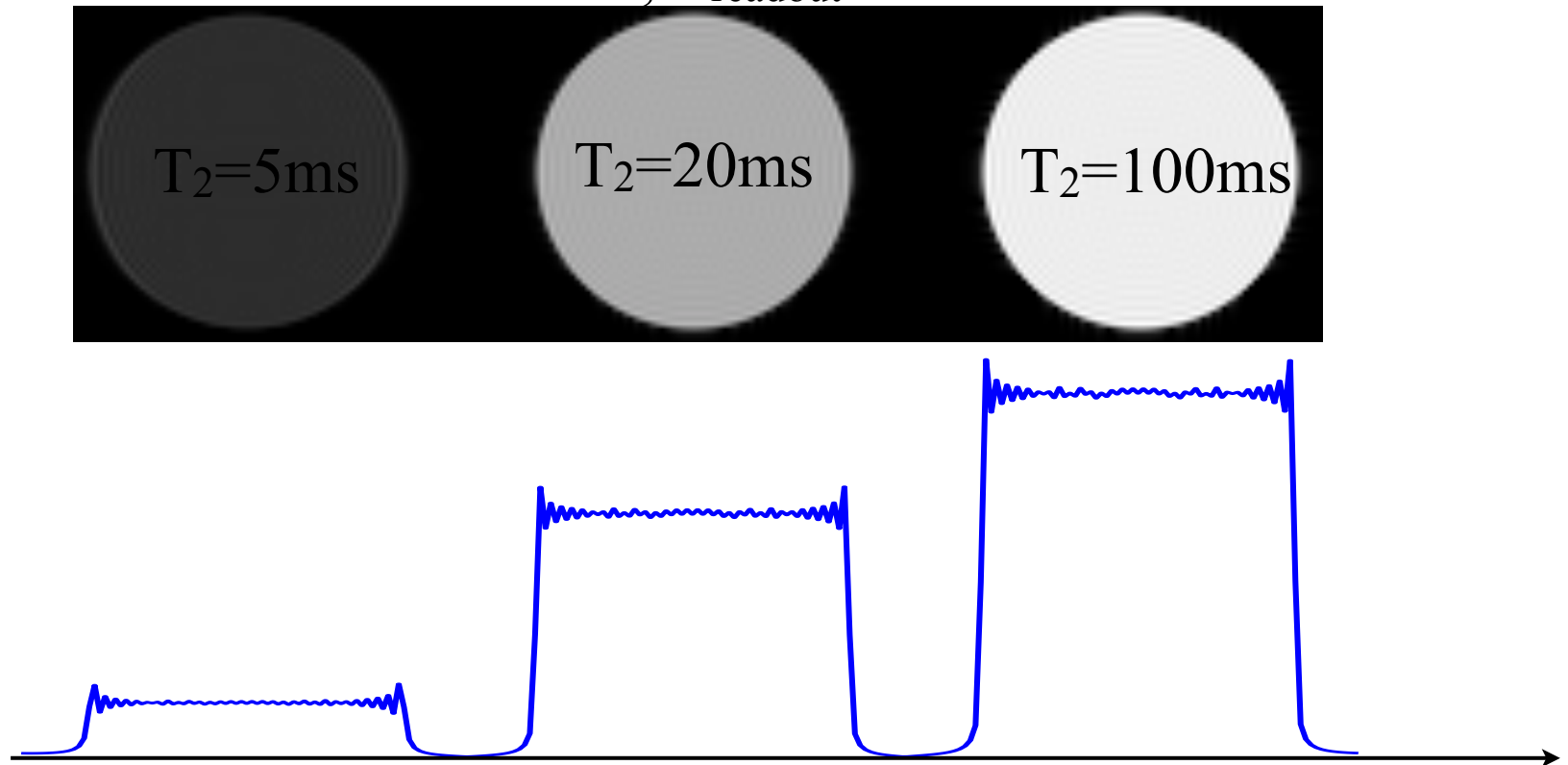
$TE=10\text{ms}$, $T_{\text{readout}}=20\text{ms}$



Effect of T2 Decay on Imaging

- Two effects:
 - Signal loss by $\exp(-TE/T_2)$ (T2 weighting)
 - Apodization - causes blurring in readout

$TE=10\text{ms}$, $T_{\text{readout}}=20\text{ms}$



Off - Resonance

- So far, assumed B_0 constant. But B_0 varies due to:
 - Main field inhomogeneity (~ 1 ppm)
 - Object magnetic susceptibility
 - Chemical shift

Main Field Inhomogeneity

- Magnet is designed to be homogeneous over a (spherical) volume
- Typical numbers:
 - Bare magnet ~10-100 ppm
 - Shimmed magnet ~1 ppm
@3T 1ppm is 127Hz
- Generally, main magnet inhomogeneity is not a limitation

Object Susceptibility

- Most biological objects perturb the field

$$\Delta B_{z,\text{tissue}} \approx \chi B_{0,\text{freespace}}$$

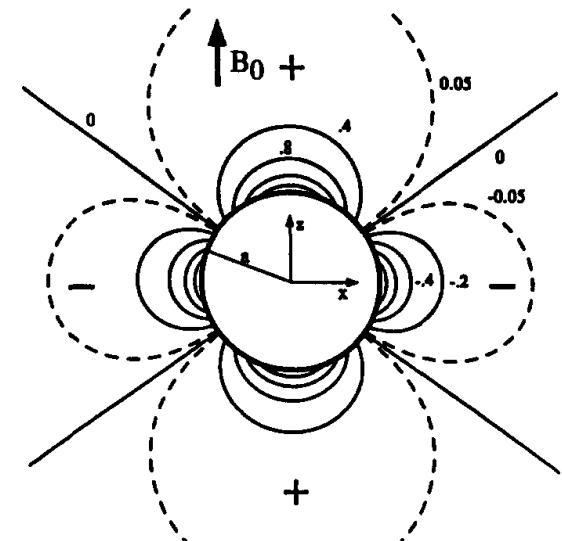
- χ is the magnetic susceptibility
- Larmor frequency lower in tissue than air

$\chi_{\text{water}} = -9.05\text{ppm}$ w.r.t free-space

$\chi_{\text{air}} = 0.36\text{ ppm}$ w.r.t free-space

for a sphere:

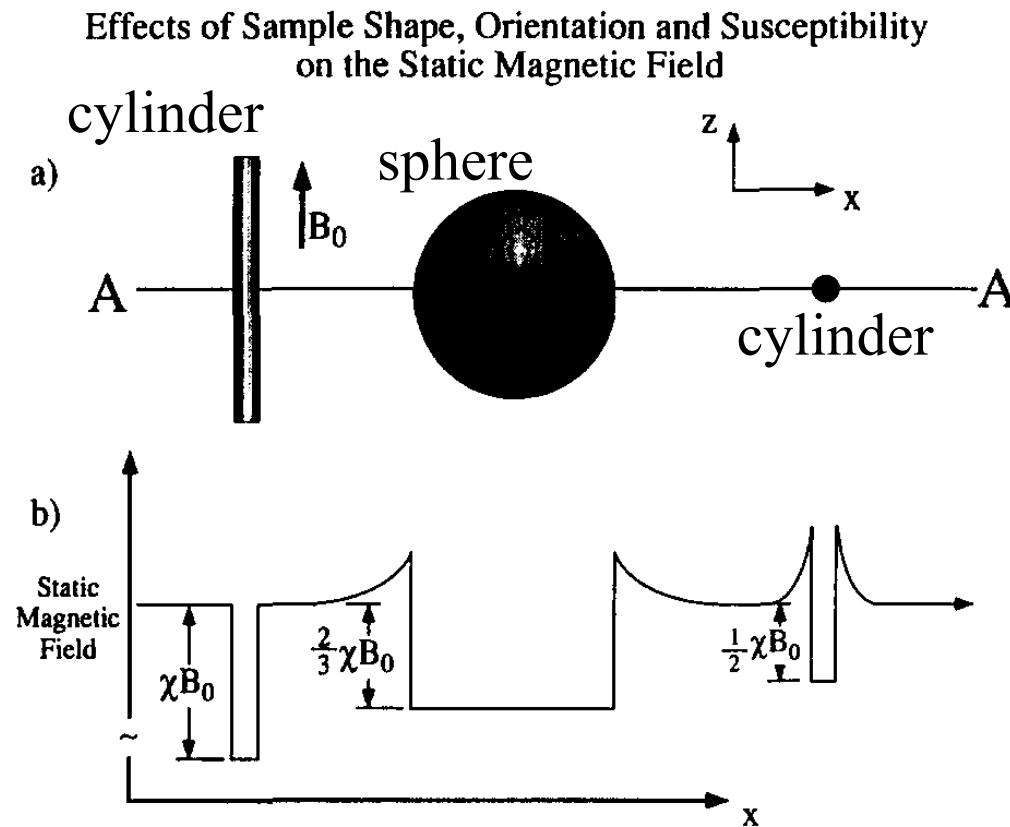
$$\Delta B_z = \frac{\Delta\chi B_0}{3} \left(\frac{a}{r}\right)^3 (3 \cos^2 \theta - 1)$$



John F. Schenck: Review article: Role of magnetic susceptibility in MRI

Object Susceptibility

- Complex behavior at boundaries.
 - Depends on $\Delta\chi$ and geometry
 - Typical $\Delta B_0 \pm 3$ ppm ($-12 < \chi < -6$)

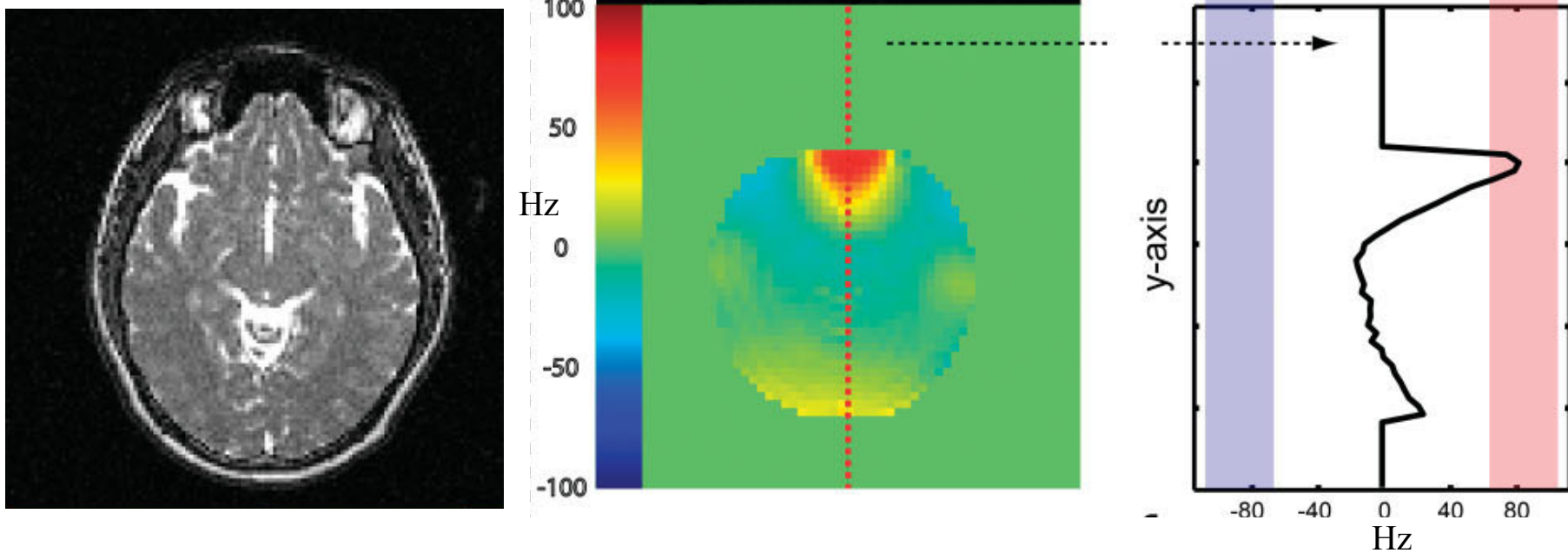


John F. Schenck: Review article: Role of magnetic susceptibility in MRI

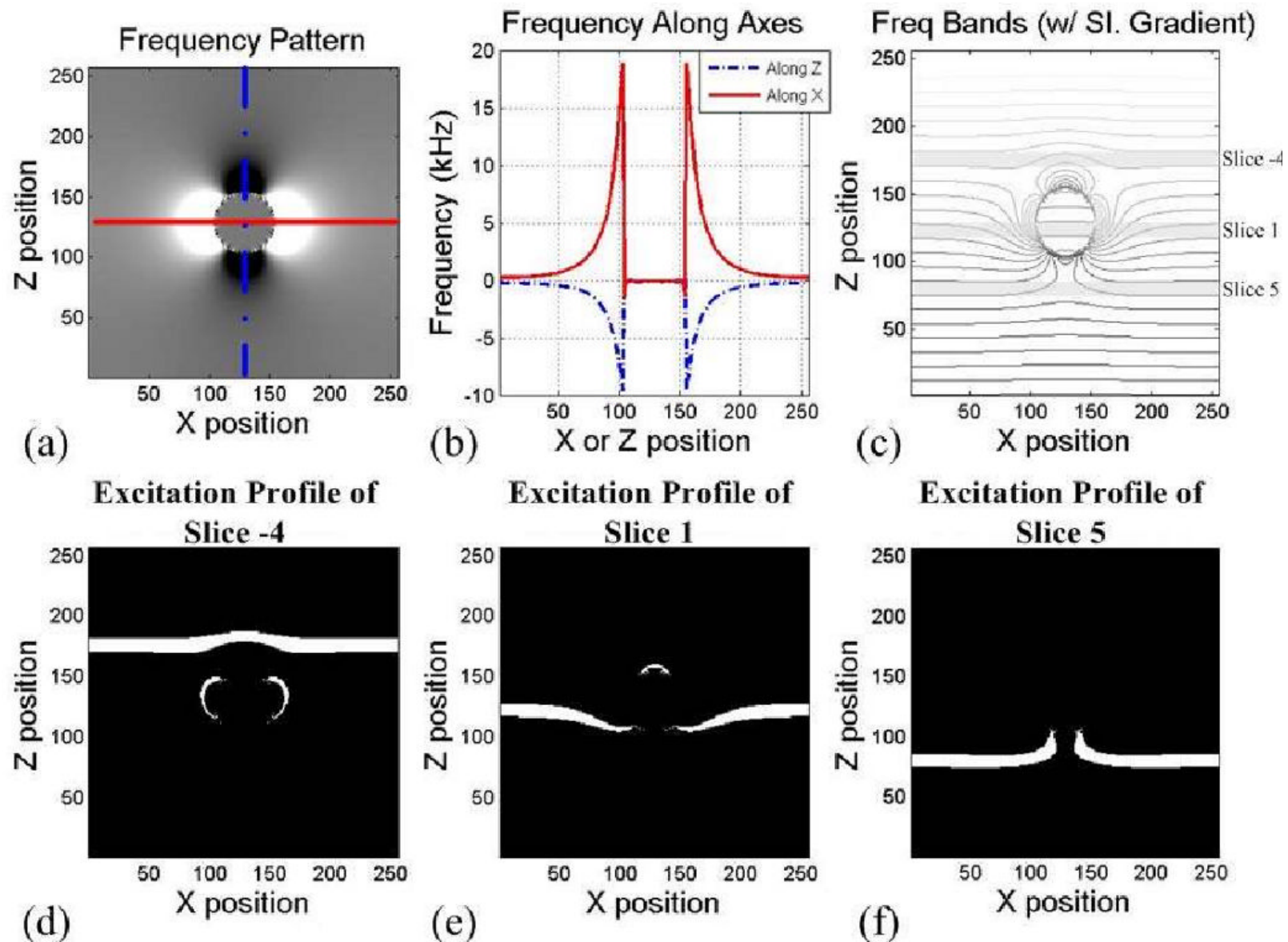
Macroscopic Effect

- Problem areas:
 - Brain above sinuses, auditory canals
 - Heart surrounded by lungs
 - Abdomen

Field inhomogeneity brain@1.5T



Macroscopic Effects - Metal Artifacts



Magn Reson Med. 2009 July ; 62(1): 66–76. doi:10.1002/mrm.21967.

Macroscopic Effects - Metal Artifacts

metal artifact



corrected (SEMAC)



- $\chi_{\text{titanium}} = 182$

- $\chi_{\text{stainless steel (nonmagnetic)}} = 3520-6700$

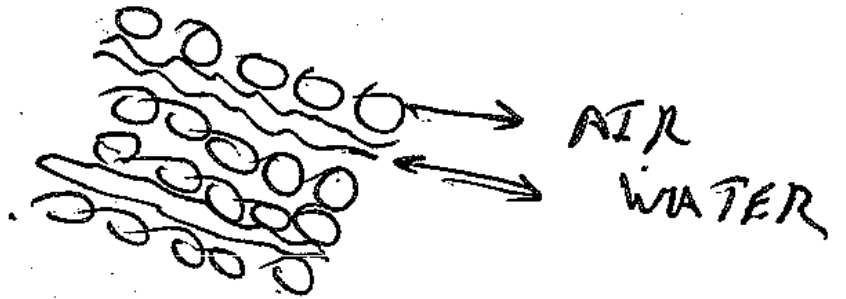
could be a project...

Magn Reson Med. 2009 July ; 62(1): 66–76. doi:10.1002/mrm.21967.

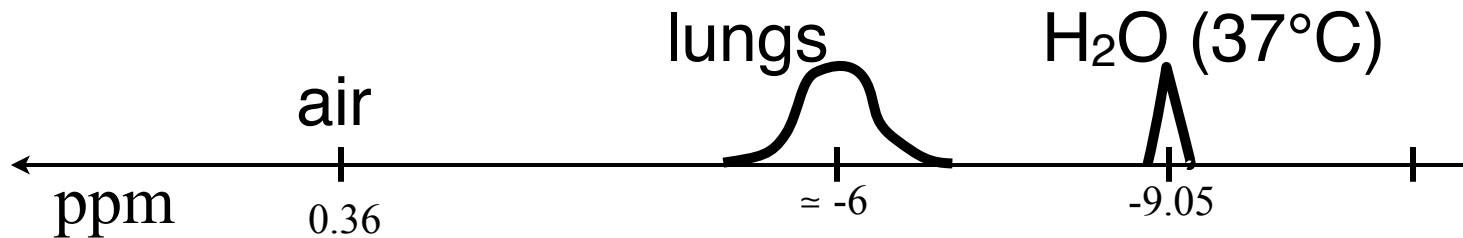
M. Lustig, EECS UC Berkeley

Microscopic Effects

- Lungs:
 - Approx: 1/6 tissue, 5/6 air



- Result:
 - Distribution of field/frequencies



- Tells a lot about microstructure of tissue

Blood

- +Water $\chi=-9.05$
- +Hemoglobin molecule (deoxy) $\chi=0.15$
- +Red Blood cells (deoxy) $\chi=-6.52$
- *Deoxy blood $\chi=-8.77$
- *Oxy blood $\chi=-9.05$

*Magnetic Resonance in Medicine 68:863–867 (2012)

+ **John F. Schenck: Review article: Role of magnetic susceptibility in MRI**

Chemical Shift

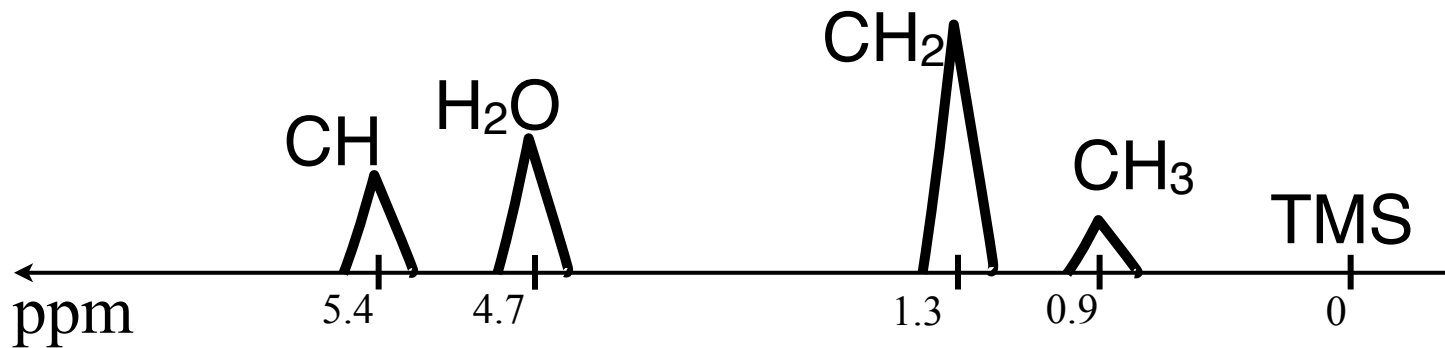
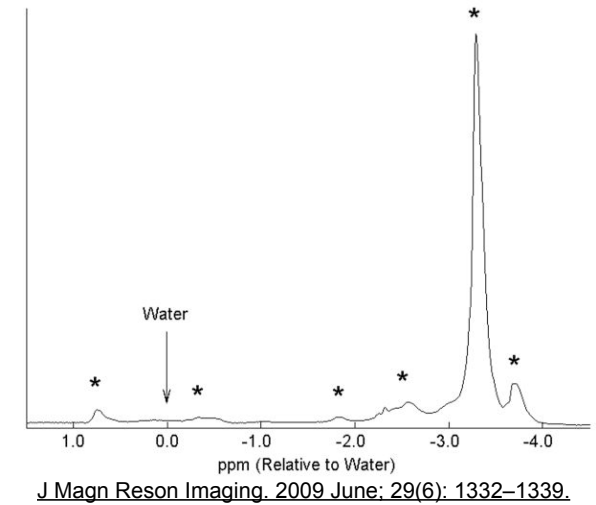
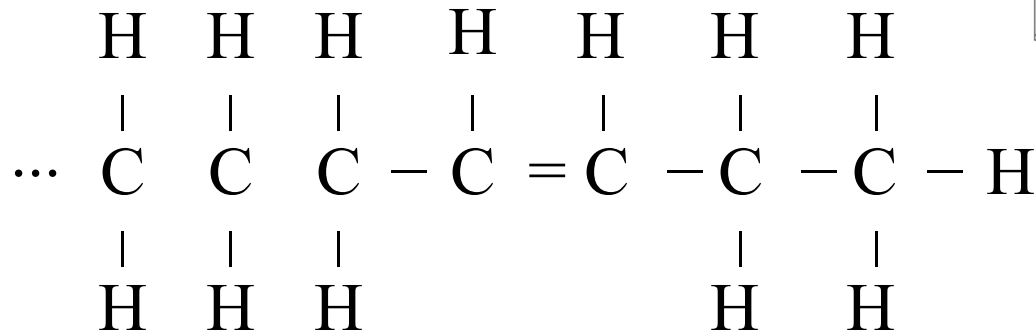
- Protons in complex molecules are “shimmed” by adjacent spins and electrons

$$B_{cs} = B_0(1 - \sigma)$$

- σ is a shielding constant - depends on molecular structure
- Example: Lipids

Chemical Shift

- Example: Lipids



Lipids from CH₂ is 3.4ppm below water
@3T ≈ 440 Hz

Heterogeneous Tissue

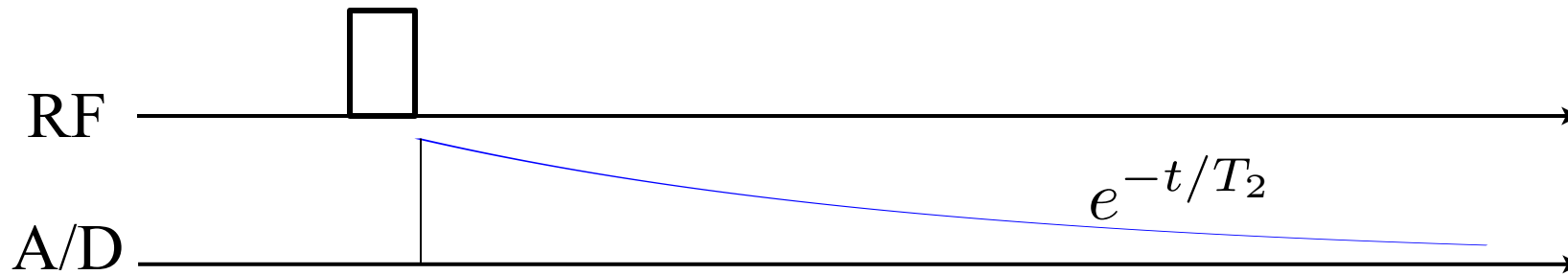
- Tissue is a combination of
 - Chemical shift
 - Susceptibility
 - Geometry
- Results are complex
 - Interesting cases:
 - Blood (fMRI)
 - Lungs
 - Trabecular bone
 - Iron in brain / liver

Effect on Imaging

- Magnitude
 - Phase
 - Geometric
 - Blurring
-
- All depend on spatial scale we look at,
and acquisition strategy

Off Resonance Effect on FID

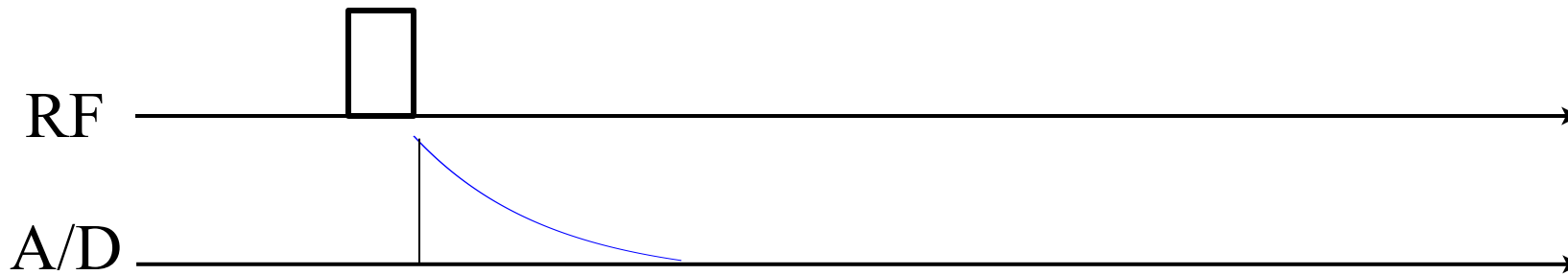
- Simple spectroscopy experiment
 - On resonance



Signal decays with T_2

Off Resonance Effect on FID

- Simple spectroscopy experiment
 - With Susceptibility variation, FID is:



Why?

Off Resonance Effect on FID

- The field is:

$$B(\vec{r}) = B_0 + E(\vec{r})$$

uniform error-field

- Error is spatial and spectral
- In the rotating frame ω_0

$$\vec{B} = E(\vec{r})\hat{k}$$
$$\omega_E(\vec{r}) = \gamma E(\vec{r})$$
$$f_E(\vec{r}) = \frac{\gamma}{2\pi} E(\vec{r})$$

Off Resonance Effect on FID

$$m_{x,y}(\vec{r}, t) = m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r})t} e^{-\frac{t}{T2(\vec{r})}}$$

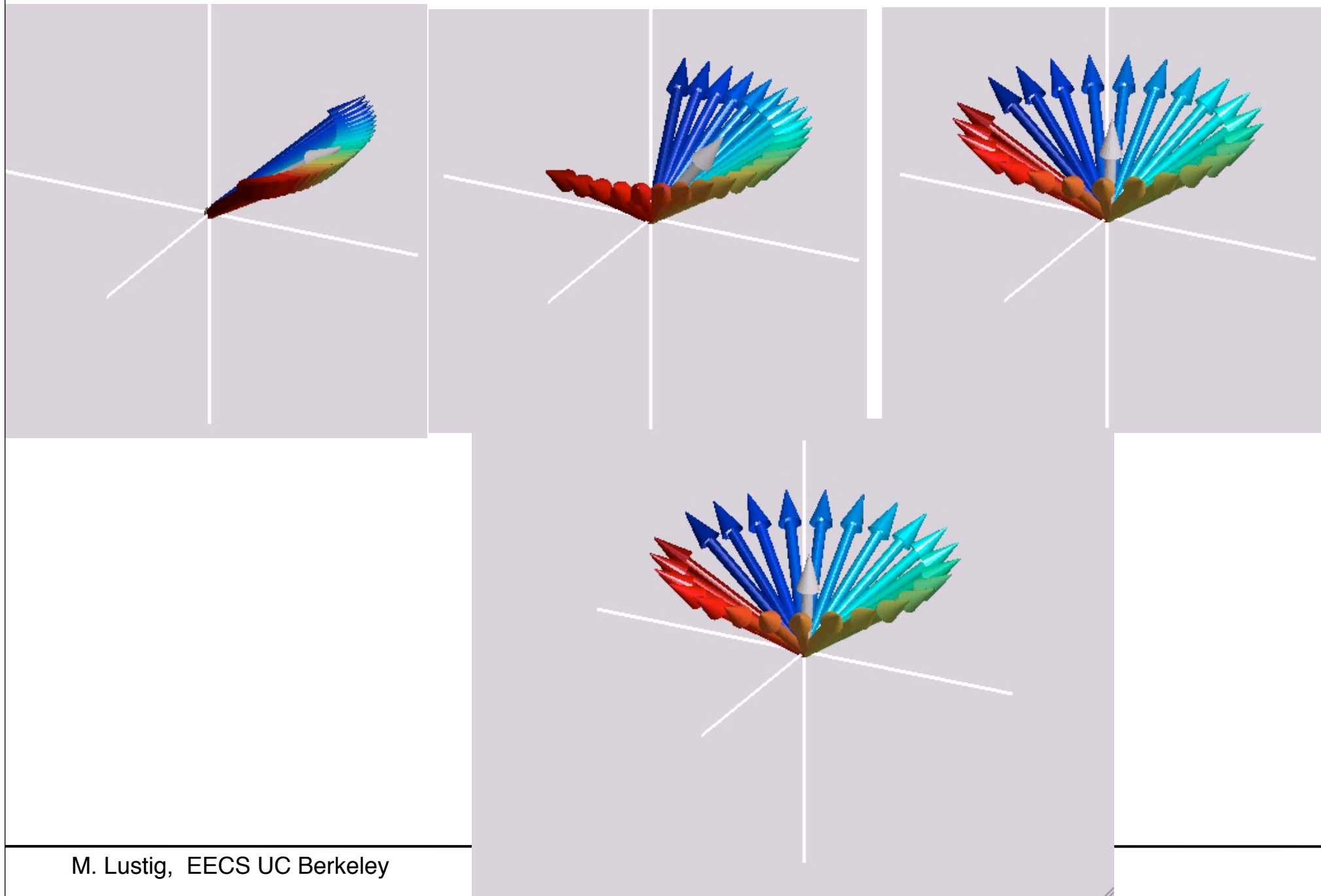
- Received signal is:

$$s(t) = \int_{\vec{R}} m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r})t} e^{-\frac{t}{T2(\vec{r})}} d\vec{r}$$

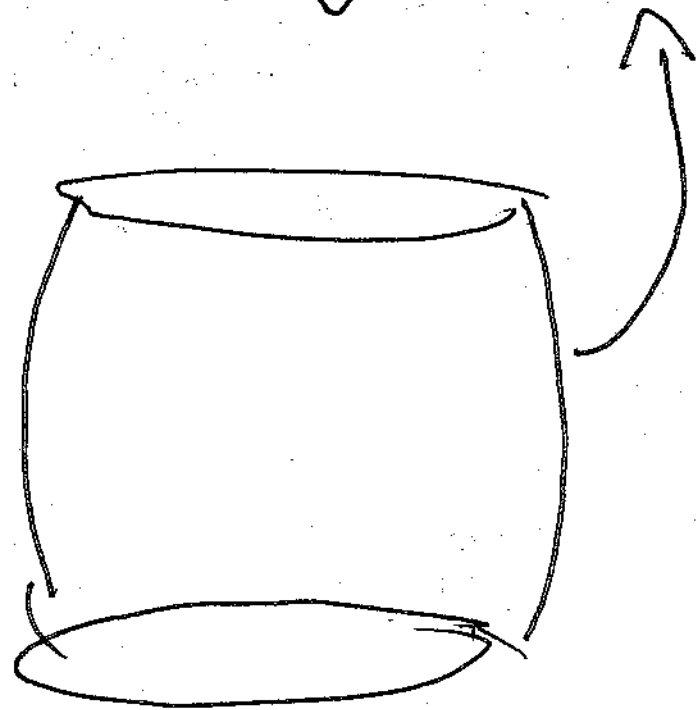
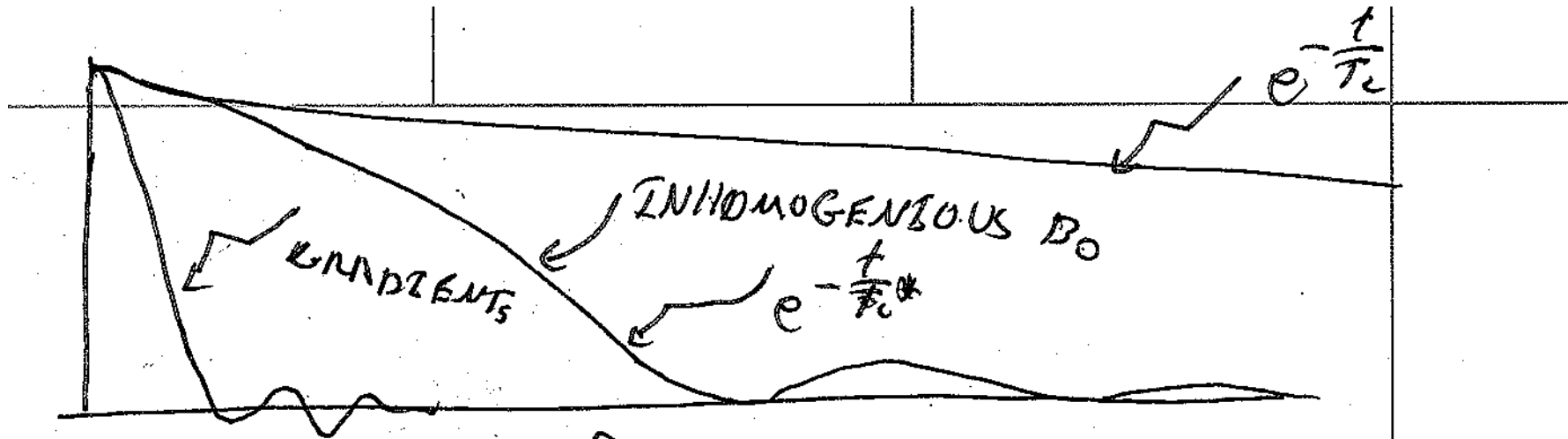
magnetization dephasing relaxation

- with time, phase dispersion causes signal cancellation and loss.

Off - Resonance Effects on FID



Off Resonance Effects on FID



SIGNAL FROM ENTIRE VOLUME

Off Resonance Effects on FID

GRADIENTS ARE VERY STRONG INHOMOGENEITY!

INHOMOGENEITY \Rightarrow SIGNAL LOSS KNOWN AS T_2^*

APPROXIMATE AS EXPONENTIAL

$$s(b) / \text{voxel} = \left[\int_{\text{voxel}} m_{\text{sig}}(r, 0) e^{-i\omega_E(r)t} e^{-t/T_L(r)} d\vec{r} \right]$$

$$\approx \left[\int_{\text{voxel}} m_{\text{sig}}(r, 0) d\vec{r} \right] e^{-\frac{t}{T_2^*(r)}}$$

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

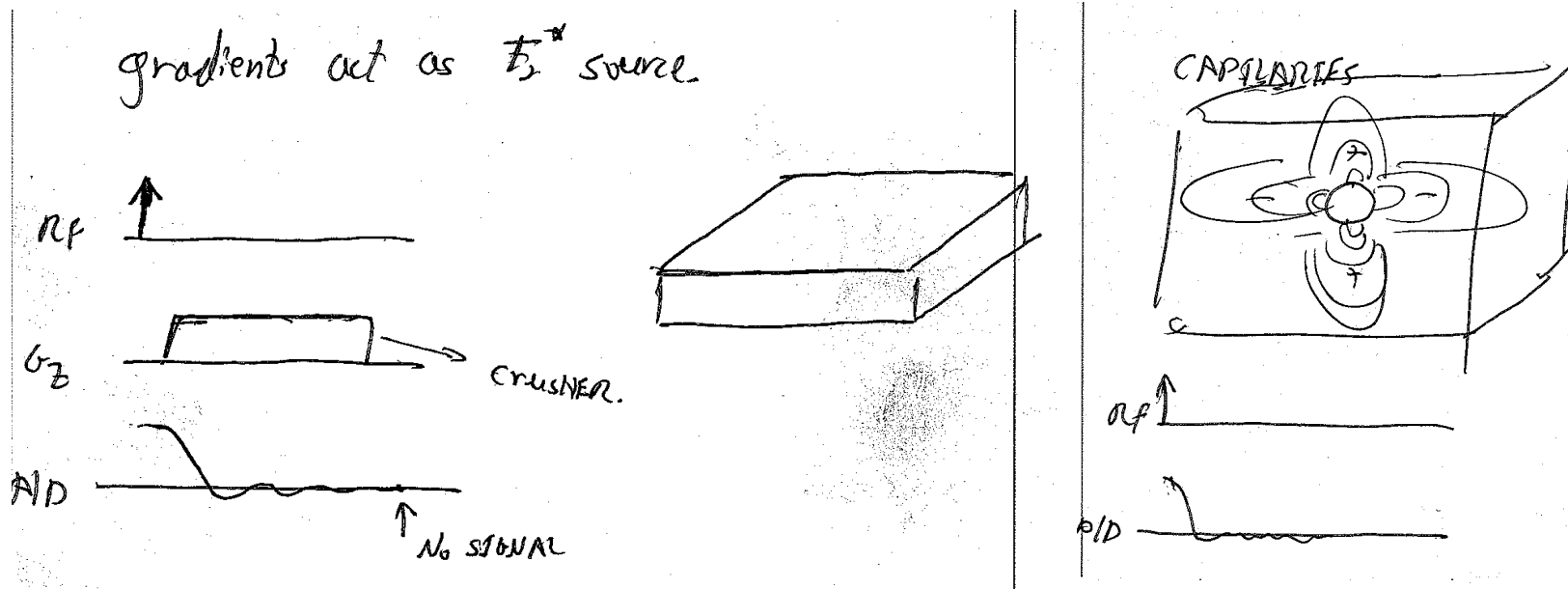
\downarrow
 RELAX

\downarrow
 DEPHASE

$$R_2^* = R_2 + R_2'$$

Off Resonance Effects on FID

- T_2^* : Not a good model for Large-Scale variations (dephasing near sinuses)
- T_2^* : Is a good model for small scale distributed variations (dephasing near capillaries)



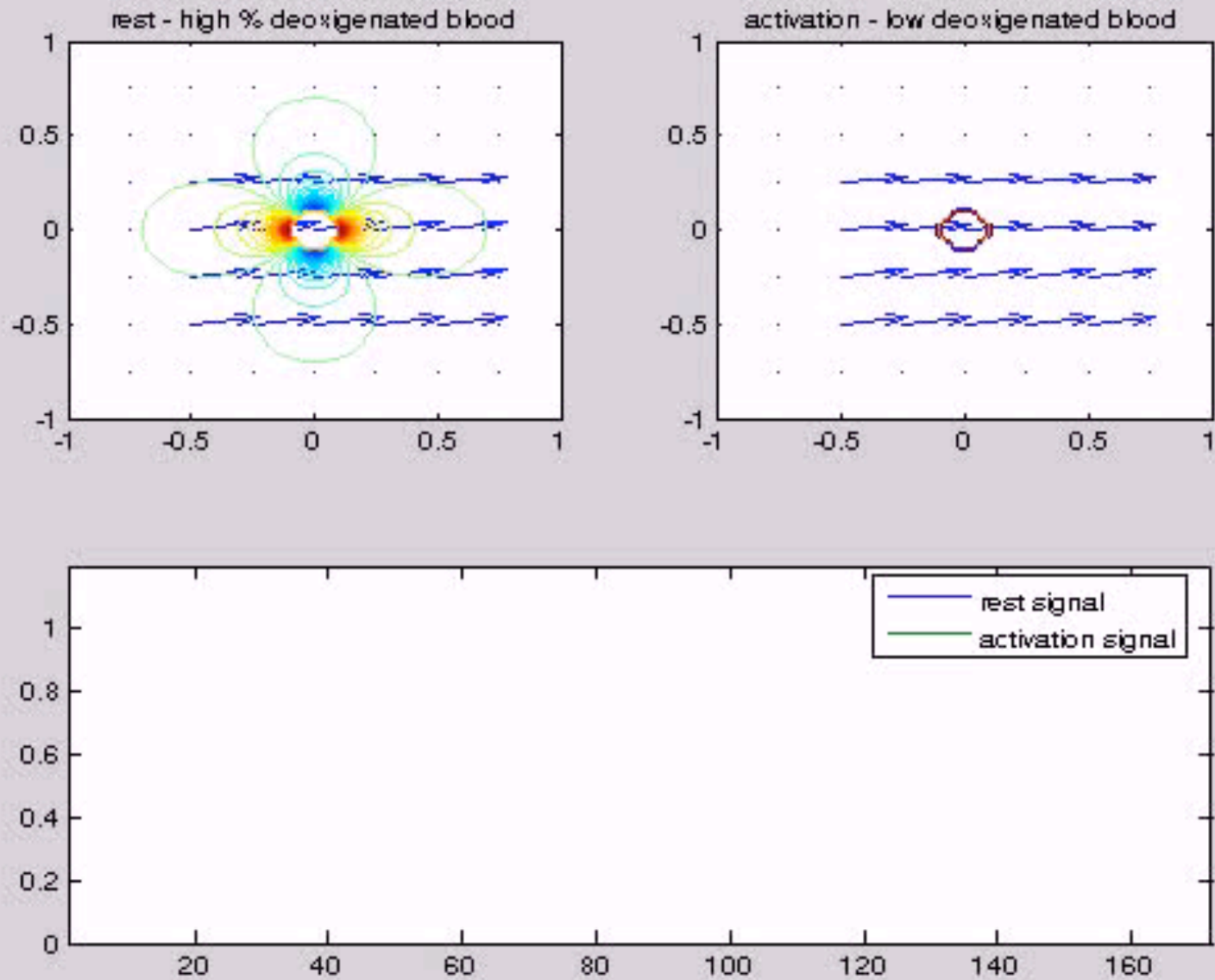
Twinkle, Twinkle, T_2^*

T_2^* (“tee-two-star”): time constant for the decay of signals measured by NMR

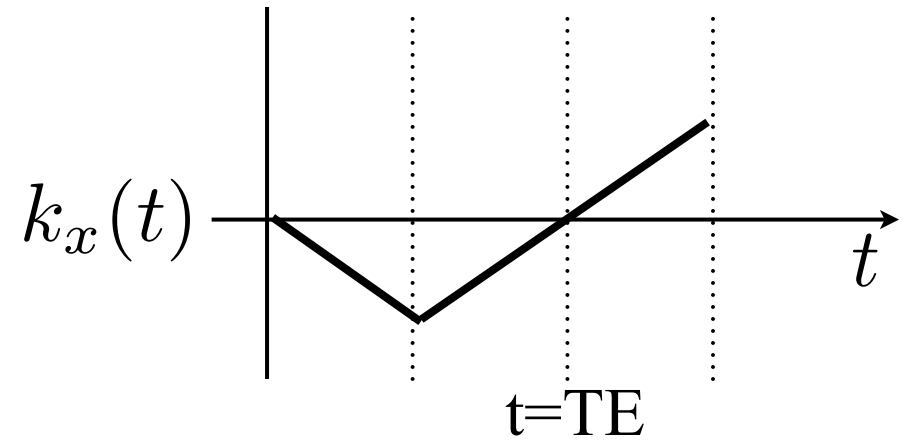
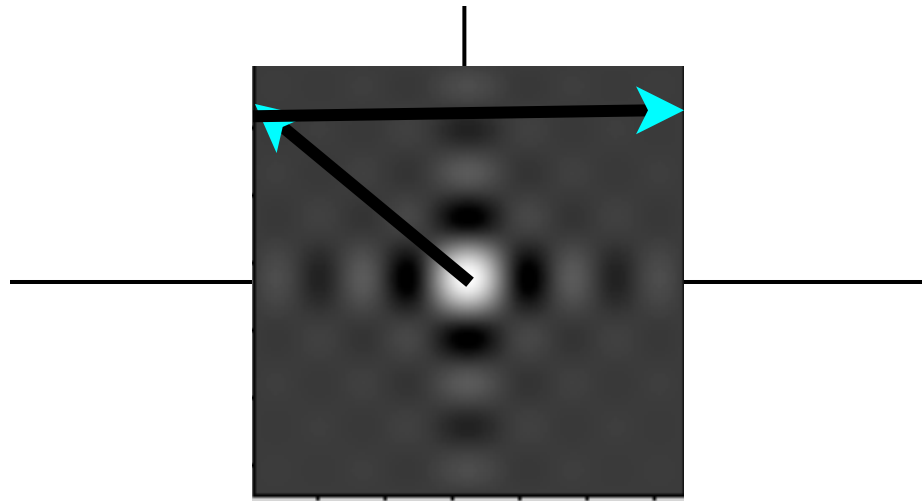
T_2^* depends on . . .

- random spin-spin interactions (which govern T_2)
- inhomogeneities in the magnetic field (which make T_2^* shorter than T_2)

Example fMRI



Off -Resonance Effects on Imaging



$$k_x = \frac{\gamma}{2\pi} G_x (t - TE) \Rightarrow t = \frac{k_x}{\frac{\gamma}{2\pi} G_x} + TE$$

The x-verse magnetization is:

$$m_{xy}(\vec{r}, t) = m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r})t} e^{-\frac{t}{T_2(\vec{r})}} e^{-i2\pi k_x(t)x} d\vec{r}$$

Off -Resonance Effects on Imaging

neglecting T_2 and substituting for t :

$$m_{xy}(\vec{r}, t) = m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r}) \left(\frac{k_x(t)}{\frac{\gamma}{2\pi} G_x} + TE \right)} e^{-i2\pi k_x(t)x} d\vec{r}$$

Or,

$$m_{xy}(\vec{r}, t) = m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r})TE} e^{-i2\pi k_x(t) \left(x + \frac{\omega_E(\vec{r})}{\gamma G_x} \right)} d\vec{r}$$

phase/dephasing displacement

Off -Resonance Effects on Imaging

$$m_{xy}(\vec{r}, t) = m_{xy}(\vec{r}, 0) e^{-i\omega_E(\vec{r})TE} e^{-i2\pi k_x(t) \left(x + \frac{\omega_E(\vec{r})}{\gamma G_x} \right)} d\vec{r}$$

phase/dephasing displacement

An on-resonance spin at position:

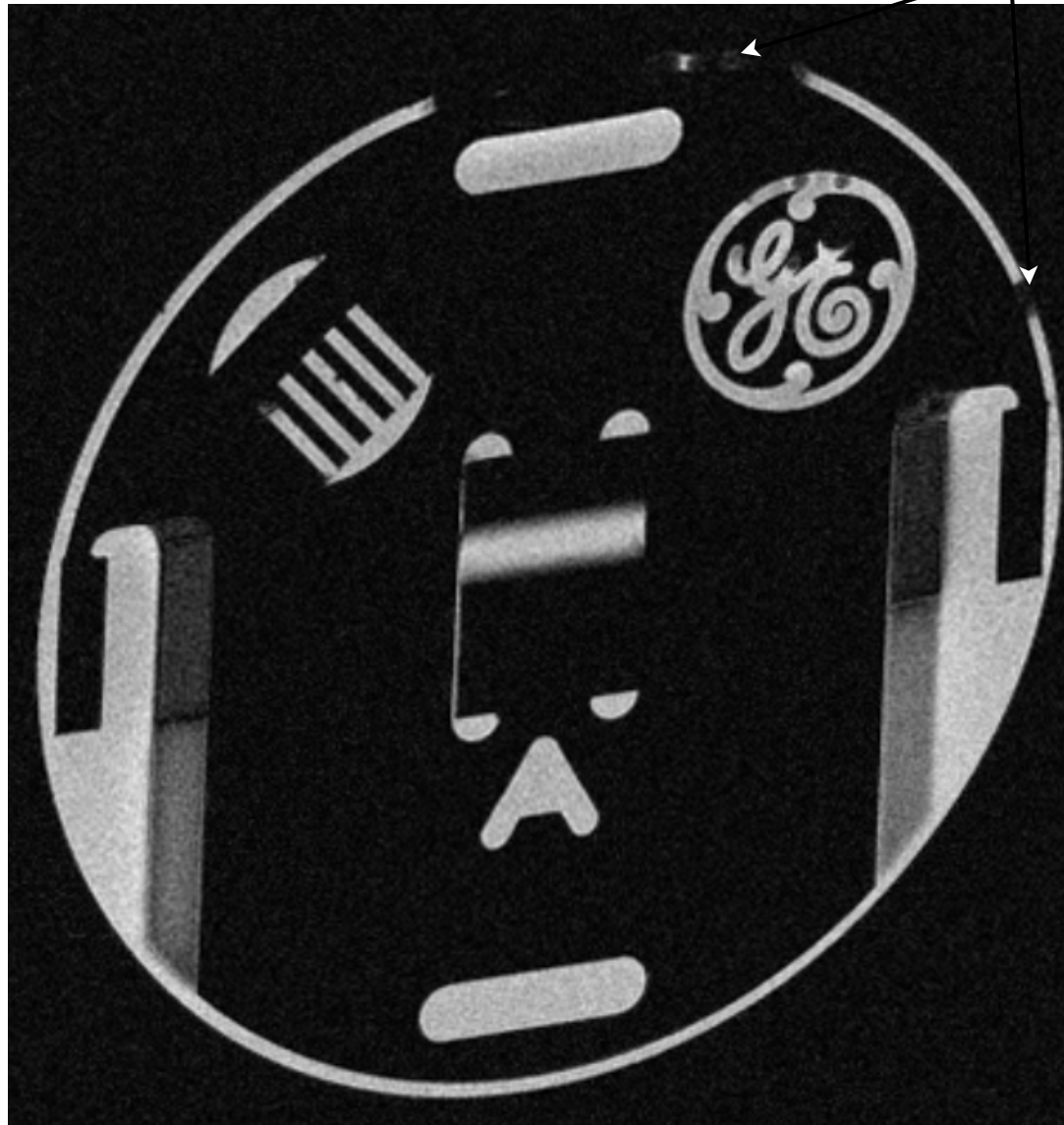
$$x' = x + \frac{\omega_E(\vec{r})}{\gamma G_x}$$

Produces the same signal as a spin at x , with off-resonance!

Examples:

shape

readout
direction?

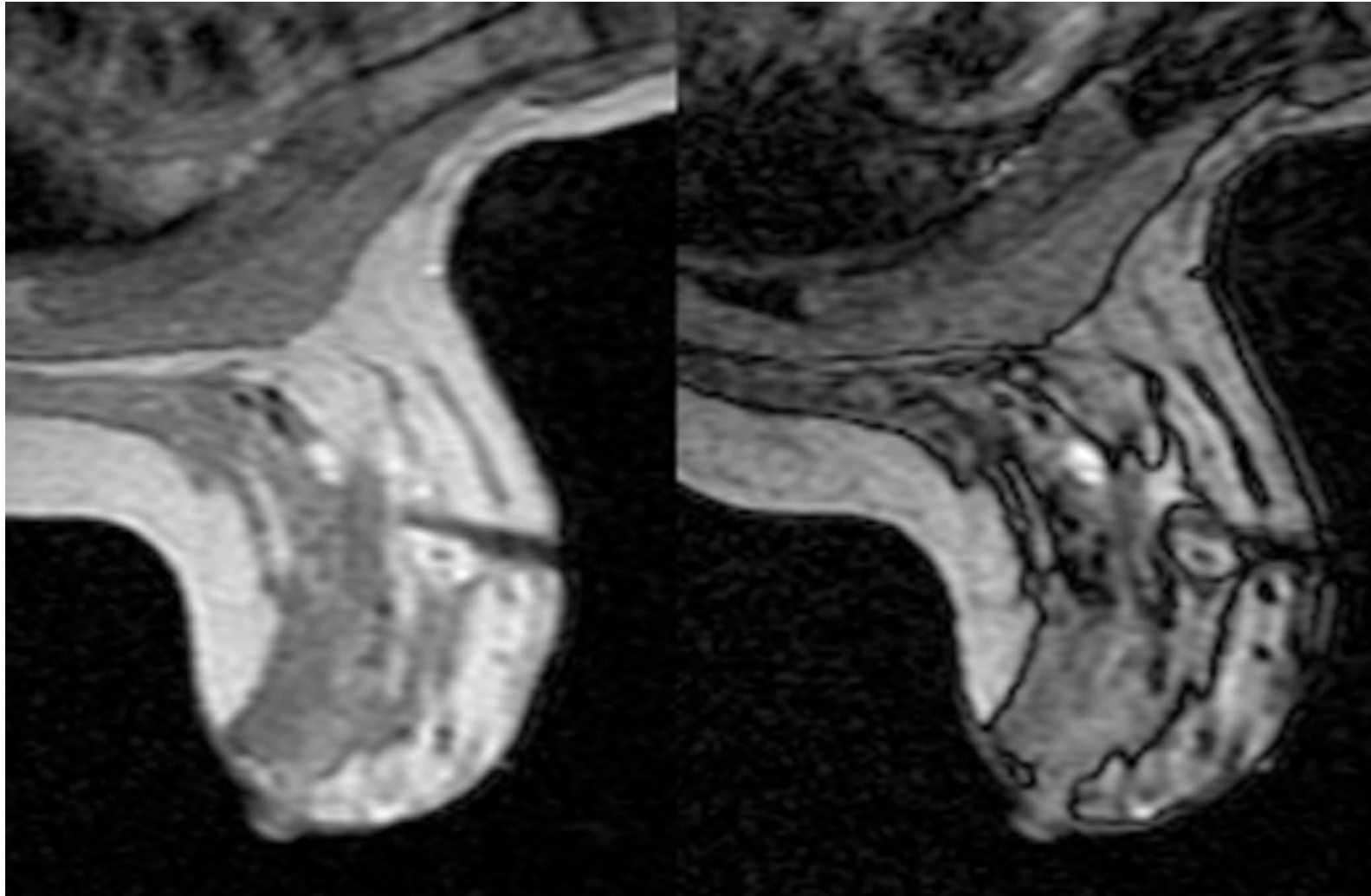


signal loss

Examples: Phase vs Echo Time

water and lipids in phase

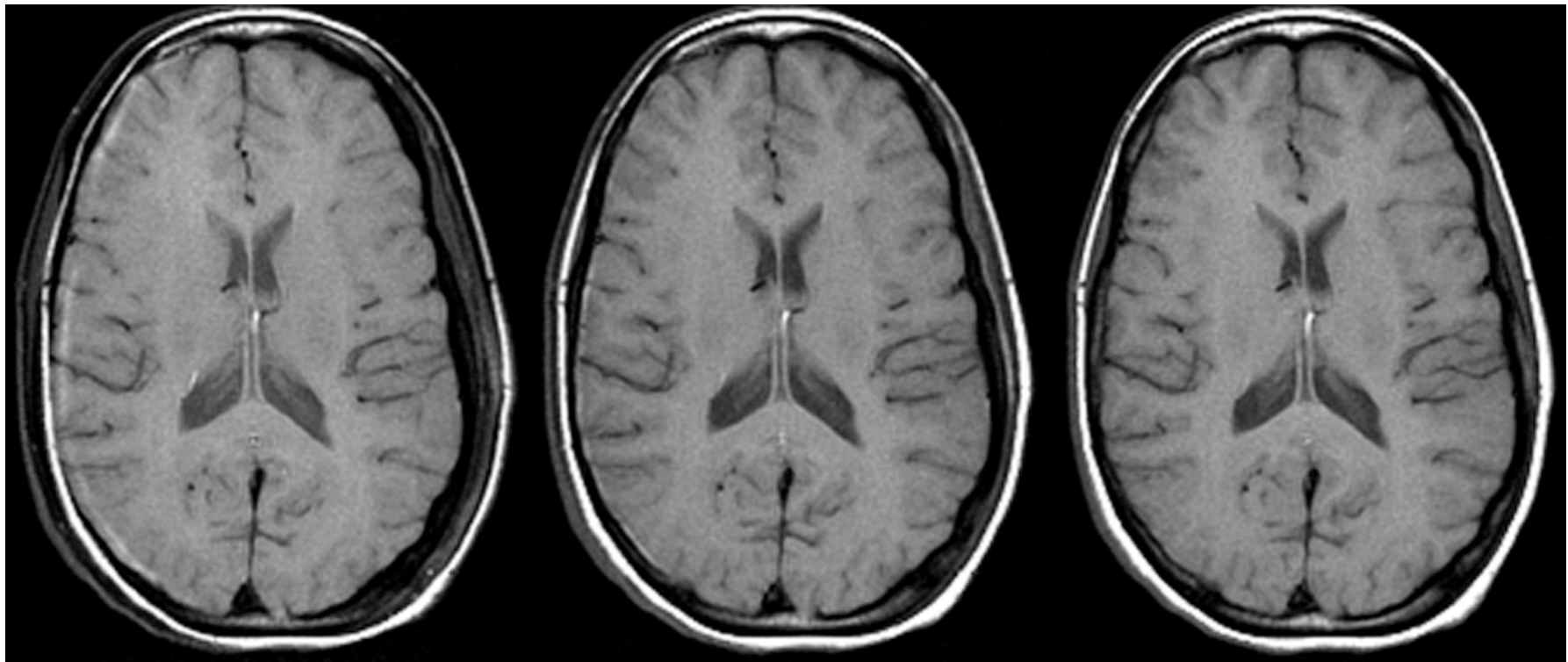
water and lipids out-of-phase



Example: Chemical Shift

$$x' = x + \frac{\omega_E(\vec{r})}{\gamma G_x}$$

Increasing G_x reduces chemical shift artifact →



Example:

Let,

$$\frac{\gamma}{2\pi} G_x = 1 \text{ KHz/cm}$$

$$E(\vec{r}) = 3 \text{ ppm}$$

$$\omega_E(\vec{r}) = 2\pi \cdot 200 \text{ Hz}$$

$$\Delta x = x' - x = \frac{\omega_E(\vec{r})}{\gamma G_x} = \frac{\cancel{2\pi} \cdot 200 \text{ Hz}}{\cancel{2\pi} \cdot 1 \text{ KHz/cm}} = 0.2 \text{ cm}$$

At:

$$\delta_x = 1 \text{ mm} \quad \text{this is a shift of two pixels!}$$

Effects in Spin-Warp

- Off-Resonance

- Modest spatial distortions (few pixels)
- Relatively benign artifacts
- Reduce artifacts with large G_x

- Chemical-Shift

- Fat shift of -220Hz @ 1.5T
- Fat image is displaced from Water
- In practice F/W shift limited to ~2pixels

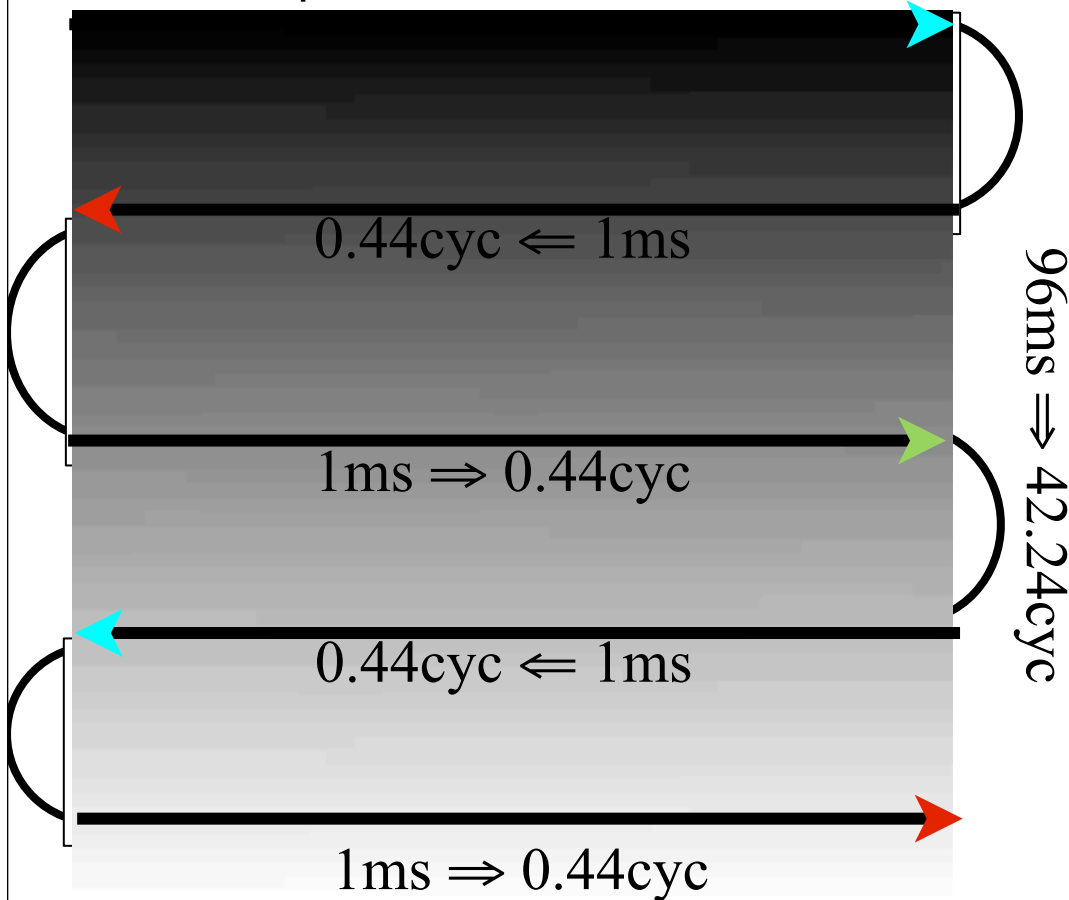
2 pixels shift are two cycles of linear phase across k-space

$$\frac{2 \text{ cyc}}{220 \text{ Hz}} \approx 9.1 \text{ ms}$$

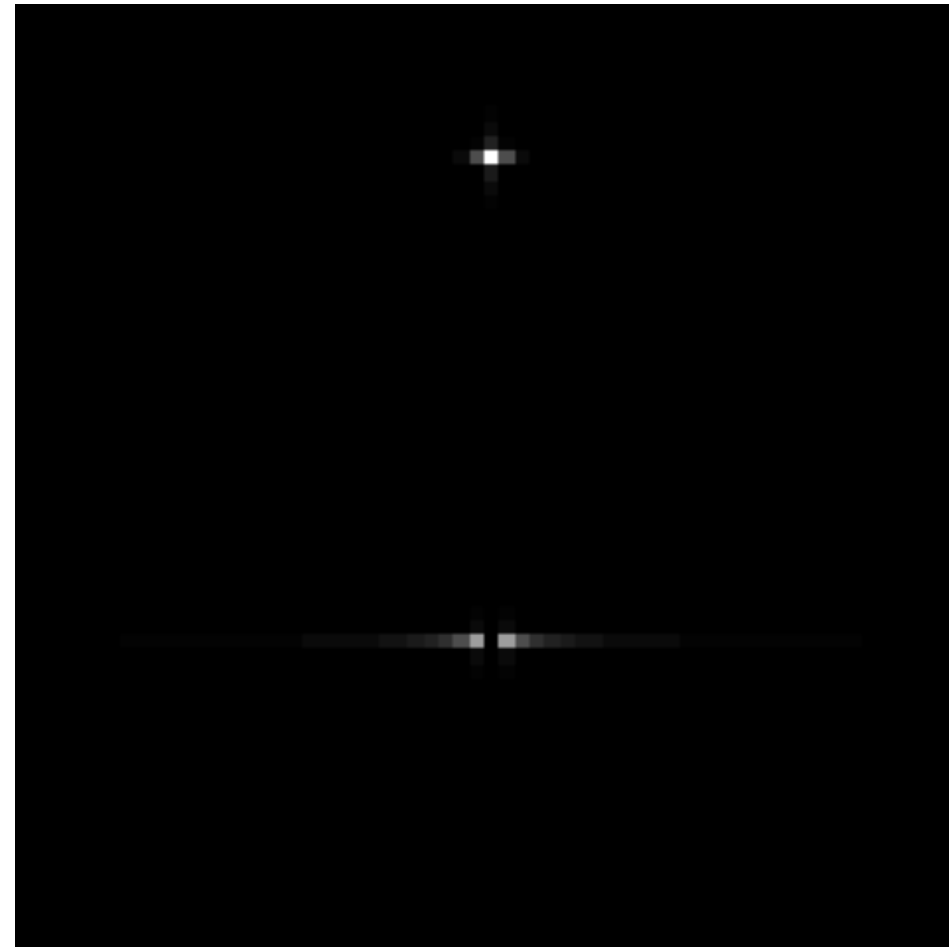
Off-Resonance in EPI

Example: Lipids

phase accumulation

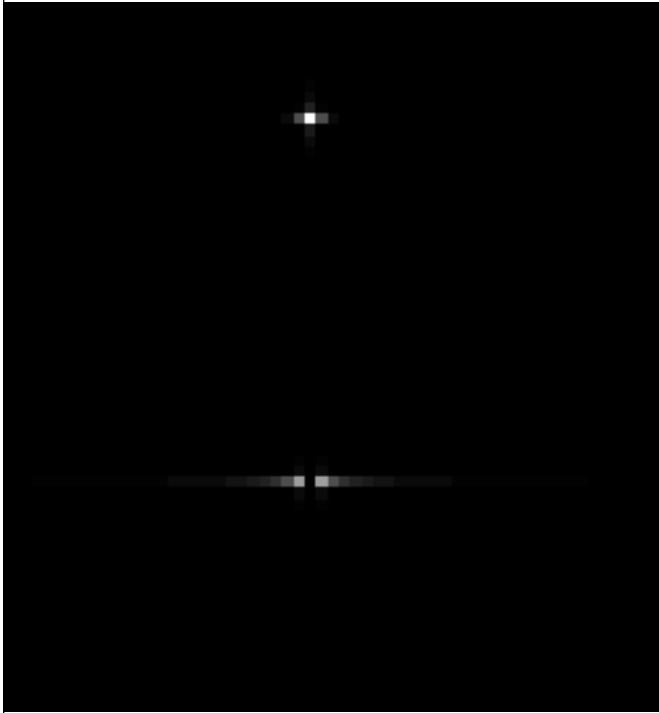


point-spread function

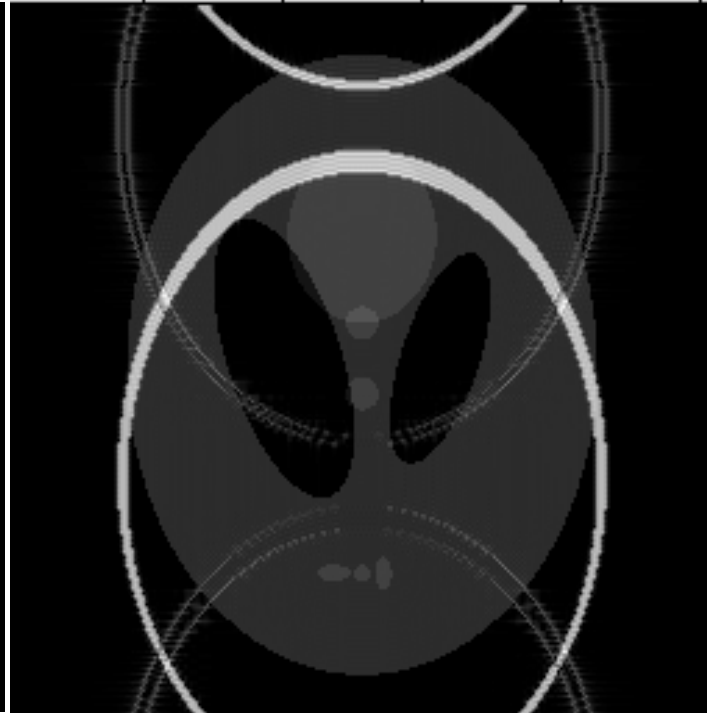


Off-Resonance in EPI

point-spread function



simulation

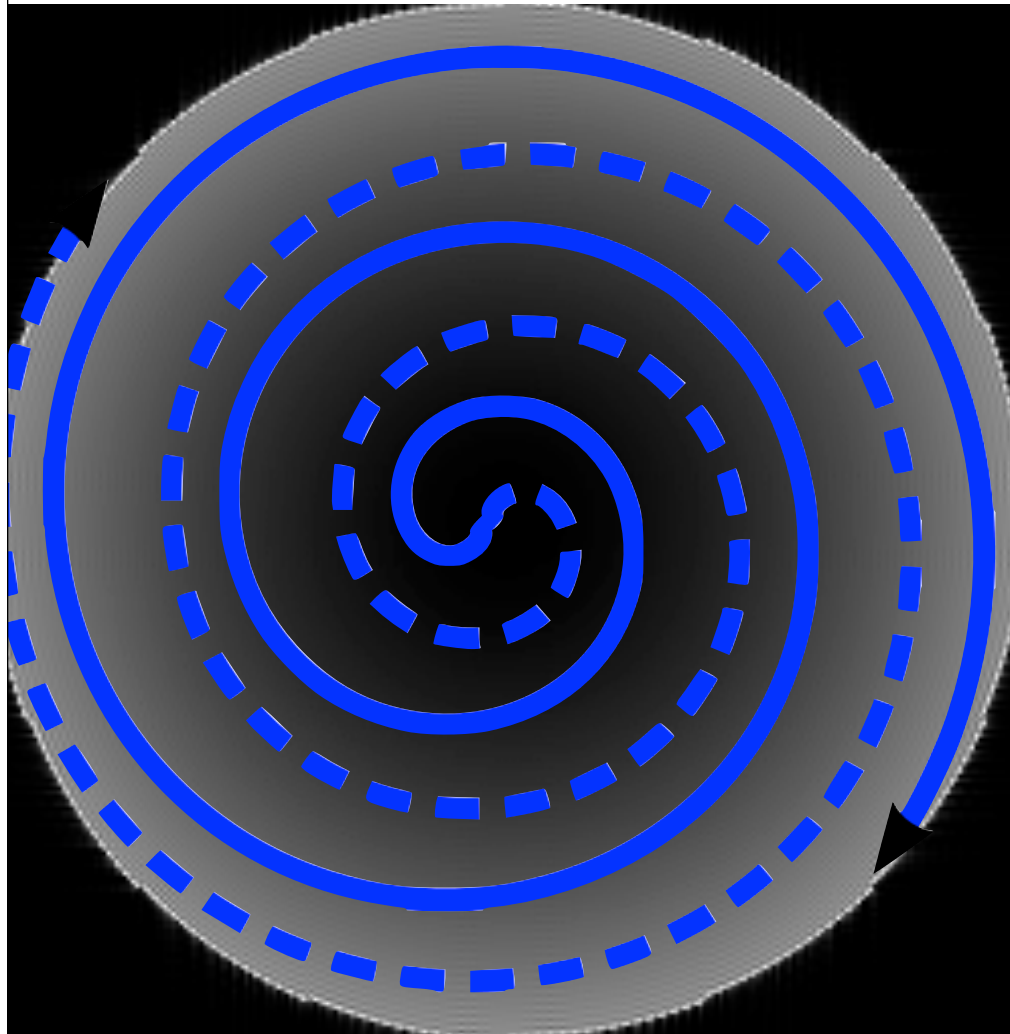


Leg

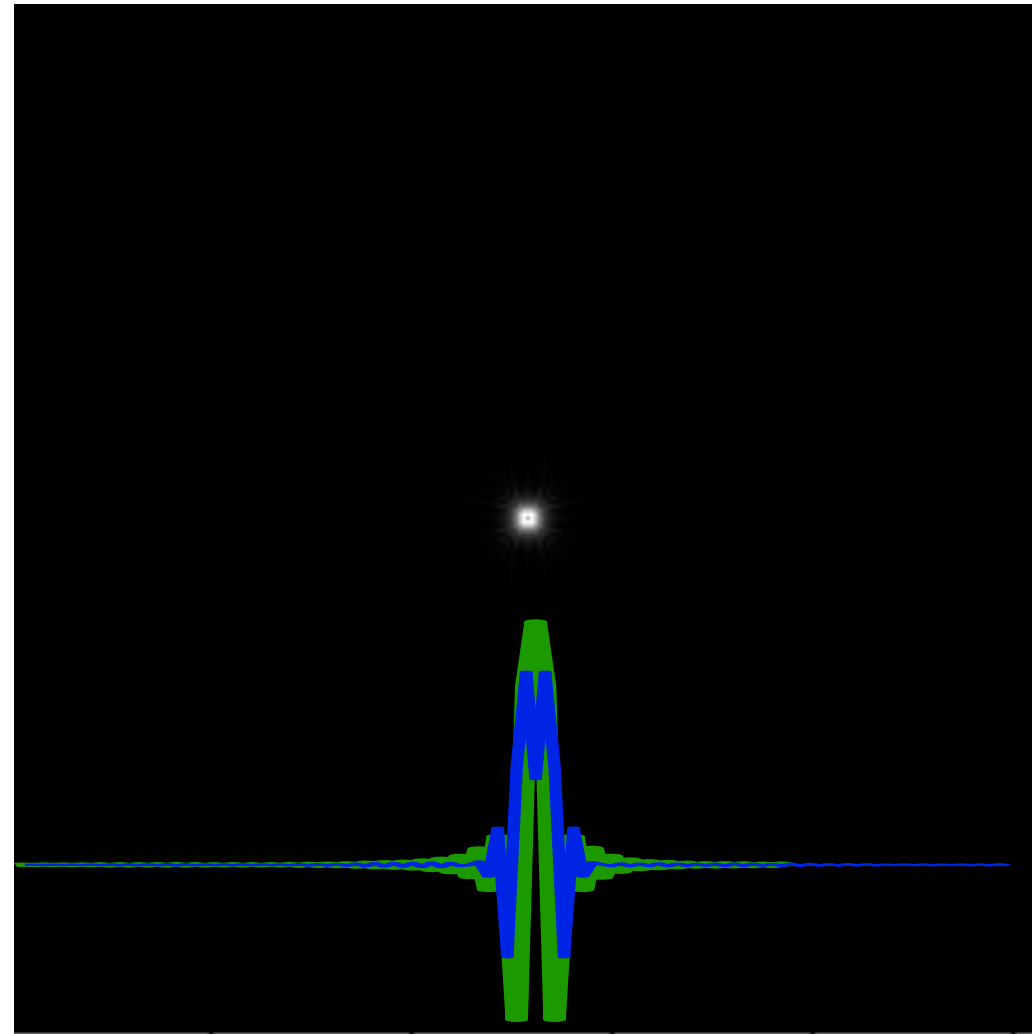


Off-Resonance in Spiral

phase accumulation



point-spread function



$2 \text{ ms} \Rightarrow 0.88 \text{cyc}$

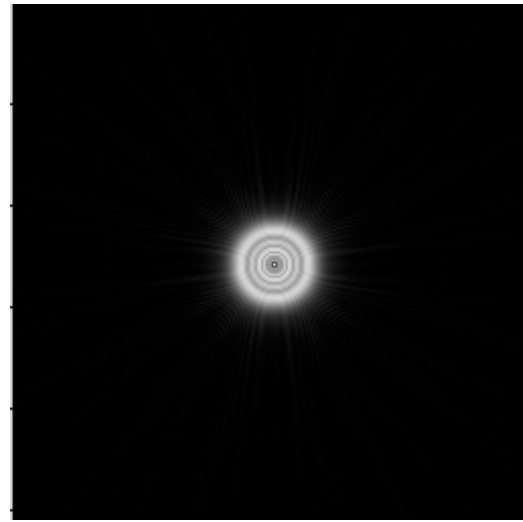
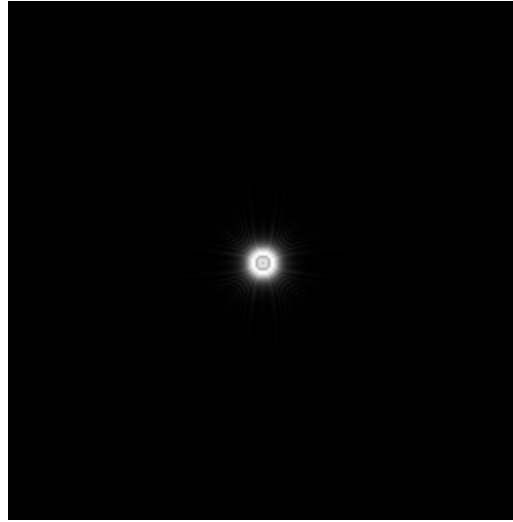
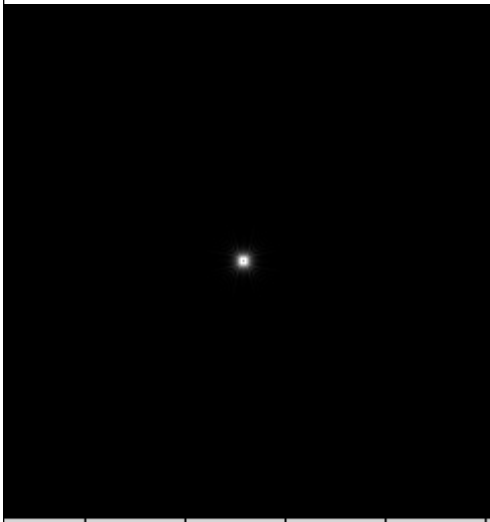
Off-Resonance in Spiral

Readout time @3T:

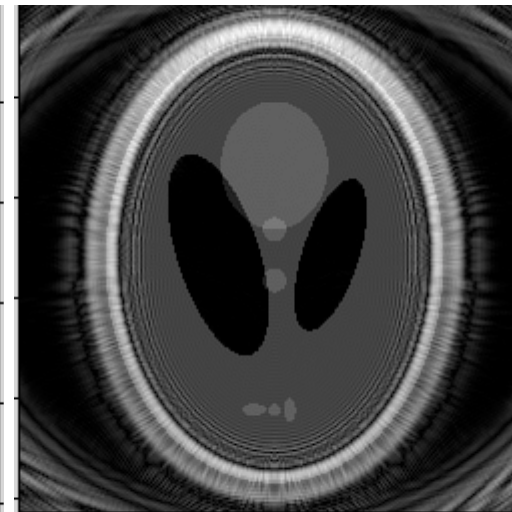
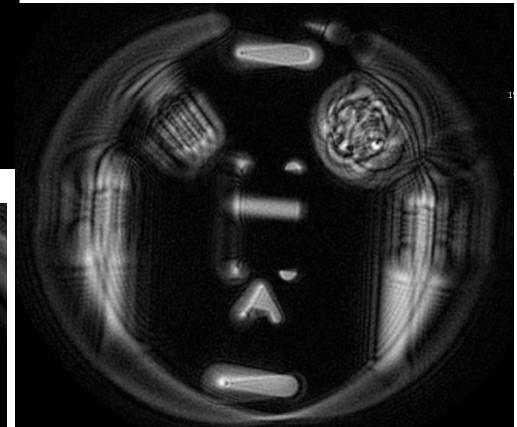
2ms

5ms

13ms

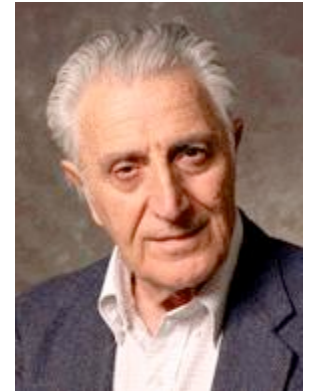
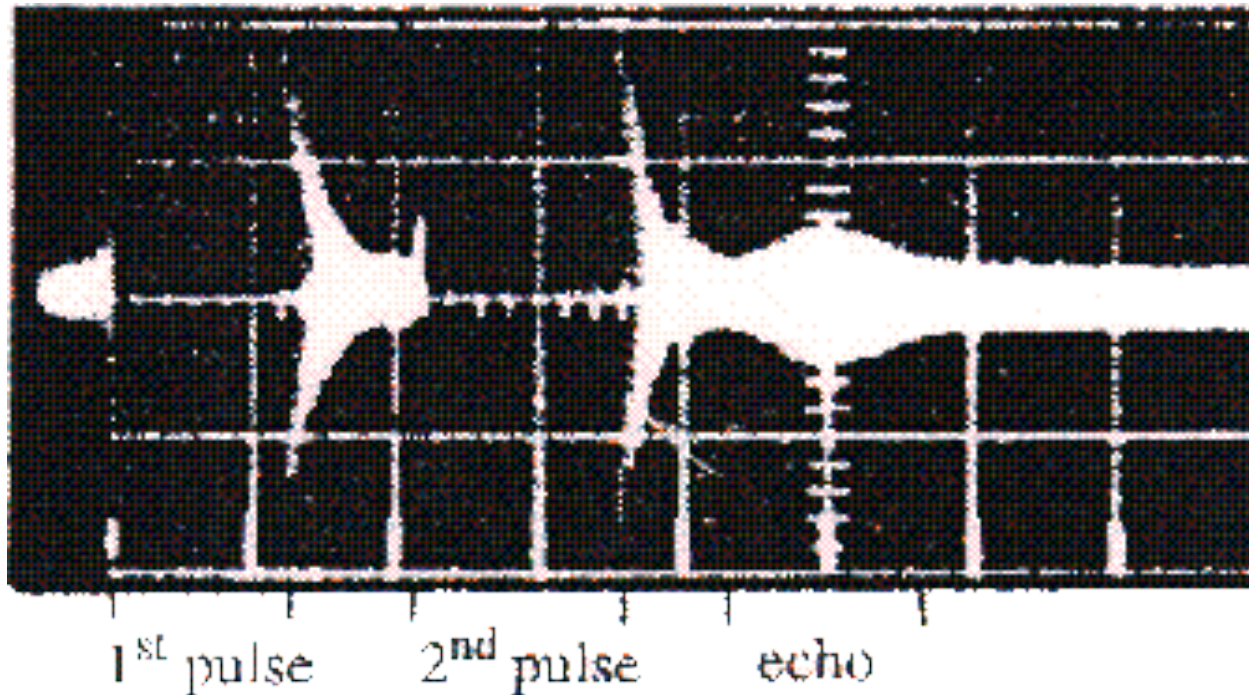


Spiral scan
with linear off-resonance



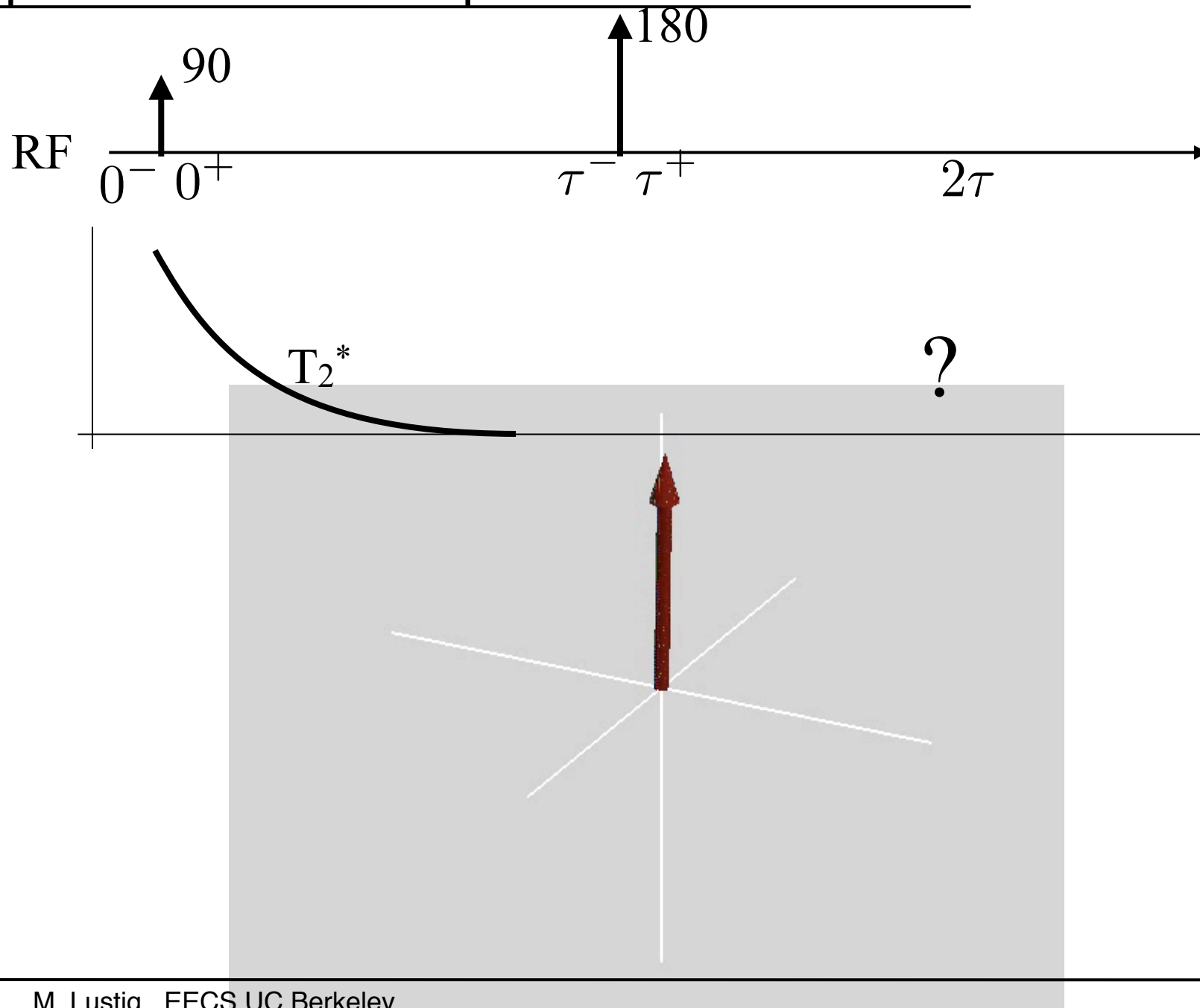
Echoes

- Early in NMR people noticed the following odd behavior (Hann, 1950)

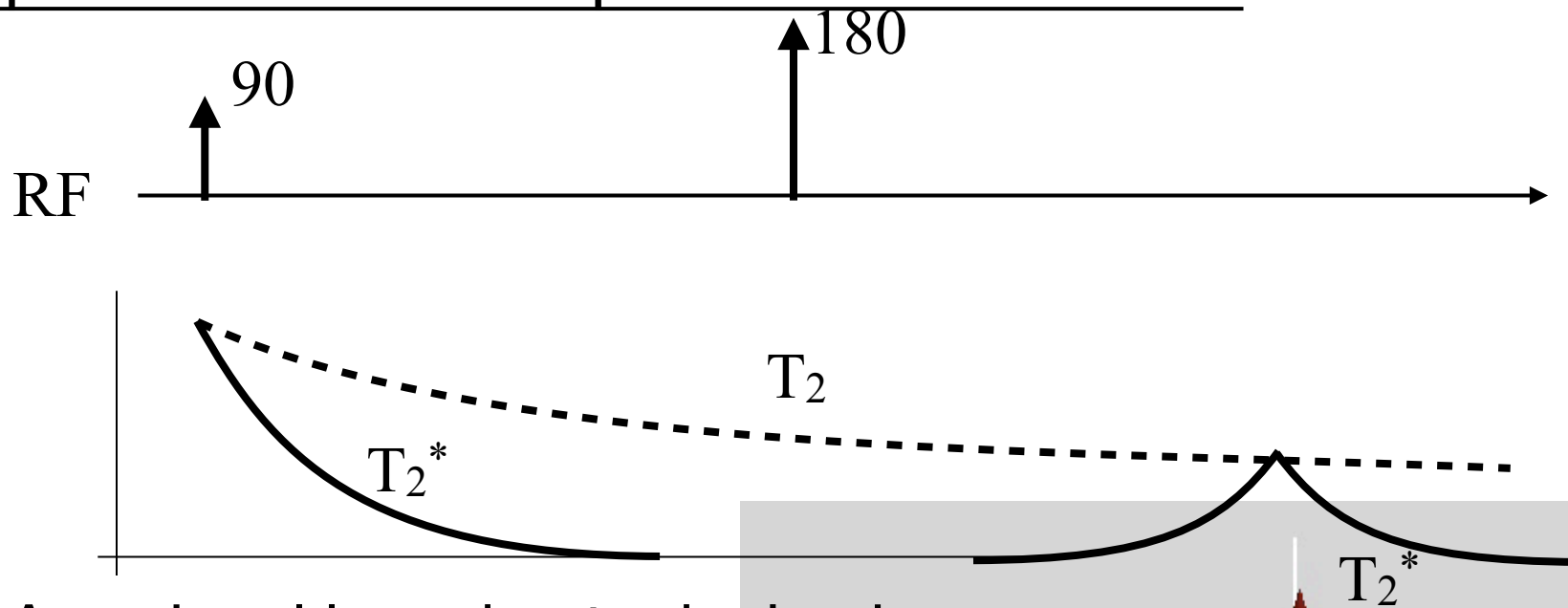


Two 90° excitations, separated by T cause large signal to form at $2T$, Why?

Spin Echo Pulse Sequence



Spin Echo Pulse Sequence

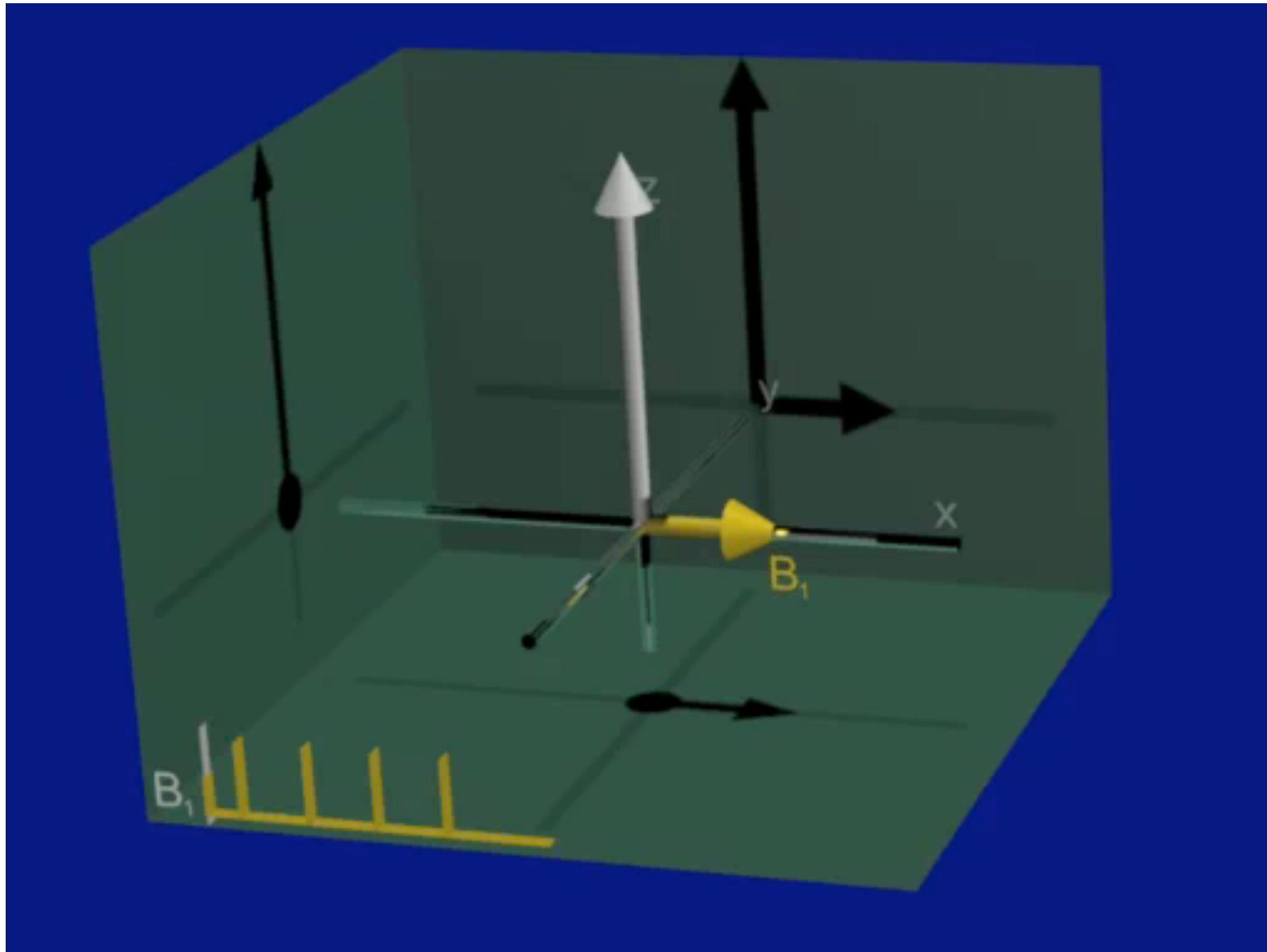


Any signal loss due to dephasing is recovered at the spin-echo

If $wE(r,t)$ changes over time, refocusing is not perfect

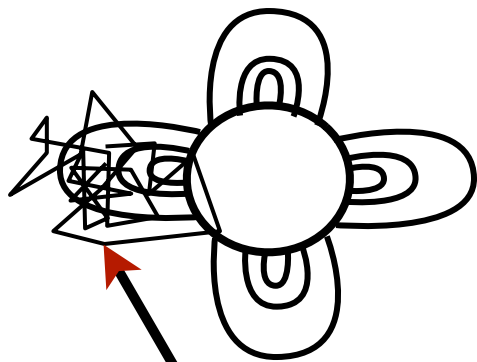
Provides probe to measure molecular motion

Multi-Spin Echo - CPMG

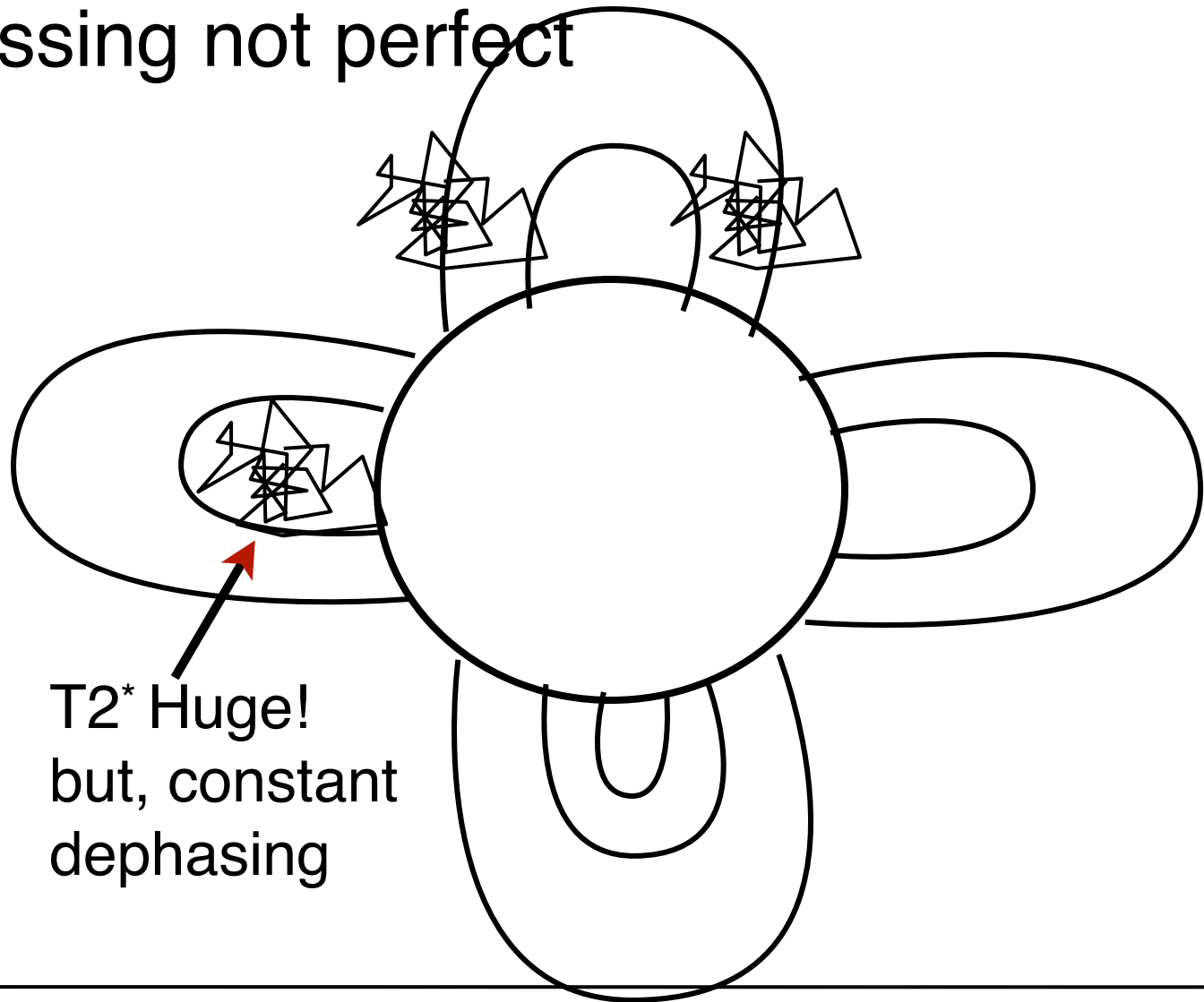


Non uniform dephasing

- Any signal due to dephasing $\omega_E(r)t$ is refocussed
- If $\omega_E(r,t)$ refocussing not perfect

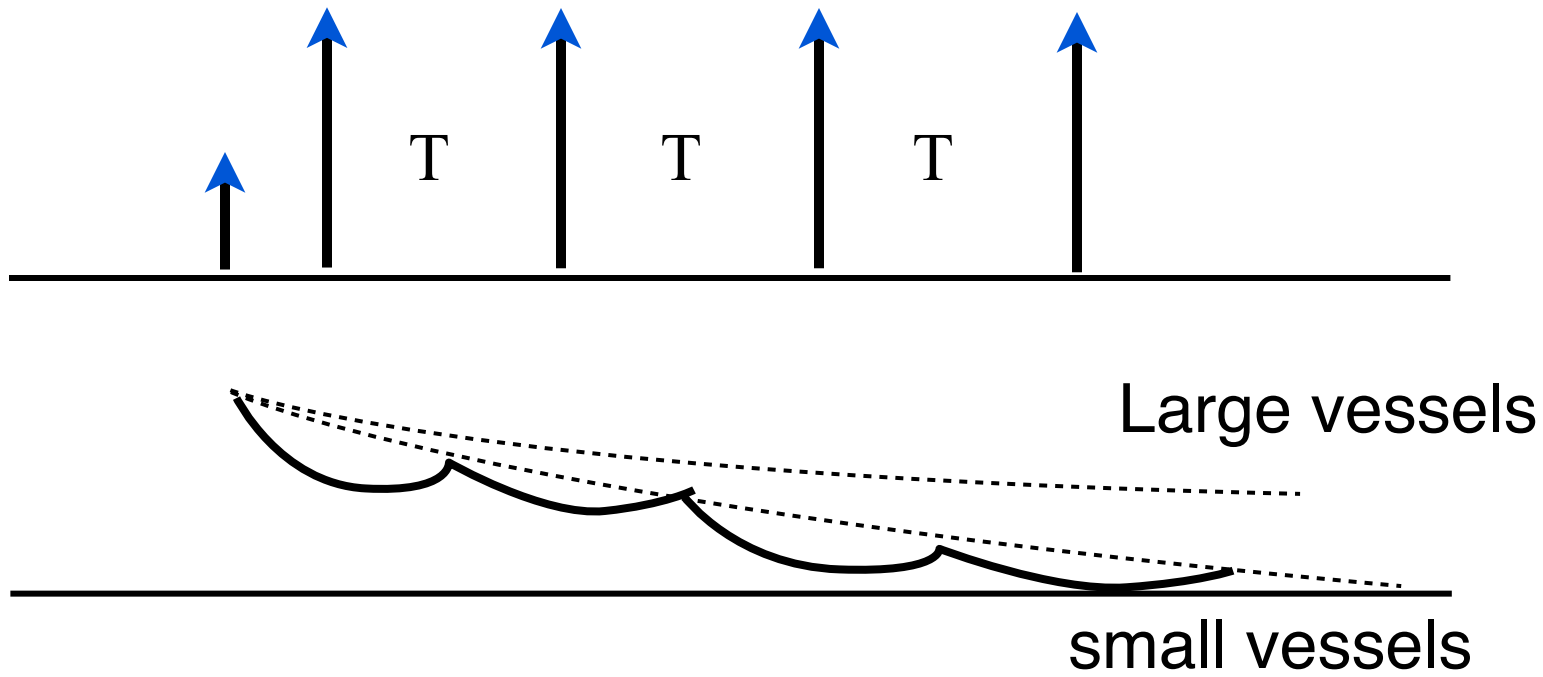


time-varying
dephasing
shows as T_2



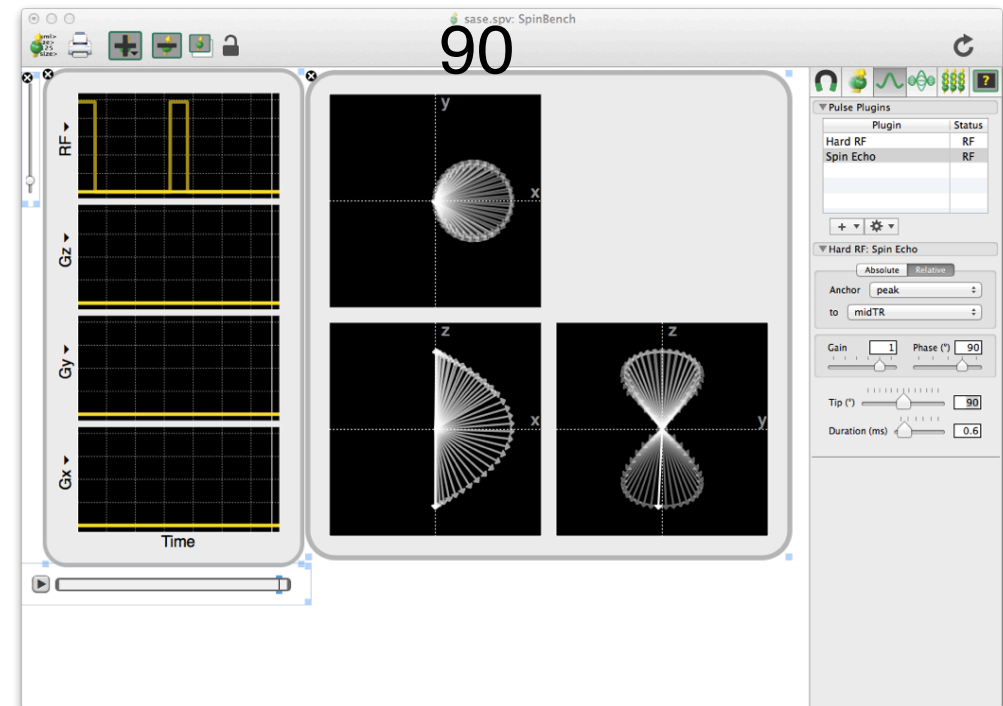
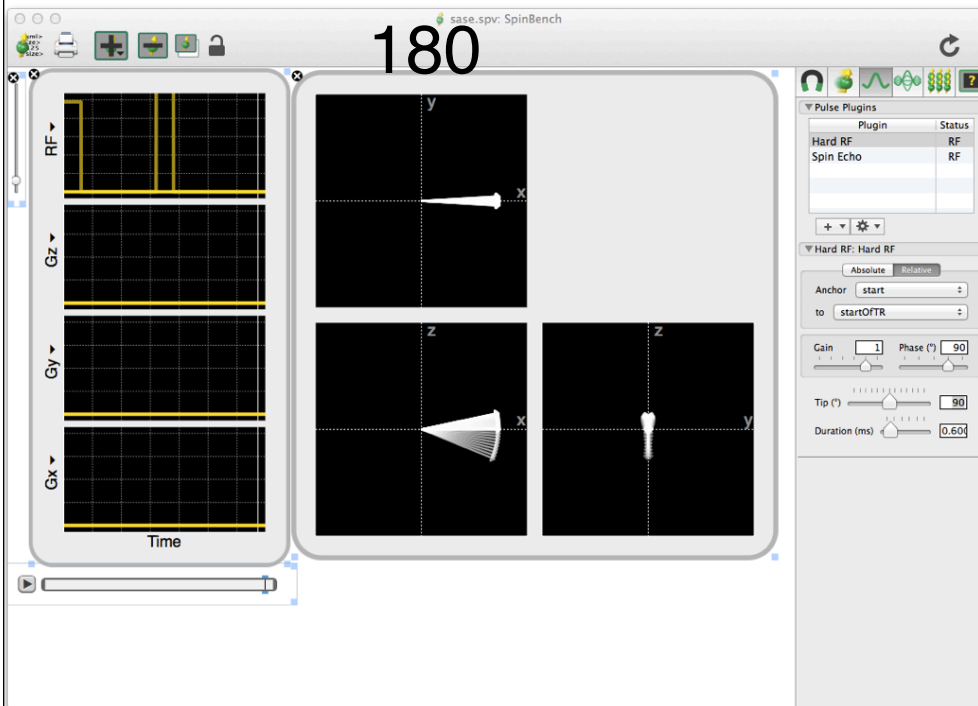
T_2^* Huge!
but, constant
dephasing

$T_2(T)$



Low Flip Angle Spin Echoes

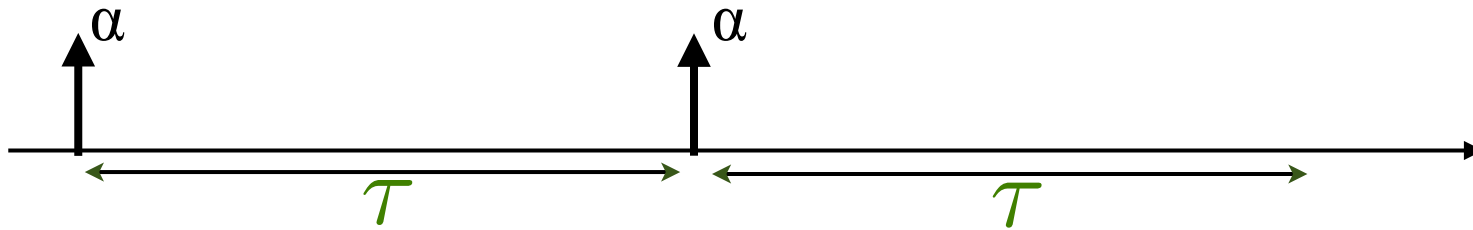
- Any pulse can produce a spin-echo
 - 180 maximally refocuses magnetization
 - Lower refocusing power with lower flip angle
 - 90 refocusses 1/2 (see homework)



Low Flip Angle Spin Echoes

- Refocusing RF result can be decomposed to several components:
 - Pass through
 - Refocused
 - Parasitic excitation
 - Magnetization stored in M_z

Low Flip Angle Spin Echoes



$$M = R_z(\omega_E \tau) R_x(\alpha) R_z(\omega_E \tau) R_x(\alpha) M_0$$

For 90 degrees:

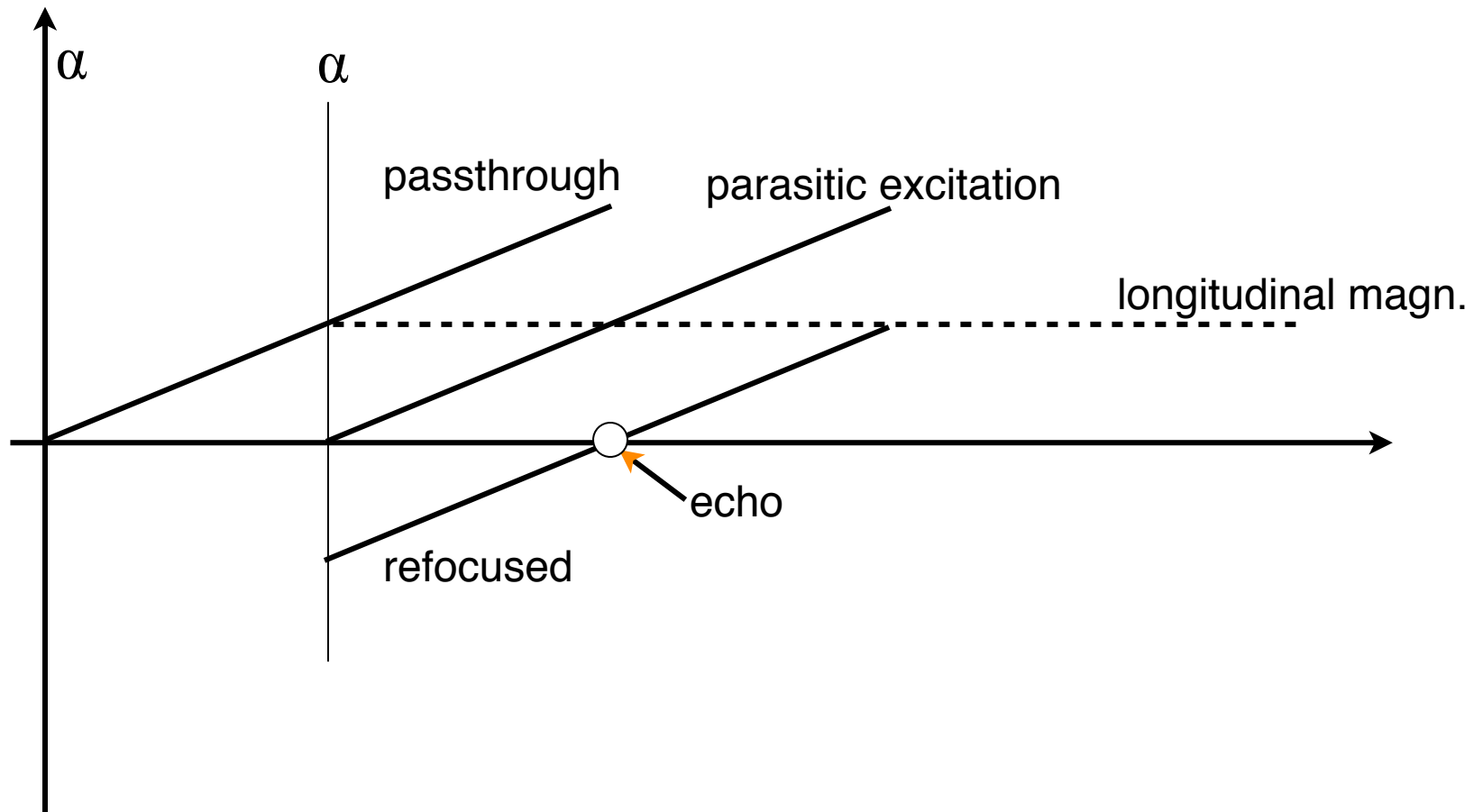
$$M_{xy} = \frac{1}{2} \sin(\omega_E 2\tau) M_0 - i \sin^2(\omega_E \tau) M_0$$

In general.... I think

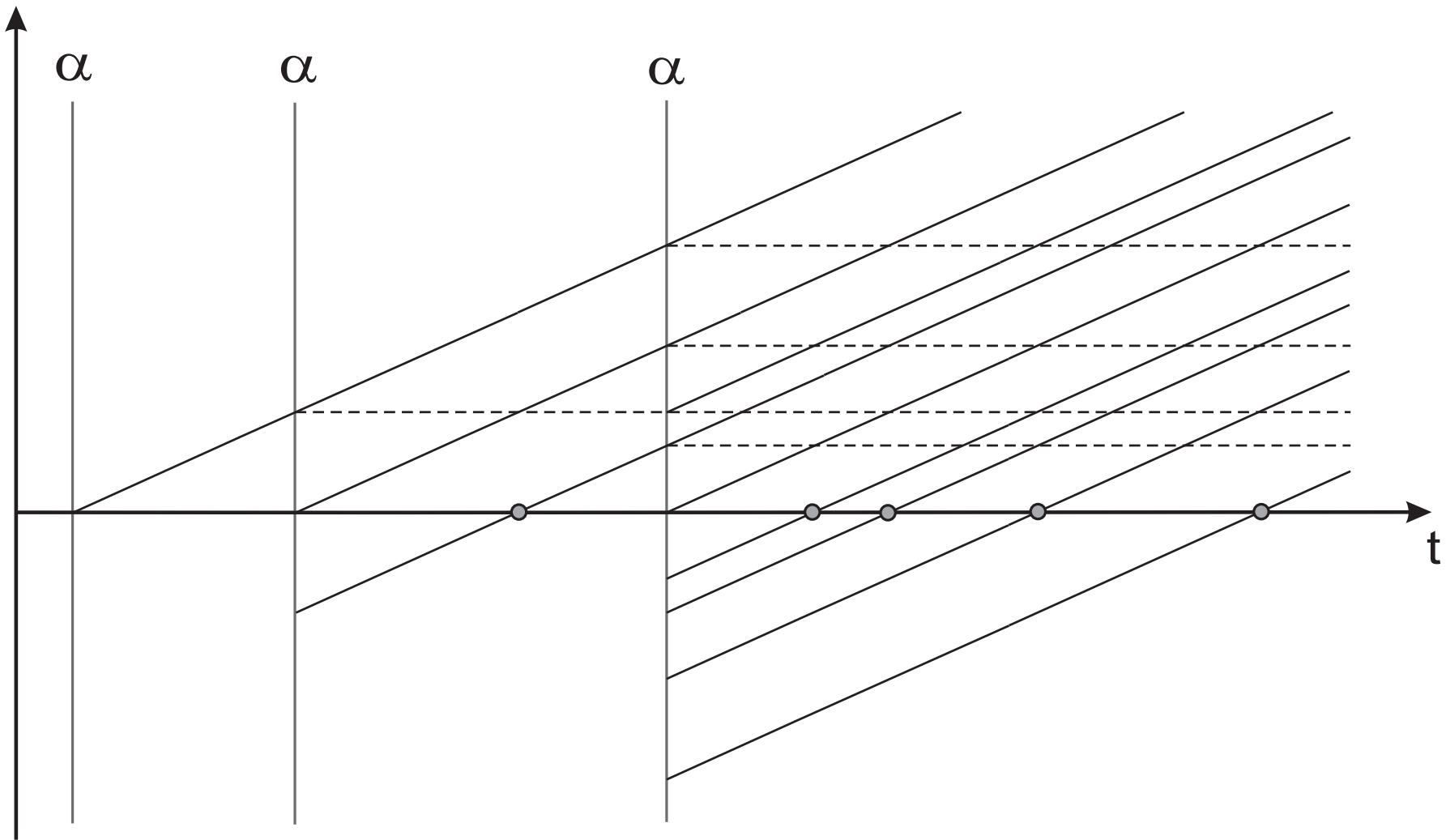
$$M_{xy} = \underbrace{A_1 \cos(2\omega_E \tau) + A_2 \sin(2\omega_E \tau)}_{\text{passthrough}} + \underbrace{B_1 \cos^2(\omega_E \tau) + B_2 \sin^2(\omega_E \tau)}_{\text{refocused}} + \underbrace{C_1 \cos(\omega_E \tau) + C_2 \sin(\omega_E \tau)}_{\text{parasitic}}$$

Phase Graphs

- Useful to find echo formations



Phase Graphs



* from document by Matthias Weigel

Extended Phase Graphs

- Each component has different magnitude and phase
- Echo consist of contribution from many pathways
- Can track to calculate magnitude & Phase of echo (Beyond scope... see document by Matthias Weigel)

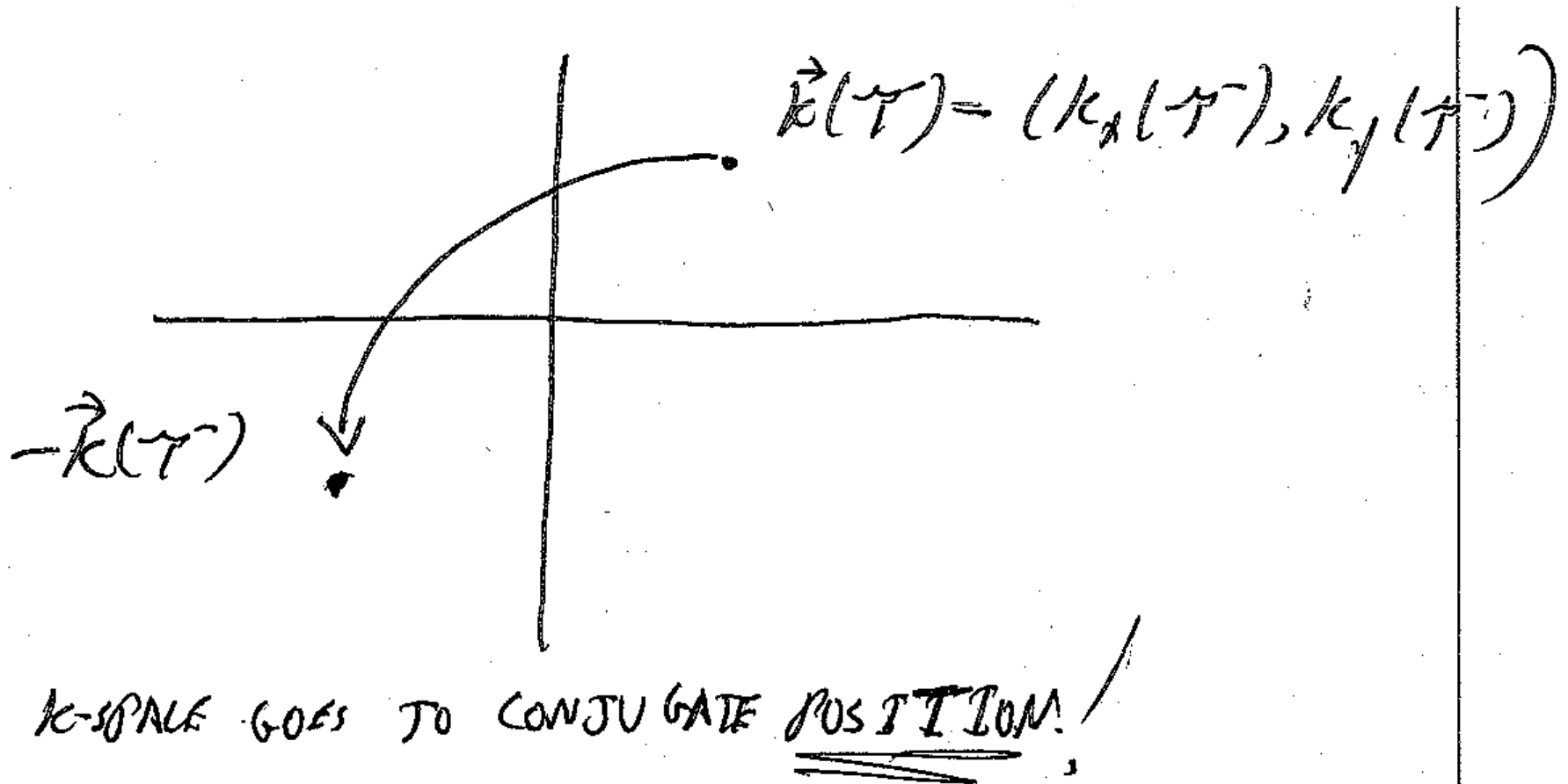
Spin Echo Imaging

- Spin echo negates phase -- Conjugation
- How to incorporate in pulse sequence?

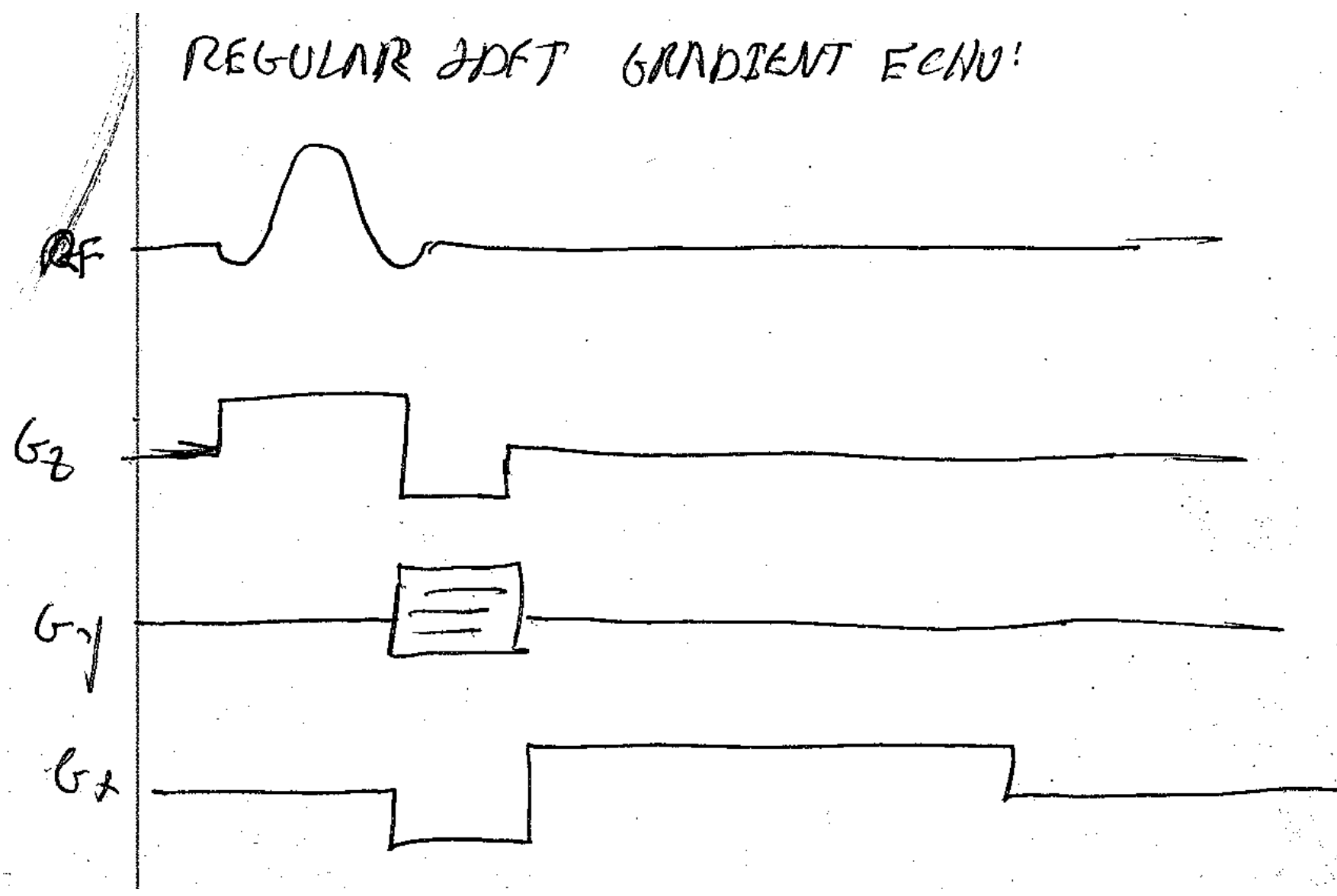
$$M_{xy}(\vec{r}, t) = M_{xy}(\vec{r}, 0)e^{-i2\pi(k_x(t)x + k_y(t)y)}$$

Spin Echo Imaging

- Spin echo negates phase -- Conjugation
- How to incorporate in pulse sequence?

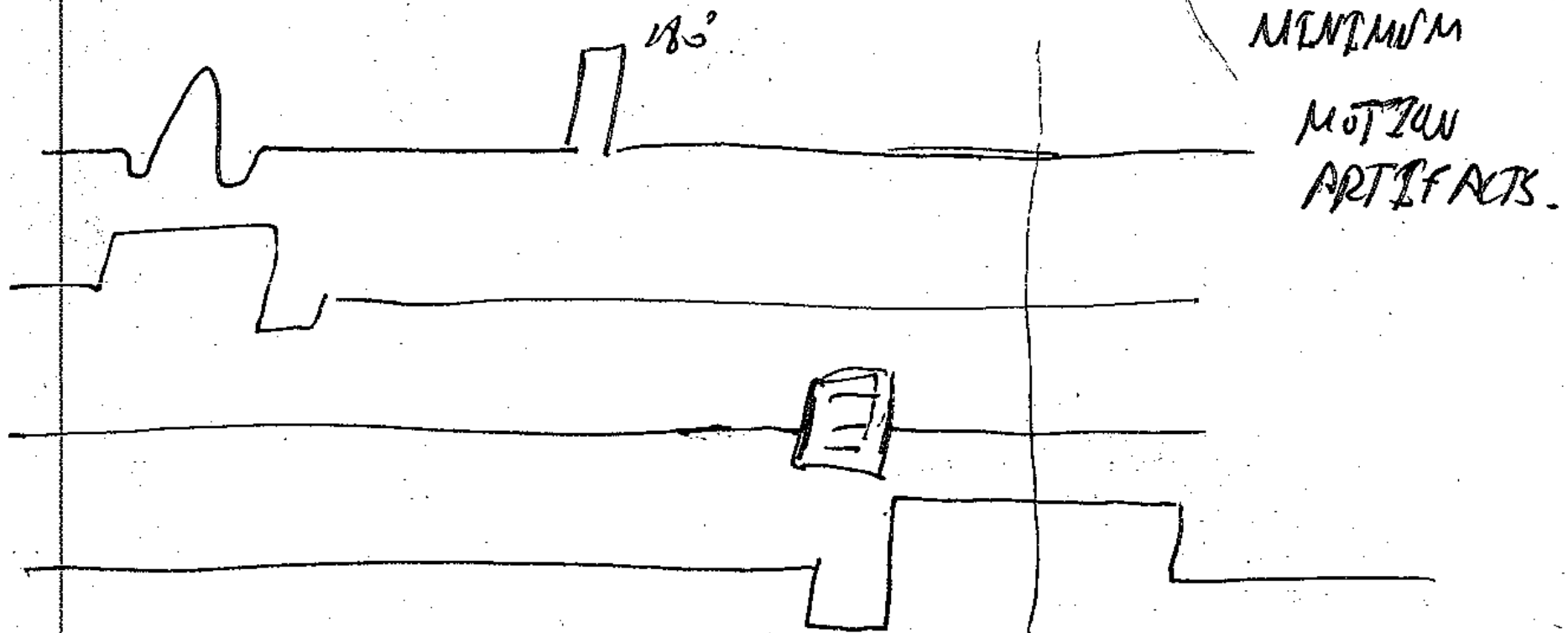


REGULAR JDT GRADIENT ECHO!



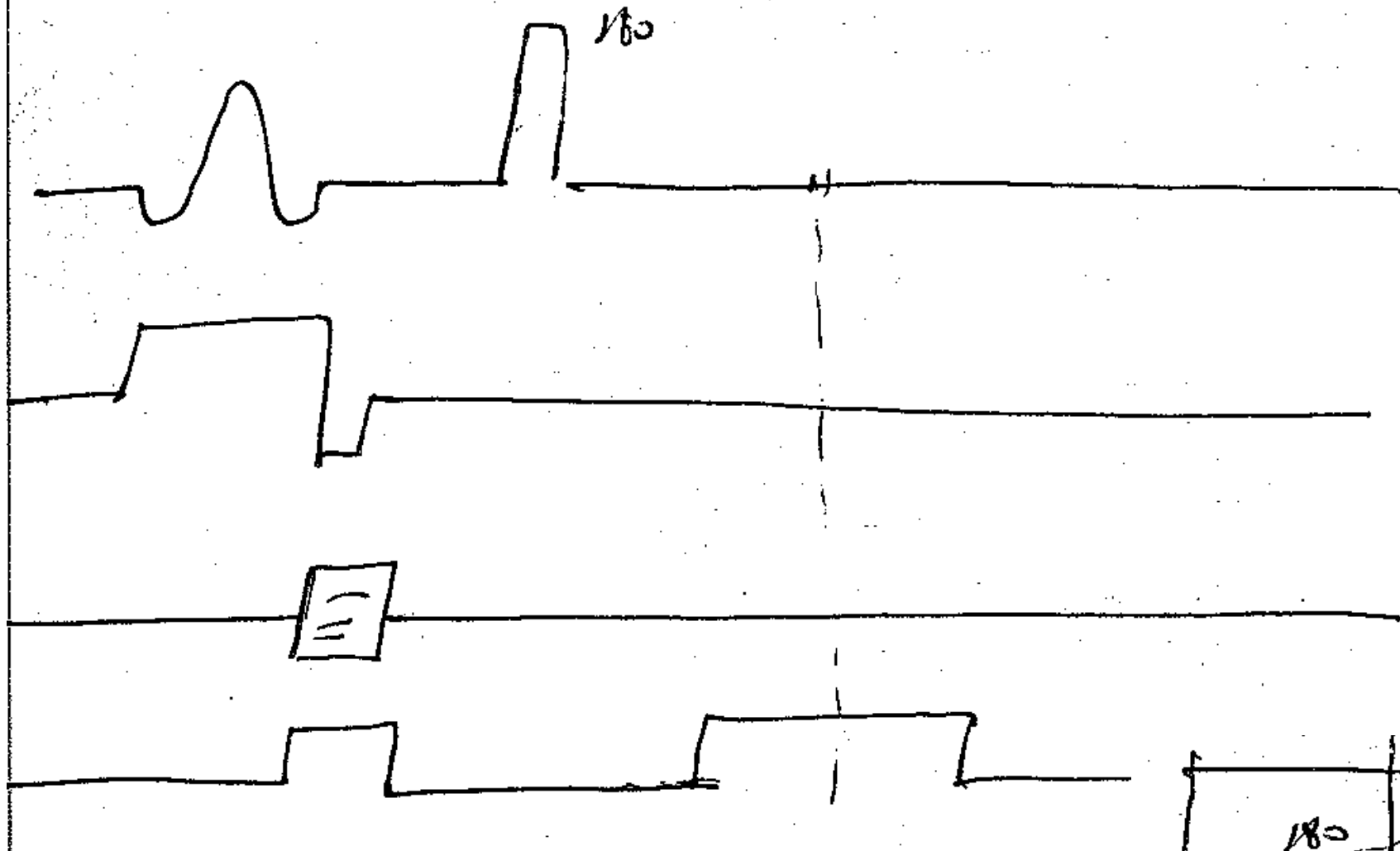
NEGATIVE GRADIENT LEFT OF 180

TRIVIAL ~~ADAPTATION~~ ADAPTATION:

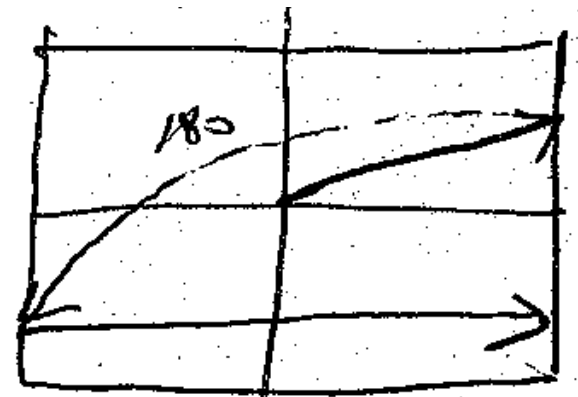


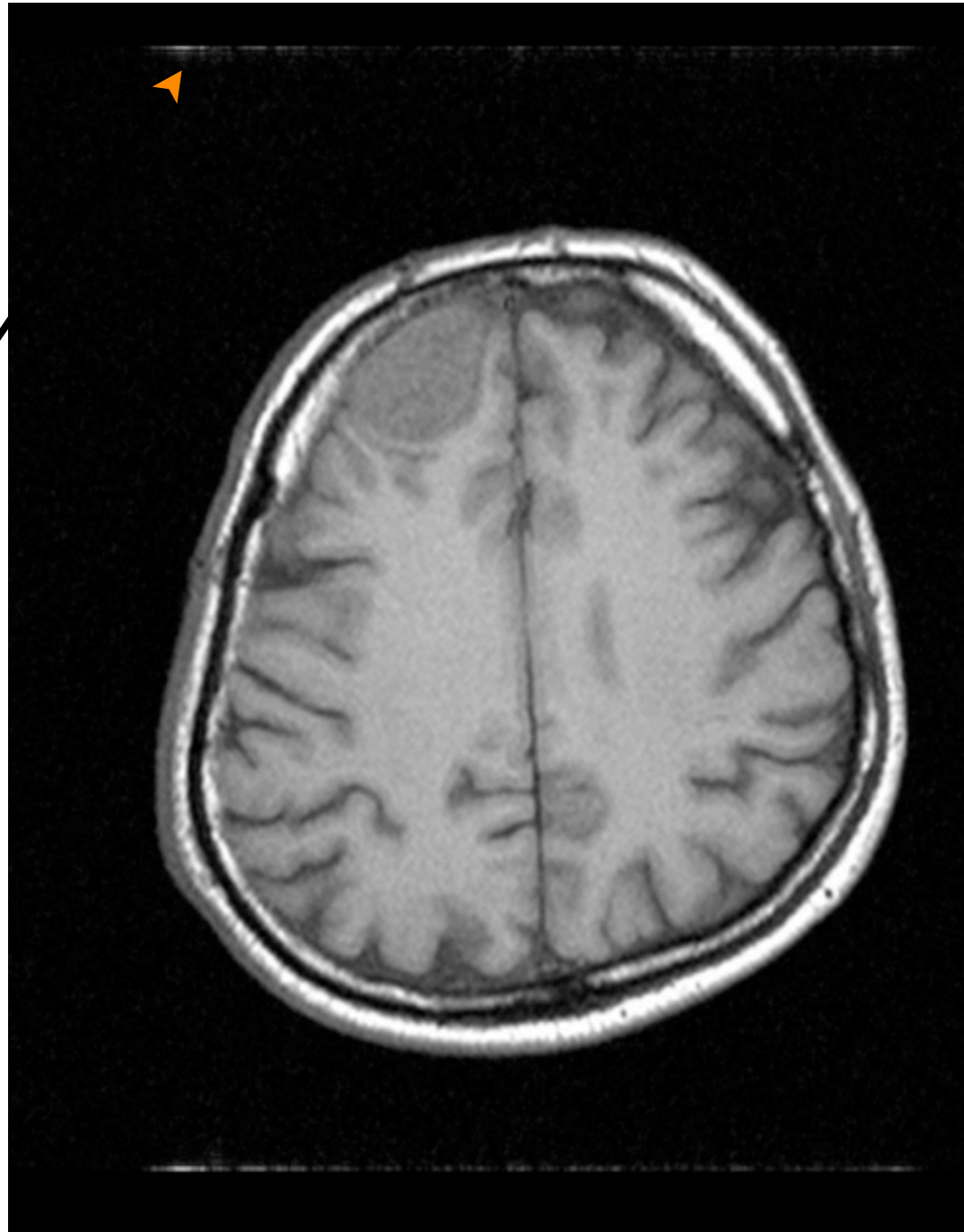
WANT GRADIENT ECHO TO HAPPEN WITH
SPIN-ECHO.

SHORTEST ECHO TIME



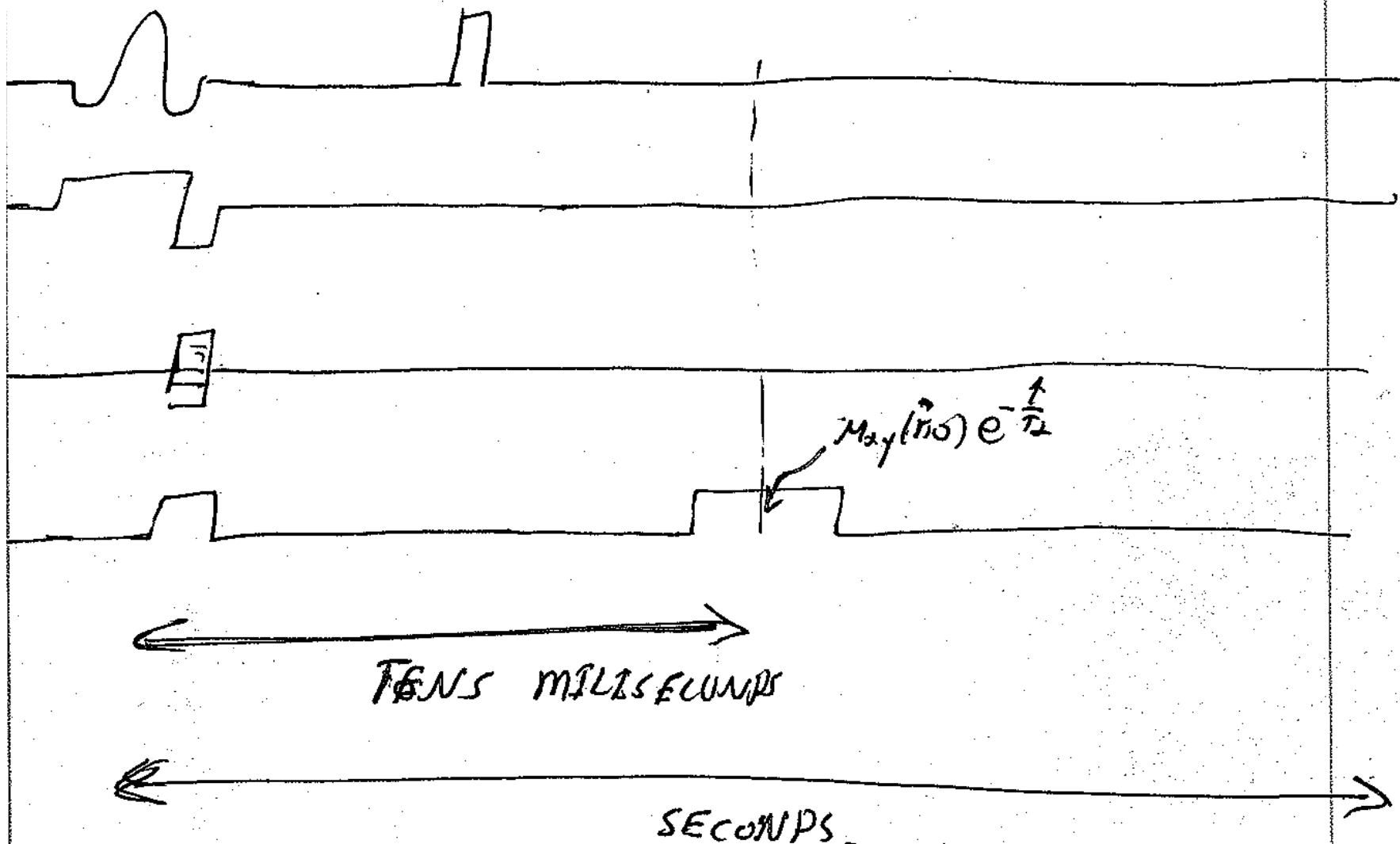
imperfect 180 causes parasitic
which is not phase encoded (HW)



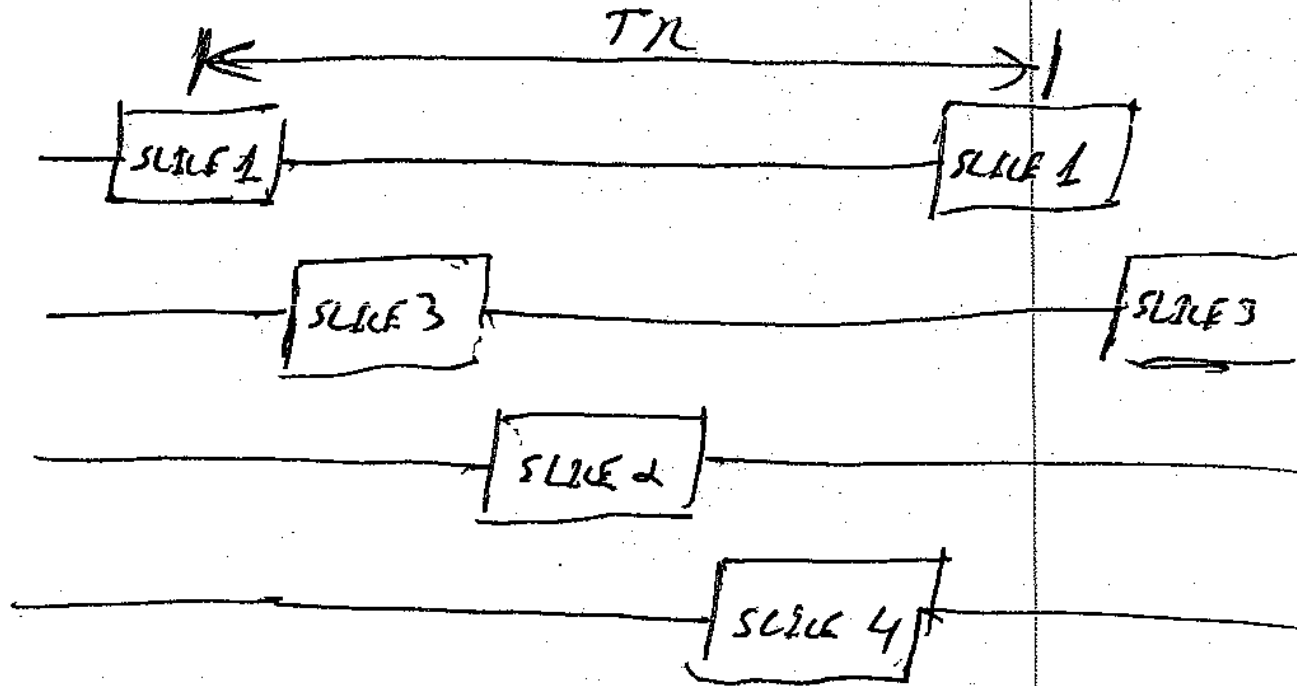
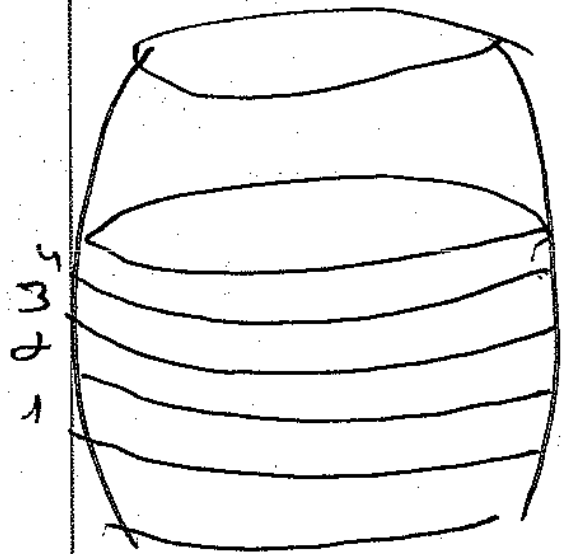


MULTI-SLICE

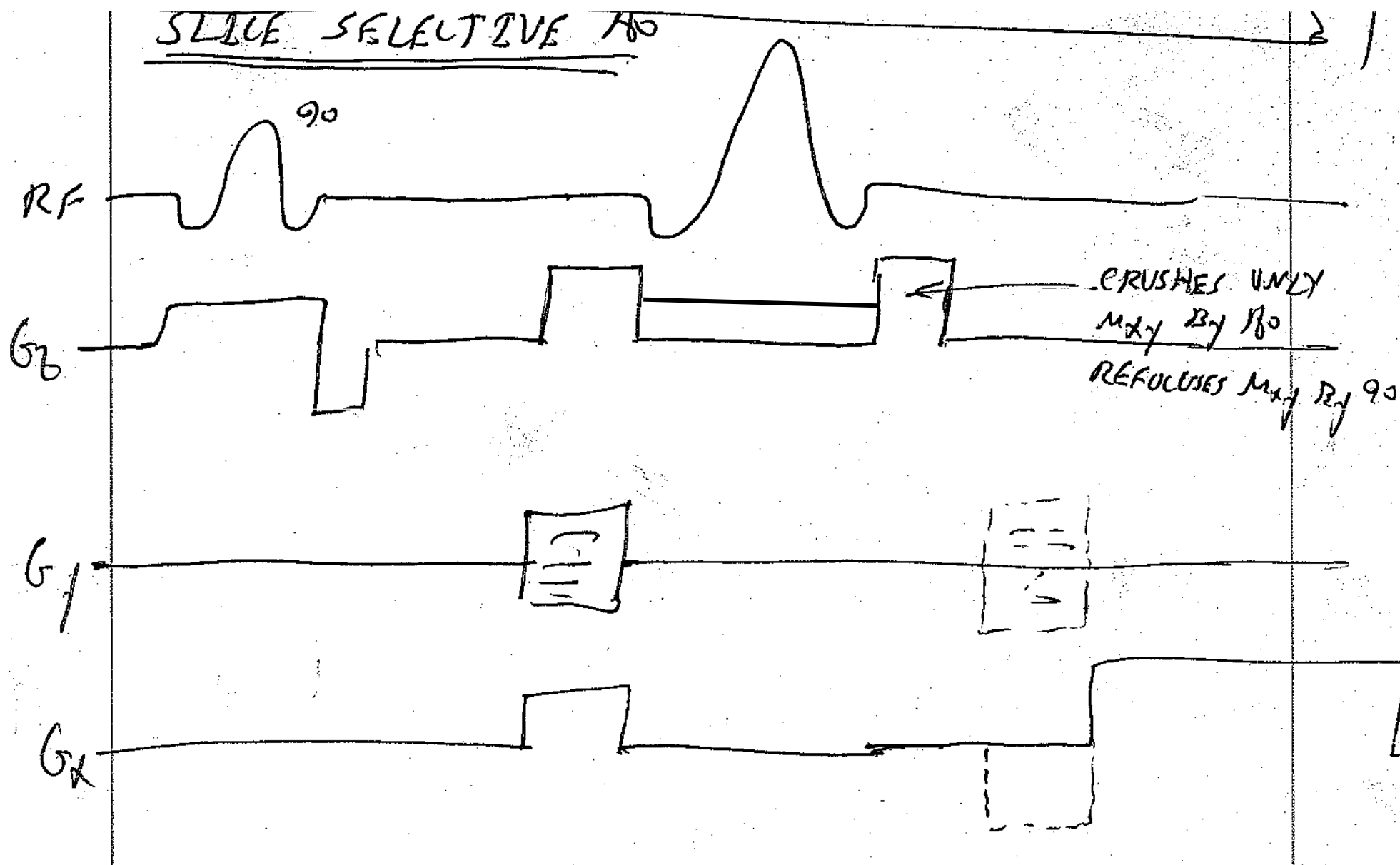
SAMPLE B_2 - WEIGHTED SPIN - ECHO.



INEFFICIENT! ADD MULTISLICE MAKE I/O SELECTIVE

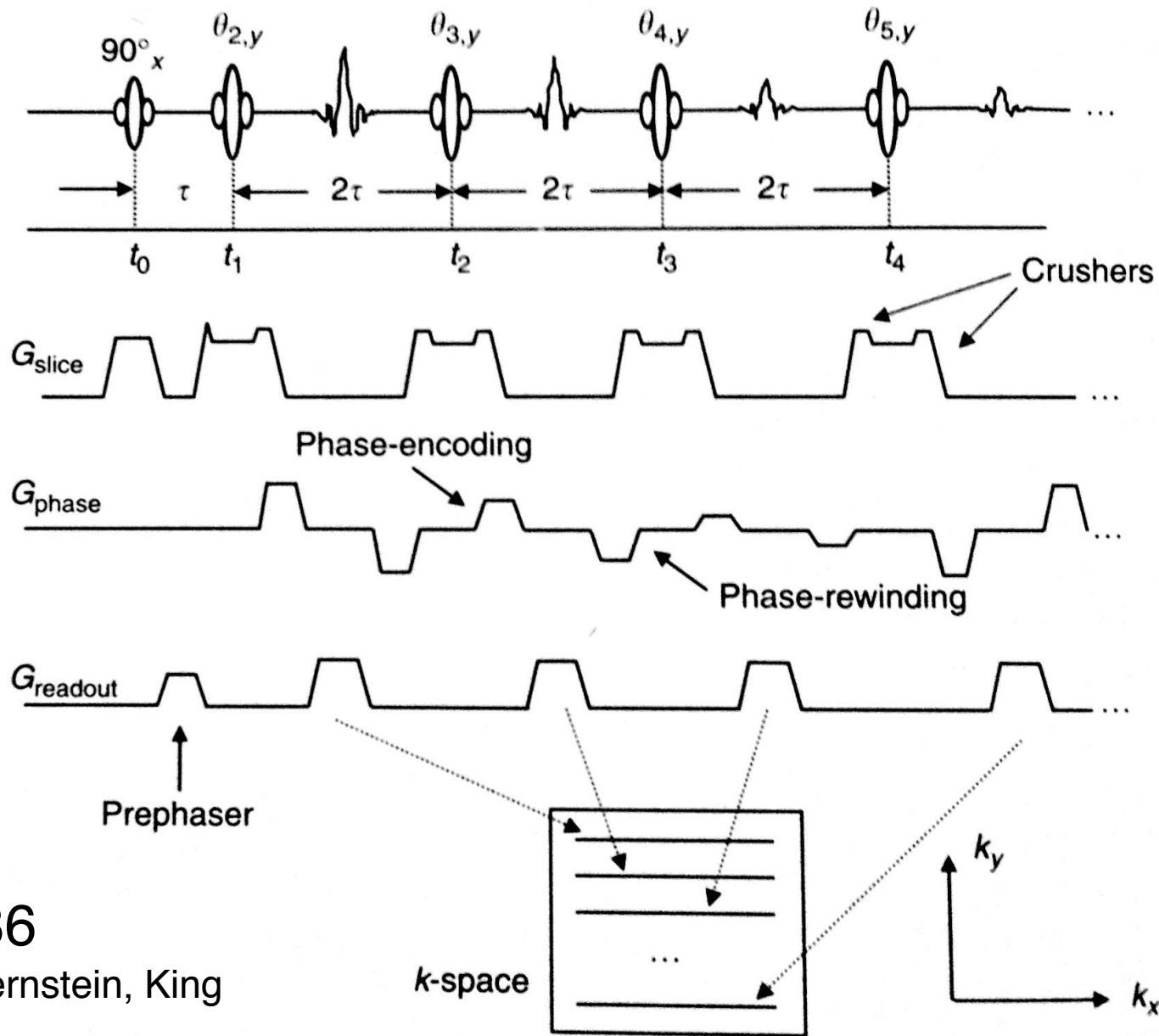


WHEN IMAGING ONE SLICE, OTHER RECOVER!
VERY EFFICIENT ALWAYS MIRRORING DATA.



what's the phase graph for all pathways?

Fast/Turbo Spin-Echo (RARE)



Hennig '86

figure from Bernstein, King and Zhou

Other non-idealities

- RF Inhomogeneity
 - Affects Excitation
 - Spatially varying flip-angle
 - Spatially varying phase
 - Can cause problem for fat saturation
 - Worse @ high field (wave effects)
 - Affects receive

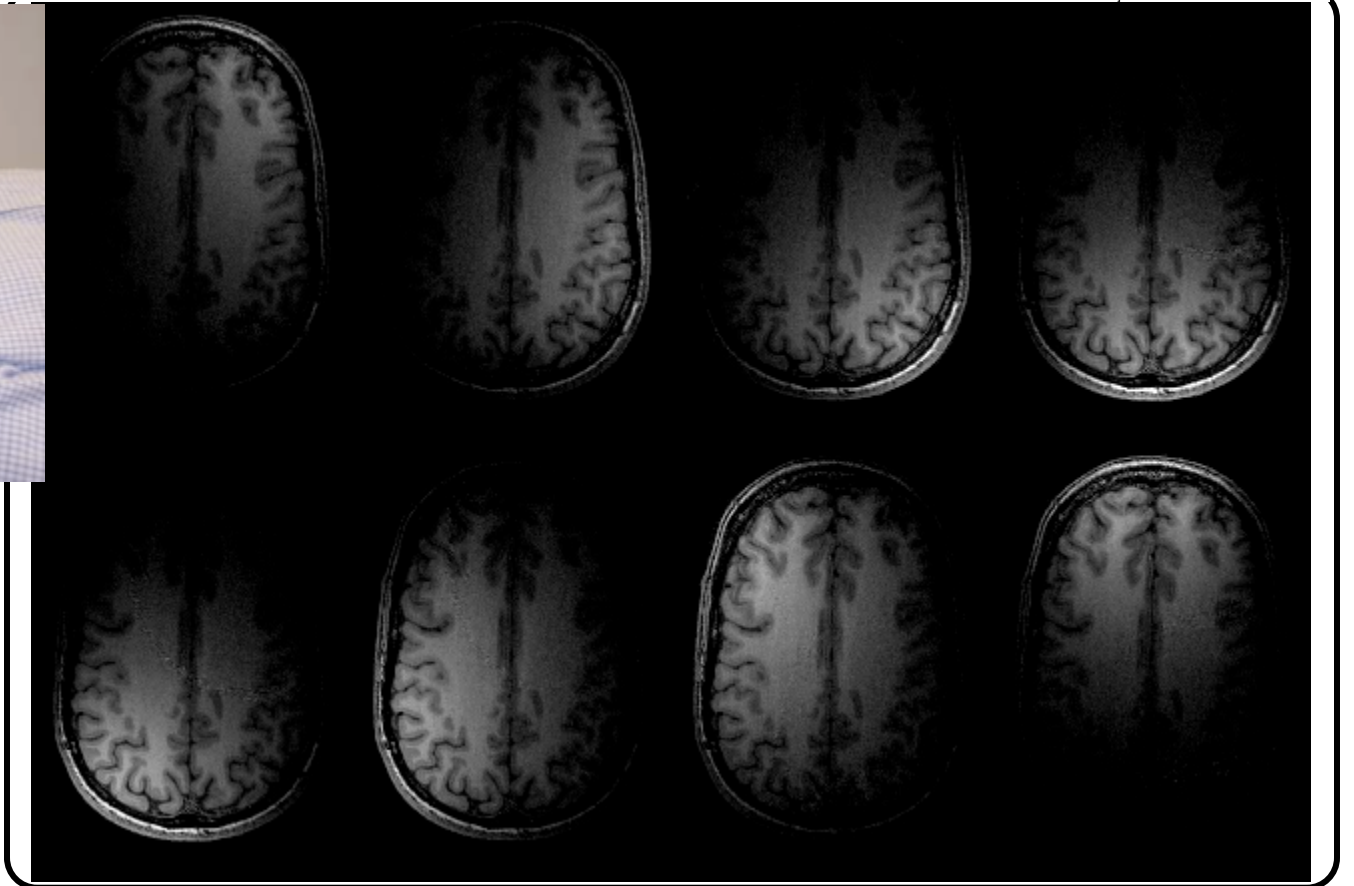
RF excitation inhomogeneity



AFFECTS RECEIVE :

(-) VARYING COMPLEX SENSITIVITY

$$s(t) = \iint_{\vec{r}} c(\vec{x}, y) m(\vec{x}, y) e^{-i2\pi(\vec{k} \cdot \vec{r})} d\vec{r}$$



CAN BE A GOOD THING: SENSITIVITY ENCODING.



$$S_1(b) = \int_{\mathcal{R}} m(\vec{r}, 0) C_1(\vec{r}) e^{-i2\pi(\vec{k} \cdot \vec{r})} d\vec{r}$$

$$S_2(b) = \int_{\mathcal{R}} m(\vec{r}, 0) C_2(\vec{r}) e^{-i2\pi(\vec{k} \cdot \vec{r})} d\vec{r}$$

TO RECONSTRUCT m_{C_1} & m_{C_2} need n encodings

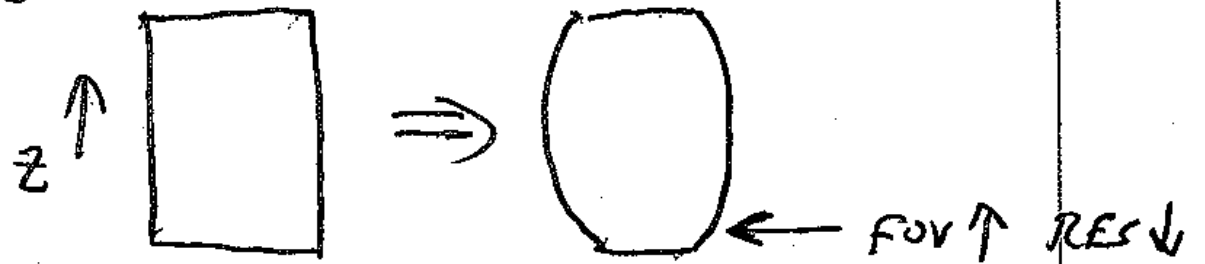
BUT TO RECONSTRUCT m need (maybe) $\frac{n}{2}$ encodings.

More later!

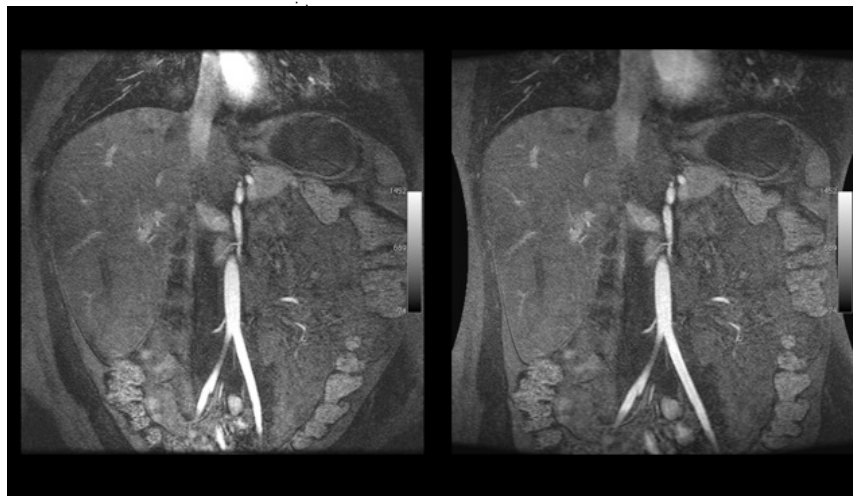
Gradient non-idealities

GRADIENTS

(→) Non Linearity - GRADIENT ROLLES OFF



EASY TO CORRECT BY INTERPOLATION



Concomitant Gradient (Maxwell Terms)

↳ CONCOMITANT GRADIENTS

IDEAL GRADIENTS VIOLATE MAXWELL EQU.

FOR CYLINDRICAL COILS USED IN MRI

$$B = B_0 + G_x x + G_y y + G_z z + \frac{1}{2B_0} \left[\frac{G_z^2}{4} (x^2 + y^2) + (G_x^2 + G_y^2) z^2 - G_x G_z x z - G_y G_z y z \right]$$

\uparrow
main

$\underbrace{\hspace{10em}}$
gradients

EXAMPLE! AT $z = 20\text{cm}$ $G_x = 40\text{ mT/m}$ $B_0 = 1.5\text{ T}$

Concomitant Gradient (Maxwell Terms)

↳ CONCOMITANT GRADIENTS

IDEAL GRADIENTS VIOLATE MAXWELL EQU.

FOR CYLINDRICAL COILS USED IN MRI

$$B = B_0 + G_x x + G_y y + G_z z + \frac{1}{2B_0} \left[\frac{G_z^2}{4} (x^2 + y^2) + (G_x^2 + G_y^2) z^2 - G_x G_z xz - G_y G_z yz \right]$$

\uparrow
main

└──────────┘
gradients

EXAMPLE! AT $z = 20\text{cm}$ $G_x = 40\text{ mT/m}$ $B_0 = 1.5\text{ T}$

$$\frac{G_x^2 z^2}{2B_0} = \frac{(40 \cdot 10^{-3} \cdot 0.2)^2}{2 \cdot 1.5} = 2.13 \cdot 10^{-5} \text{ T} = 14.2 \text{ ppm}$$

$$G = 10 \text{ mT/m} \Rightarrow 0.889 \text{ ppm}$$

$$B_0 = 0.7 \text{ T} \Rightarrow 65.3 \text{ ppm}$$

Concomitant Gradients

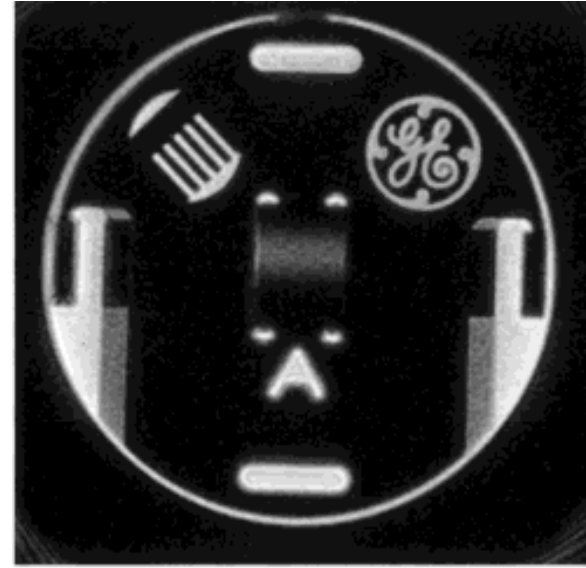
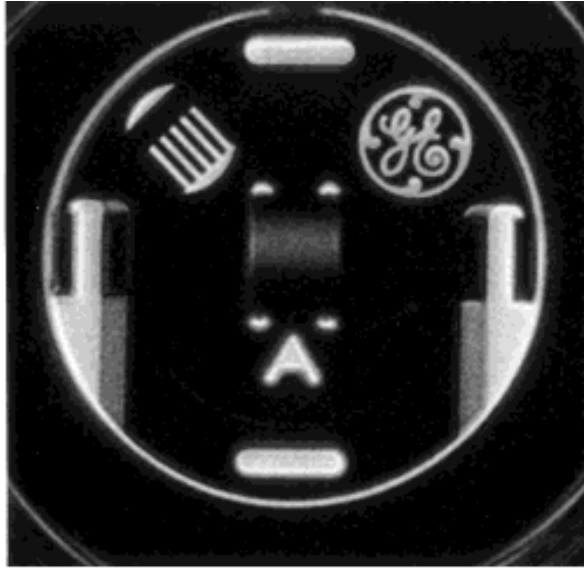
king et. al, MRM 41:103-112(1999)

$$\frac{G_x z^2}{2B_0}$$

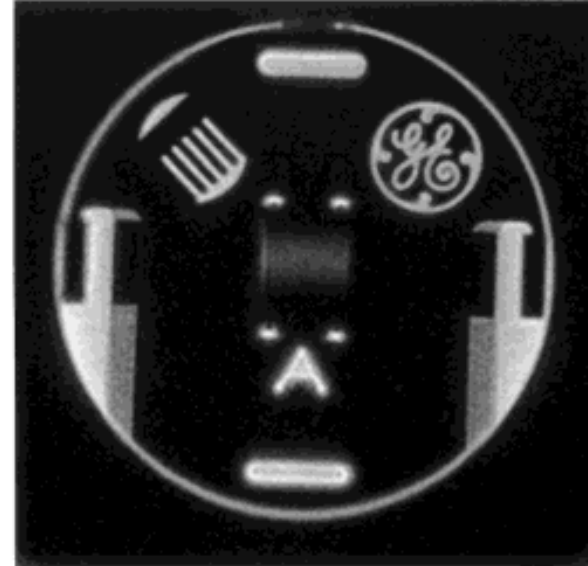
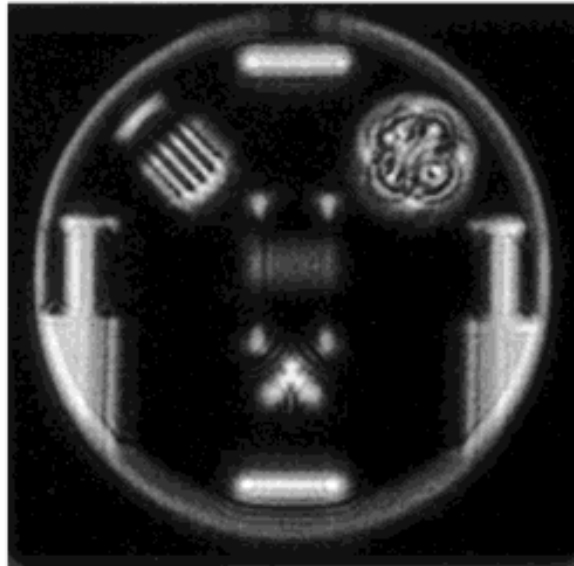
$g_m = 2.1 \text{ G/cm}$

$g_m = 0.524 \text{ G/cm}$

$z=0 \text{ cm}$



$z=10 \text{ cm}$



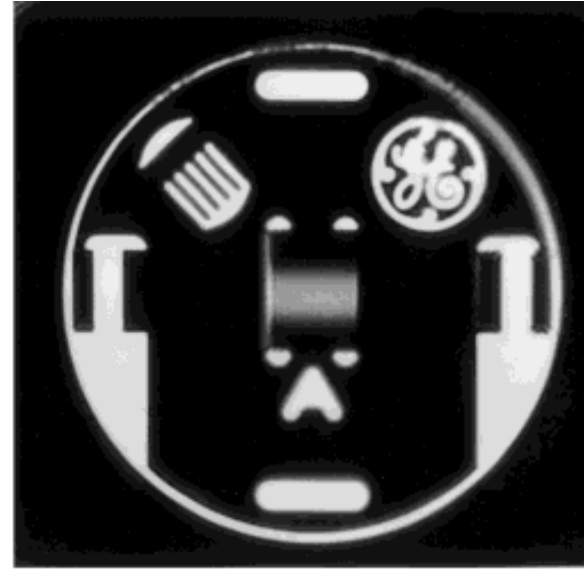
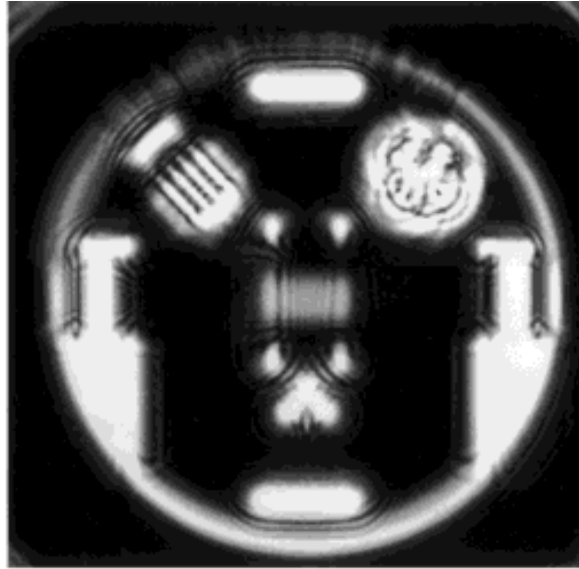
@z=10cm

king et. al, MRM 41:103-112(1999)

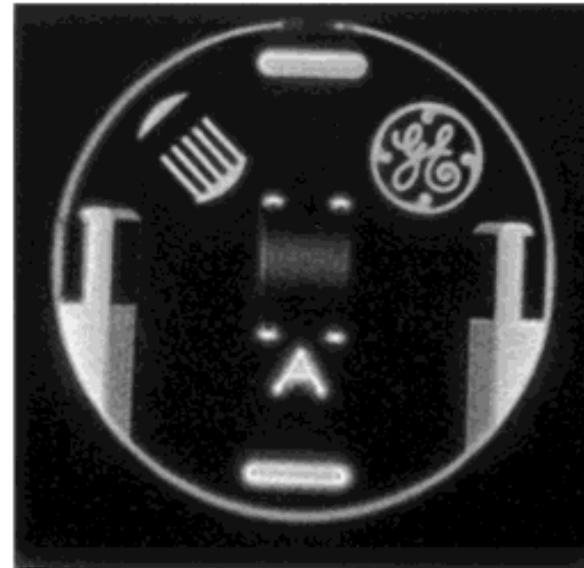
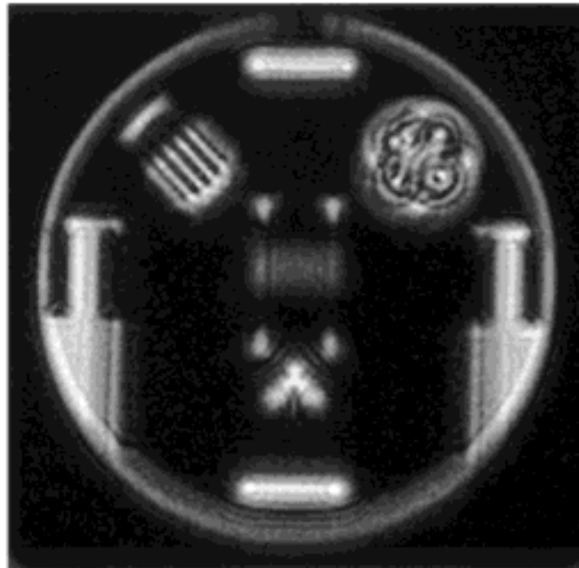
$g_m = 2.1 \text{ G/cm}$

$g_m = 0.524 \text{ G/cm}$

1.0 T



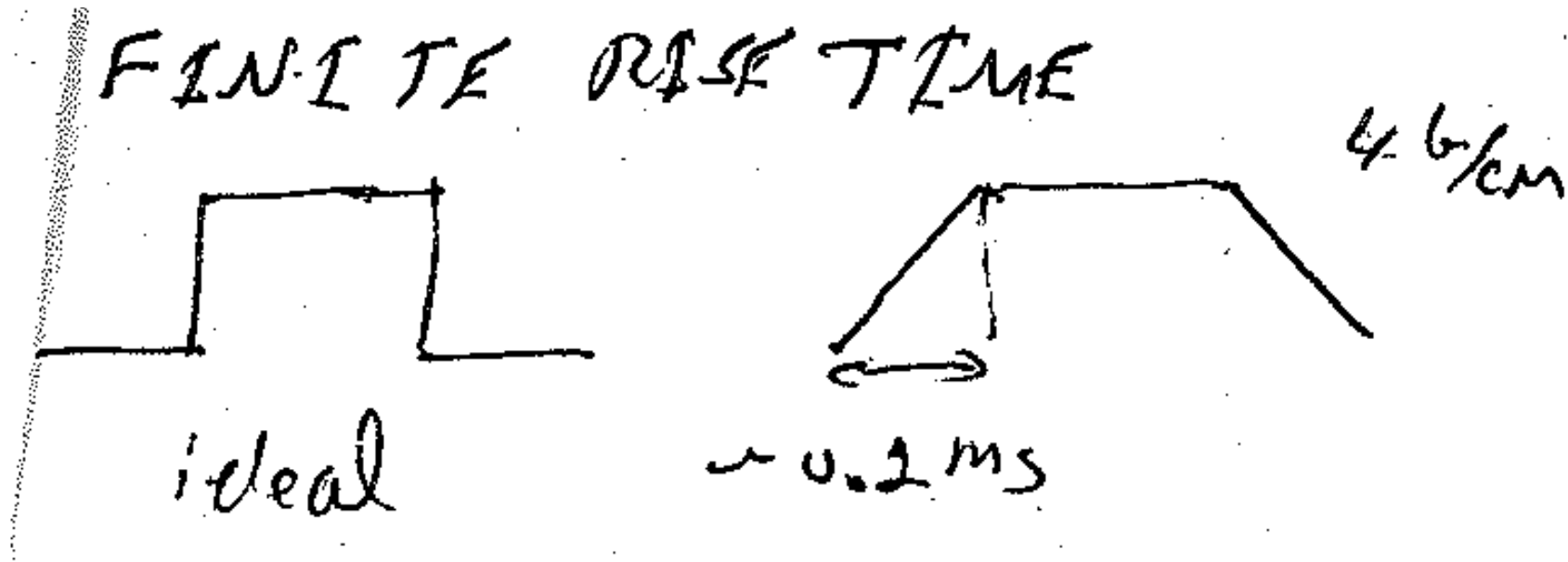
1.5 T



a

b

Gradient Finite Rise-Time

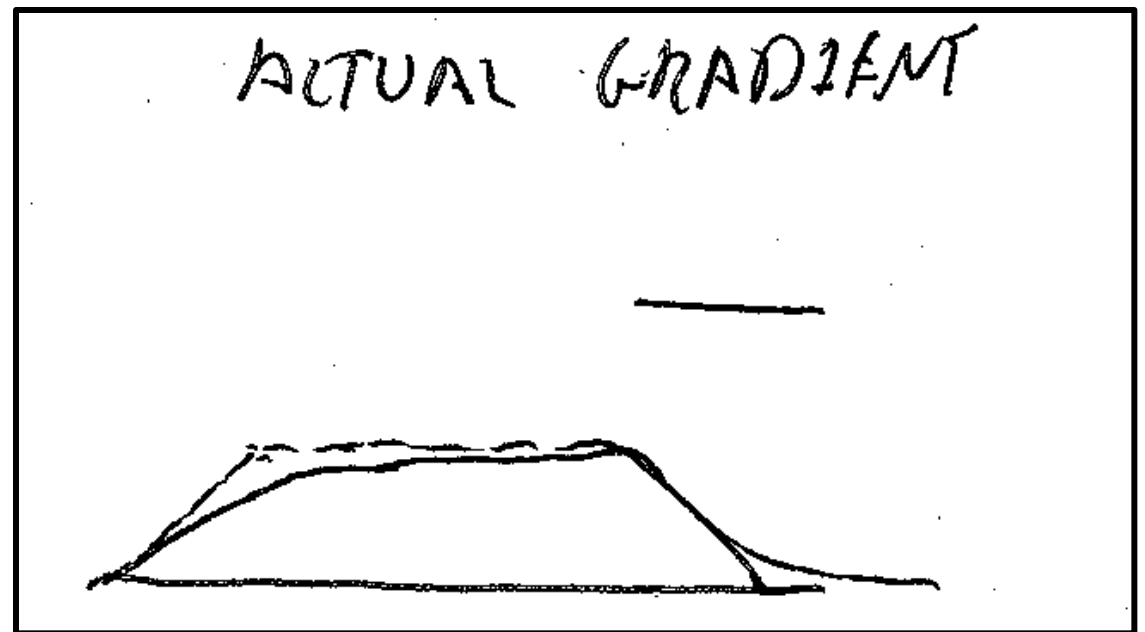
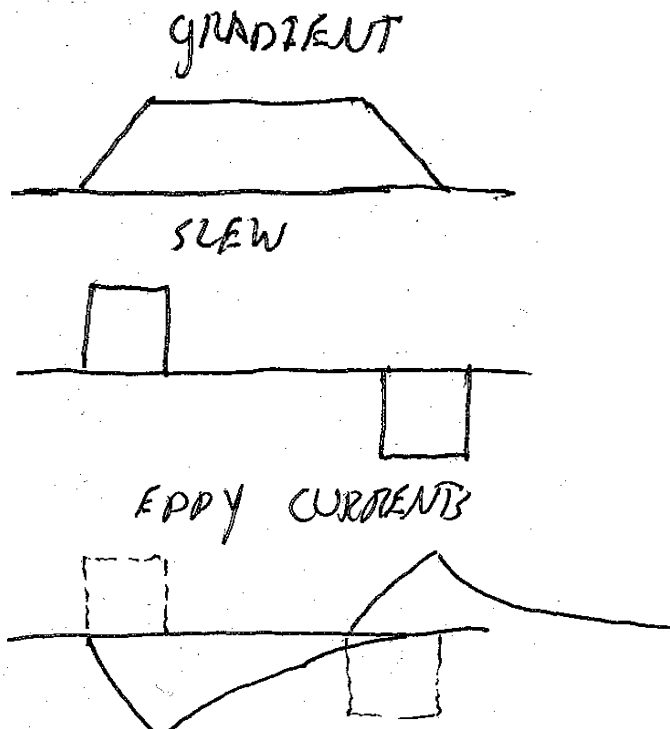


Even finite-rise-time rate! (often ignored)

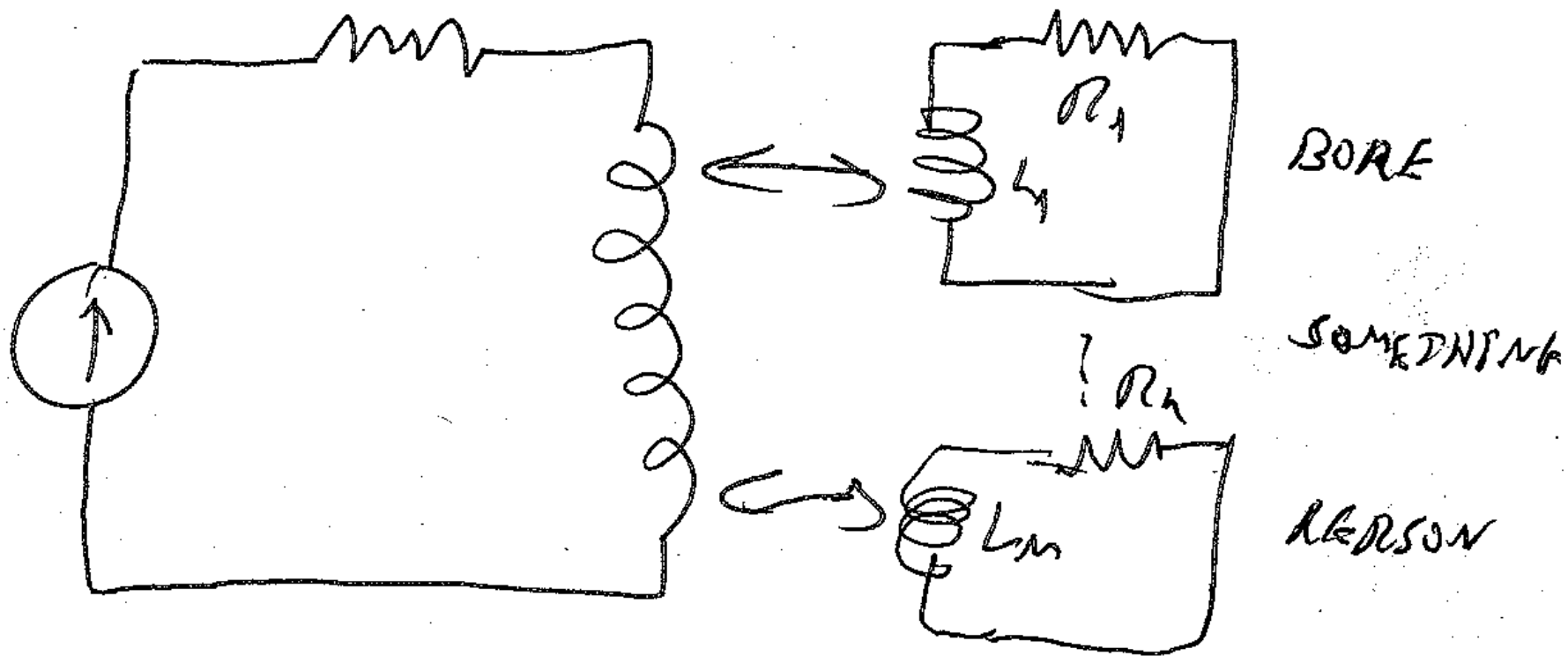
Eddy Currents

Generated By ELECTRIC FIELD FROM CHANGING
MAGNETIC FLUX.

BUILD UP IN TIME VARYING GRADIENTS AND
DELAY IN CONSTANT \propto SLEW RATE



$$B_e(\vec{r}, t) = b_0(t) + \vec{r} \cdot \vec{g}(t) + \dots +$$



Eddy Currents

$$g(t) = - \frac{dB}{dt} \otimes e(t)$$

$$e(t) = H(t) \sum \alpha_n e^{-\frac{t}{\tau_n}}$$

↑ step function

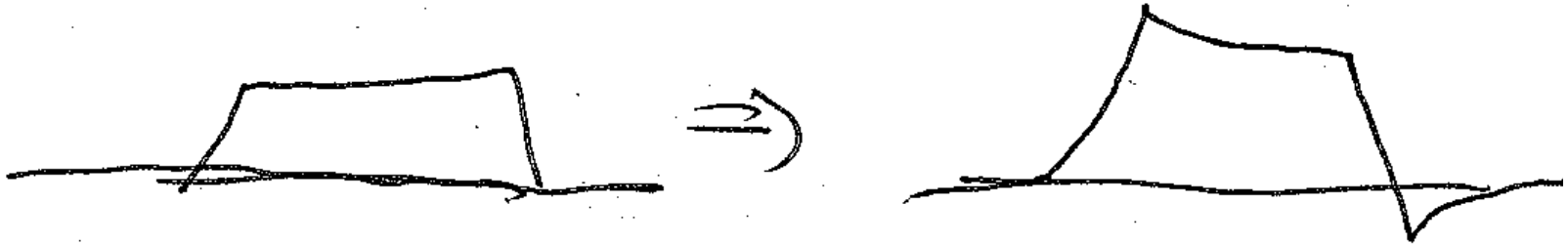
↑ intensity

↑ time constant

Pre-emphasis

PREEMPHASIS!

$\beta_0 \rightarrow$ change editer phase
gradient



FOR SHORT TIME constants

$$G_{\text{net}}(t) = G_{\text{applied}}(t - \alpha \tau)$$

Benstein ch. 10.3