
Homework 1
Due: Tuesday, September 8, at 12:30pm

Topic Probability review.

Reading Durrett, “Review of probability” (unnumbered first chapter); Gallager, chapter 2

Problem 1 (*events*) (*Durrett, Exercise 1.13*)

Give an example of three events A, B , and C such that (i) A and B are independent, (ii) A and C are independent, but (iii) A and $B \cup C$ are *not* independent. Can this happen if B and C are disjoint?

Problem 2 (*poisson random variable*)

Let X have a Poisson distribution with parameter $\lambda > 0$. We suppose that λ itself is a random variable with exponential probability density function (PDF) with parameter θ .

- (a) Determine the probability mass function (PMF) of X .
- (b) Determine the conditional PDF of λ given that $X = k$.

Problem 3 (*max and min of i.i.d. random variables*)

Let X be a random variable with cumulative distribution function (CDF) $F_X(x)$. Find the CDFs of the following random variables:

- (a) The maximum of n i.i.d. random variables each with the same distribution as X .
- (b) The minimum of n i.i.d. random variables each with the same distribution as X .
- (c) The difference between the random variables defined in Parts (a) and (b), respectively. You can assume for this part that X has a PDF.

Problem 4 (*expectation and variance*)

- (a) Express the expectation of the sum of n random variables in terms of the individual expectations. Under what conditions is the expectation of the sum equal to the sum of the expectations?
- (b) Is it true that the expectation of a product of n random variables is always the product of the individual expectations? If so, prove it. If not, give a counter-example and a condition on the random variables such that you can prove that the statement is true.
- (c) Compute the variance of a sum of n random variables in terms of the variances of the individual random variables and their covariances. Under what condition is the variance of the sum equal to the sum of the variances?

Problem 5 Exercise 2.1 in Gallager’s notes.

Problem 6 Exercise 2.4 in Gallager’s notes.