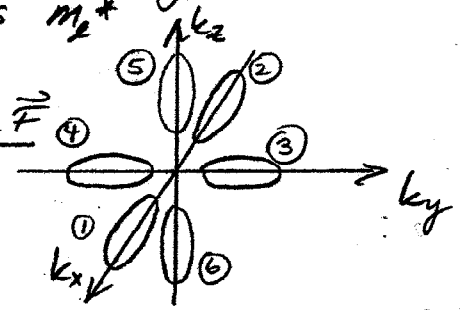
(1) Effective Mass

a) 6 "ellipsoidal conduction band minima in silicon
each minimum: two-fold symmetric transverse effective mass m_t^* and a longitudinal effective mass m_l^*



<u>Show that the acceleration $\frac{d\vec{v}}{dt}$ is always along the direction of the force $\vec{F}$</u>

$$\vec{F} = \overleftrightarrow{M}\vec{a} = \overleftrightarrow{M}\frac{d\vec{v}}{dt}$$

find effective mass tensor $\overleftrightarrow{M}$

$$[M^{-1}]_{ij} = \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_i \partial k_j} = \frac{1}{\hbar}\frac{\partial}{\partial k_j}\left(\frac{1}{\hbar}\frac{\partial E(k)}{\partial k_i}\right) = \frac{1}{\hbar}\frac{\partial v_i}{\partial k_j} \qquad \left(v_i = \frac{1}{\hbar}\frac{\partial E}{\partial k_i}\right)$$

average over the 6 conduction bands:

$$[M^{-1}]_{ij} = \frac{1}{6}\sum_{k=1}^{6}\frac{\partial^2 E(k)}{\partial k_i \partial k_j}$$

bands ① & ②: $E(k) = E_c + \frac{\hbar^2 k_x^2}{2m_l^*} + \frac{\hbar^2(k_y^2+k_z^2)}{2m_t^*} = E_c + \frac{p_x^2}{2m_l^*} + \frac{p_y^2+p_z^2}{2m_t^*}$

bands ③ & ④: $E(k) = E_c + \frac{\hbar^2(k_x^2+k_z^2)}{2m_t^*} + \frac{\hbar^2 k_y^2}{2m_l^*}$

bands ⑤ & ⑥: $E(k) = E_c + \frac{\hbar^2(k_x^2+k_y^2)}{2m_t^*} + \frac{\hbar^2 k_z^2}{2m_l^*}$

$$\overleftrightarrow{M}^{-1} = \frac{1}{6}\left\{2\begin{bmatrix}\frac{1}{m_l^*} & 0 & 0\\ 0 & \frac{1}{m_t^*} & 0\\ 0 & 0 & \frac{1}{m_t^*}\end{bmatrix} + 2\begin{bmatrix}\frac{1}{m_t^*} & 0 & 0\\ 0 & \frac{1}{m_l^*} & 0\\ 0 & 0 & \frac{1}{m_t^*}\end{bmatrix} + 2\begin{bmatrix}\frac{1}{m_t^*} & 0 & 0\\ 0 & \frac{1}{m_t^*} & 0\\ 0 & 0 & \frac{1}{m_l^*}\end{bmatrix}\right\}$$

$$= \frac{1}{3}\left(\frac{2}{m_t^*} + \frac{1}{m_l^*}\right)\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\vec{F} = \overleftrightarrow{M}\frac{d\vec{v}}{dt} = 3\left(\frac{2}{m_t^*} + \frac{1}{m_l^*}\right)^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\frac{d\vec{v}}{dt} = 3\left(\frac{2}{m_t^*} + \frac{1}{m_l^*}\right)\frac{d\vec{v}}{dt}$$

$\Rightarrow$ acceleration along the direction of force

<u>"mobility" mass</u> – apply electric field $\vec{\mathcal{E}}$ and include collisions

$$\vec{F} = q\vec{\mathcal{E}} = \overleftrightarrow{M}\left(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}\right)$$

in steady state: $\vec{v} = \tau\overleftrightarrow{M}^{-1}q\vec{\mathcal{E}} = \frac{1}{3}\left(\frac{2}{m_t^*} + \frac{1}{m_l^*}\right)\tau q\vec{\mathcal{E}} =$

$$= \frac{q\tau}{m_\mu^*}\vec{\mathcal{E}} = \mu\vec{\mathcal{E}},$$

where μ is mobility and m_μ^* is "mobility" mass

$$\Rightarrow \boxed{\frac{1}{m_\mu^*} = \frac{1}{3}\left(\frac{2}{m_t^*} + \frac{1}{m_l^*}\right)}$$

b) find density of states effective mass m_N^*

density of states: $D(E)dE = \frac{dN}{V} = \frac{2}{h^3} dp_x dp_y dp_z$

nondegenerate Fermi distribution in the conduction band:

$$f_c(E) \approx \exp\left(\frac{E_F - E}{kT}\right)$$

calculate electron density in band ①; by symmetry the electron densities in all six bands are equal

$$n_{total} = 6\int_{E_c}^{\infty} f_c(E)D(E)dE = 6\int_{-\infty}^{\infty} f_c(\vec{p})D(\vec{p})d\vec{p}$$

$$= 6\iiint_{-\infty}^{\infty} \exp\left[\frac{E_F - E_c - \frac{p_x^2}{2m_l^*} - \frac{p_y^2+p_z^2}{2m_t^*}}{kT}\right]\frac{2}{h^3}dp_x dp_y dp_z$$

$$= \frac{12}{h^3}\exp\left[\frac{E_F-E_c}{kT}\right]\int_{-\infty}^{\infty}\exp\left(\frac{-p_x^2}{2m_l^*kT}\right)dp_x\int_{-\infty}^{\infty}\exp\left(\frac{-p_y^2}{2m_t^*kT}\right)dp_y\int_{-\infty}^{\infty}\exp\left(\frac{-p_z^2}{2m_t^*kT}\right)dp_z$$

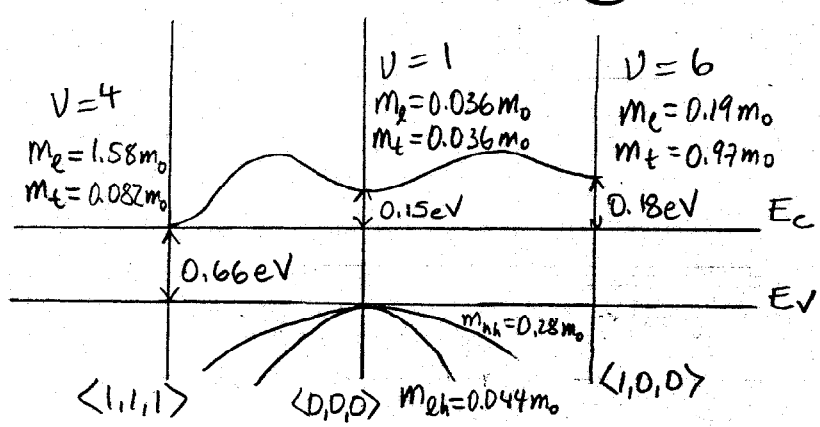
$$= \frac{12}{h^3}\exp\left[\frac{E_F-E_c}{kT}\right]\sqrt{2\pi m_l^* kT}\left(\sqrt{2\pi m_t^* kT}\right)^2$$

$$= 2\left(\frac{2\pi 6^{2/3} m_l^{*1/3} m_t^{*2/3}}{h^2}\right)^{3/2}\exp[E_F - E_c]$$

$$= 2\left(\frac{2\pi m_N^* kT}{h^2}\right)^{3/2}\exp[E_F - E_c]$$

$$\Longrightarrow \boxed{m_N^* = 6^{2/3}\left(m_t^{*2} m_l^*\right)^{1/3}}$$ density of states effective mass

② Germanium band structure



note:
<1,1,1> valleys
- 8 valleys, each contributes 1/2 ellipsoid → 4 equivalent <1,1,1> valleys

a) calculate n_i to an accuracy better than 1% at 300K and 900K, assuming that the band edges do not shift with temperature

$$n_i^2 = np$$

n_i - intrinsic carrier density
n - total electron density in all three conduction bands
p - total hole density in both valence bands

$$n = n_1 + n_2 + n_3 \quad , \quad p = p_1 + p_2$$

$$n_j = N_{cj}\, e^{-(E_{cj}-E_F)/kT} \qquad N_{cj} = \frac{1}{4}\left(\frac{2m_{cj}^* kT}{\pi\hbar^2}\right)^{3/2} \quad j = 1,2,3$$

$$p_i = N_{vi}\, e^{+(E_{vi}-E_F)/kT} \qquad N_{vi} = \frac{1}{4}\left(\frac{2m_{vi}^* kT}{\pi\hbar^2}\right)^{3/2} \quad i = 1,2$$

E_F - Fermi level

E_{cj} - energy of jth conduction band edge
E_{vi} - energy of ith valence band edge

density of states effective mass for each conduction band:

$$m_{cj} = \nu_j^{2/3}\left(m_{xj}\, m_{yj}\, m_{zj}\right)^{1/3} \qquad \nu_j \text{ - \# of equivalent ellipsoids for } j^{th} \text{ band}$$

$\langle 1,1,1\rangle$: $m_{c1}^* = 4^{2/3}(1.58\times 0.082^2)^{1/3} m_0 = 0.5539\, m_0$

$\langle 0,0,0\rangle$: $m_{c2}^* = 1^{2/3}(0.036^3)^{1/3} m_0 = 0.036\, m_0$

$\langle 1,0,0\rangle$: $m_{c3}^* = 6^{2/3}(0.19\times 0.97^2)^{1/3} m_0 = 1.860\, m_0$

light holes: $m_{v1}^* = m_{lh} = 0.044\, m_0$

heavy holes: $m_{v2}^* = m_{hh} = 0.28\, m_0$ $\qquad E_{v1} = E_{v2} = E_v$

$$n_i^2 = np = \left(N_{c1}e^{-E_{c1}/kT} + N_{c2}e^{-E_{c2}/kT} + N_{c3}e^{-E_{c3}/kT}\right)e^{E_F/kT}\left(N_{v1}+N_{v2}\right)e^{(E_v-E_F)/kT}$$

$$n_i^2 = \frac{1}{16}\left(\frac{2kT}{\pi\hbar^2}\right)^3\left(m_{v1}^{*3/2} + m_{v2}^{*3/2}\right)\left[m_{c1}^{*3/2}e^{-E_{g1}/kT} + m_{c2}^{*3/2}e^{-E_{g2}/kT} + m_{c3}^{*3/2}e^{-E_{g3}/kT}\right]$$

$E_{g1} = 0.66\,eV$, $E_{g2} = 0.66 + 0.15\,eV = 0.81\,eV$, $E_{g3} = 0.66 + 0.18\,eV = 0.84\,eV$

at 300K: $kT = (8.62\times 10^{-5}\,eV/K)\,300K = 0.02586\,eV$
900K: $kT = (8.62\times 10^{-5}\,eV/k)\,900K = 0.07758\,eV$

at T = 300K

$$n_i = \sqrt{\frac{1}{2}\left(\frac{1.38\times 10^{-23}\,J/K\cdot 300K}{\pi\times(1.0546\times 10^{-34}\,J\cdot s)^2}\,9.11\times 10^{-31}\,kg\right)^3\left(0.044^{3/2}+0.28^{3/2}\right)\left(0.5539^{3/2}e^{\frac{-0.66}{0.02586}} + 0.036^{3/2}e^{\frac{-0.81}{0.02586}} + 1.86^{3/2}e^{\frac{-0.84}{0.02586}}\right)}$$

$$\boxed{n_i = 1.84\times 10^{13}\,cm^{-3}}$$

similarly at T = 900K

$$\boxed{n_i = 5.98\times 10^{17}\,cm^{-3}}$$

b) position of E_F at 300K and 900K
charge neutrality in intrinsic semiconductor implies $n = p$

$$\left(N_{c1}e^{-E_{c1}/kT} + N_{c2}e^{-E_{c2}/kT} + N_{c3}e^{-E_{c3}/kT}\right)e^{E_F/kT} = \left(N_{v1}+N_{v2}\right)e^{E_v/kT}e^{-E_F/kT}$$

$$E_F = -\frac{kT}{2} \ln\left(\frac{e^{-E_v/kT}}{N_{v1}+N_{v2}} \left(N_{c1} e^{-E_{c1}/kT} + N_{c2} e^{-E_{c2}/kT} + N_{c3} e^{-E_{c3}/kT} \right) \right)$$

energies measured relative to valence band edge ($E_v = 0$)

$$E_F = -\frac{kT}{2} \ln\left(\frac{1}{m_{v1}^{*3/2} + m_{v2}^{*3/2}} \left(m_{c1}^{*3/2} e^{-E_{c1}/kT} + m_{c2}^{*3/2} e^{-E_{c2}/kT} + m_{c3}^{*3/2} e^{-E_{c3}/kT} \right) \right)$$

at $T = 300K$

$$E_F = -\frac{1}{2}(0.02586\,eV) \ln\left(\frac{0.5539^{3/2} e^{-\frac{0.66}{0.02586}} + 0.036^{3/2} e^{-\frac{0.81}{0.02586}} + 1.86^{3/2} e^{-\frac{0.84}{0.02586}}}{0.044^{3/2} + 0.28^{3/2}} \right)$$

$$\boxed{E_F = 0.317\,eV}$$

similarly at $T = 900K$

$$\boxed{E_F = 0.274\,eV}$$

c) for $T = 0K$, E_F lies at $0.25eV$ above conduction band minimum

electron densities in each conduction band valley

for j^{th} conduction band valley

$$n_j = \frac{1}{2\pi^2} \left(\frac{2m_j^*}{\hbar^2} \right)^{3/2} \int_0^\infty \frac{E'^{1/2}\,dE}{1+\exp[(E-E_F+E_{cj})/kT]}$$

the zero of energy at the conduction band edge

at $T = 0$

$$n_j = \frac{1}{2\pi^2} \left(\frac{2m_j^*}{\hbar^2} \right)^{3/2} \int_0^{E_F - E_{cj}} E'^{1/2}\,dE = \frac{1}{2\pi^2} \left(\frac{2m_j^*}{\hbar^2} \right)^{3/2} \frac{2}{3} (E_F - E_{cj})^{3/2}$$

results:

$\langle 111 \rangle$ valley :	$n_1 = 2.34 \times 10^{20}\ cm^{-3}$
$\langle 000 \rangle$ valley :	$n_2 = 9.79 \times 10^{17}\ cm^{-3}$
$\langle 100 \rangle$ valley :	$n_3 = 2.13 \times 10^{20}\ cm^{-3}$

Problem 3

2D density of states: $D(E) = \frac{m^*}{\pi\hbar^2}$

2D electron density $\equiv n = \frac{N}{A} = \int_0^\infty f(E)D(E)\,dE$

Use Maxwell-Boltzmann approx. for non-degenerate electron gas $\Big\rangle$ $= \frac{m^*}{\pi\hbar^2}\int_0^\infty e^{-(E-E_F)/kT}\,dE$

$$n = \frac{m^*}{\pi\hbar^2} e^{(E_F-E_c)/kT}\int_0^\infty e^{-E/kT}\,dE = \underbrace{\frac{m^* kT}{\pi\hbar^2} e^{(E_F-E_c)/kT}}$$

Average Kinetic energy

$$\langle E\rangle = \int_0^\infty E f(E) D(E)\,dE = \frac{m^*}{\pi\hbar^2} e^{\frac{(E_F-E_c)}{kT}}\int_0^\infty E e^{-E/kT}\,dE$$

$$= \frac{m^*}{\pi\hbar^2}(kT)^2 e^{E_F/kT}$$

$$\langle E\rangle = \left(\frac{m^* kT}{\pi\hbar^2} e^{(E_F-E_c)/kT}\right)kT = n\,kT \Rightarrow \boxed{\frac{\langle E\rangle}{n} = kT}$$

Problem 4

Maxwell-Boltzmann distribution: $f(\vec{p}) = e^{\frac{(E_F-E_c)}{kT}} e^{\frac{-p^2}{2m^*kT}}$

$$n = \frac{1}{4\pi^3\hbar^3}\int_{\vec{p}} e^{(E_F-E_c)/kT} e^{\frac{-p^2}{2m^*kT}}\,d\vec{p} = \frac{1}{4}\left(\frac{2m^*kT}{\pi\hbar^2}\right)^{3/2} e^{\frac{(E_F-E_c)}{kT}}$$

Flux directed outward along $\hat{z}$-direction

$$\frac{J_z}{-q} = \left\langle\frac{p_z}{m^*}\right\rangle = \frac{1}{4\pi^3\hbar^3}\frac{e^{(E_F-E_c)/kT}}{m^*}\int_{\vec{p}} p_z e^{-p^2/2m^*kT}\,d\vec{p}$$

$$= \frac{1}{4\pi^3\hbar^3}\frac{e^{(E_F-E_c)/kT}}{m^*}\,2\pi\int_0^\infty\int_0^{\pi/2} p\cos\theta\, p^2\sin\theta\,d\theta\, e^{-p^2/2m^*kT}\,dp$$

$$= \frac{2\pi}{4\pi^3\hbar^3}\frac{e^{(E_F-E_c)/kT}}{m^*}\underbrace{\int_0^{\pi/2}\cos\theta\,d(\cos\theta)}_{=1/2}\underbrace{\int_0^\infty p^3 e^{-p^2/2m^*kT}\,dp}_{=\frac{1}{2}(2m^*kT)^2}$$

$$\boxed{\frac{J_z}{-q} = \frac{1}{4}\left(\frac{2m^*kT}{\pi\hbar^2}\right)^{3/2} e^{\frac{(E_F-E_c)}{kT}}\sqrt{\frac{kT}{2\pi m^*}}}$$

(5) Literature search : Intrinsic carrier concentration (n_i) in Silicon

Papers reviewed :

(1) M.A. Green, Intrinsic Concentration, Effective Density of States, and Effective Mass in Silicon, J. Appl. Phys. 67 (6) 2944-29 Mar. 15, 1990

(2) R. Vanhemmel, W. Schoenmaker, K. De Meyer, A Unified Wide Temperature Range Model for the Energy Gap, the Effective Carrier Mass, and Intrinsic Concentration in Silicon,

Solid State Electronics 36 (10) 1379-1384, Oct. 1993

(3) A.B. Sproul and M.A. Green, Improved Value for the Silicon Intrinsic Carrier Concentration from 275 K to 375 K, J. Appl. Phys. 70 (2) 846-854, July 15, 1991

There are basically 3 experimental methods that have been used so far to determine the intrinsic carrier concentration (n_i) in silicon. Each method corresponds to one of the papers above, and will be discussed below :

(1) Based on the 1st paper, the most direct measurement of n_i involves measuring the conductivity σ of silicon at a high temperature where it has intrinsic properties. At such temperatures :

$$\sigma = e(\mu_n + \mu_p)\, n_i \qquad (1)$$

where e = electronic charge

μ_n = electron mobility ; μ_p = hole mobility

Instead of measuring σ, it is ρ = resistivity = $\frac{1}{\sigma}$ that is measured. By measuring the electron and hole conductivities, intrinsic concentration can then be calculated from eq. (1)

Advantage of this technique :

It's a simple and straight-forward procedure of getting n_i by measuring only 3 parameters ρ, μ_n and μ_p.

Weakness :

The resistivity ρ is measured at high temperature as a function of temperature T, and then a curve of the form

$$\ln \rho = A + \frac{B}{T} \quad \text{- - - - (2)}$$

is fitted to the data points to get extrapolated values of ρ at low temperatures. A and B are fitting parameters.
The mobilities (μ_n and μ_p) on the other hand are most readily measured at low temperatures where silicon has extrinsic properties. The data points are also fitted by the following expressions:

$$\left.\begin{aligned} \mu_n &= \mu_{n0} \left(T/T_0\right)^{-\alpha_n} \\ \mu_p &= \mu_{p0} \left(T/T_0\right)^{-\alpha_p} \end{aligned}\right\} \quad \text{- - - - - (3)}$$

μ_{n0}, μ_{p0}, α_p and α_n are fitting parameters.
Values for μ_n and μ_p at higher T are also extrapolated from eq. (3).

Hence, to get the value of n_i at moderate temperature, we have to rely on the extrapolated values of ρ, μ_n and μ_p, and this introduces a lot of uncertainties.

Result :

The author in this paper (paper # 1) did not report his own measurements of ρ, μ_n and μ_p. He collected measurement data accumulated over the years by several different people, and tried to extract the best data to be used to calculate n_i using eq. (1).
His best estimate for n_i at $T = 300K$ is :

$$n_i = 1.08 \times 10^{10} \ cm^{-3}$$

with an estimated uncertainty of 8%.

The author also didn't explicitly mention the experimental technique used to measure mobilities. The usual method is using the Hall effect via :

$$\mu = \frac{R_H}{\rho} \quad \text{- - - } R_H = \text{Hall coefficient}$$

(2) The second paper reports measurement of n_i based on the measurement of the conduction and valence band density of states effective masses and the band gap of silicon.
n_i is calculated using the relation :

$$n_i = \sqrt{N_c N_v} \exp\left(\frac{-e E_g}{2 kT}\right) \quad \text{------ (4)}$$

where :

$$N_c = 2\left(\frac{2\pi m_c^* kT}{h^2}\right)^{3/2} = \text{conduction band effective density of states}$$

$$N_v = 2\left(\frac{2\pi m_v^* kT}{h^2}\right)^{3/2} = \text{valence band effective density of states}$$

$$m_c^* = 6^{2/3}\left(m_t^{*2} m_l^*\right)^{1/3} = \text{CB density of state effective mass}$$

m_t^* and m_l^* are the transverse and longitudinal effective masses associated with the ellipsoidal constant energy surfaces

$$m_v^* = \left\{\left(m_{lh}^*\right)^{3/2} + \left(m_{hh}^*\right)^{3/2} + \left(m_{so}^* \exp(-\Delta/kT)\right)^{3/2}\right\}^{2/3}$$

m_v^* = VB density of states effective mass

m_{lh}^*, m_{hh}^* and m_{so}^* are the effective masses of the valence band's light-hole, heavy-hole and "split-off" bands respectively

Δ = energy difference between the energy maximum in the split-off band and the two other bands

m_t^*, m_l^*, m_{lh}^*, m_{hh}^*, and m_{so}^* are measured using the cyclotron resonance experiment.

E_g = band gap energy can be measured by optical absorption exp.

Advantage :

Nothing I can think of frankly, other than perhaps quite accurate measurements of E_g and the effective masses.

Weakness :

It's complicated because it involves the measurements of a lot of parameters (7 of them) and thus large uncertainties can accumulate in the final determination of n_i using eq. (4).

This procedure is further complicated by the fact that the effective masses and the band gap depend on the temperature. The dependence of E_g on T is due to:

- lattice spacing variation (thermal expansion)
- modified carrier lattice interaction

The dependence of m^* on T is due to:

- lattice spacing variation
- non-parabolicity of the conduction band as higher states are occupied at higher T.
- change in electron-phonon interaction

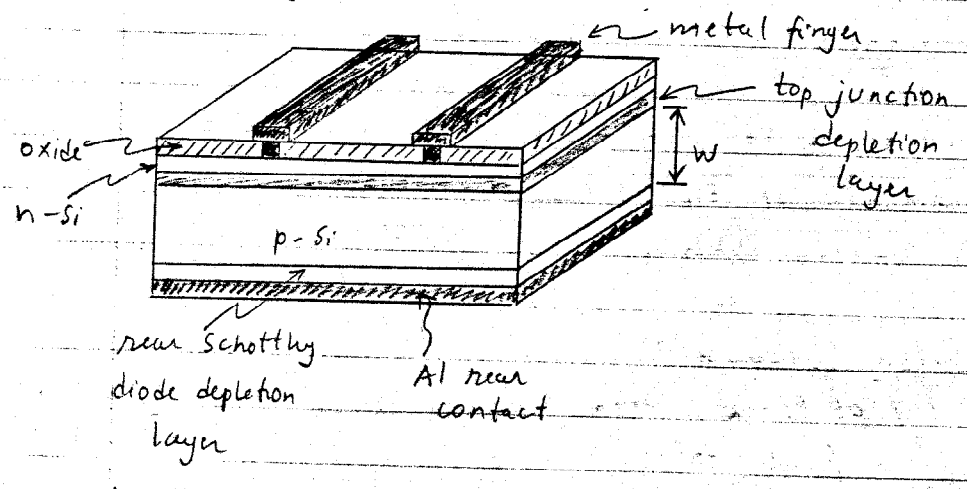
Result:

The authors took the expressions of E_g and all m^* as a function of T previously obtained by other people to compute n_i as a function of T using eq. (4). The result was then compared with experimental data of the type described in the 1st paper. They computed the result of:

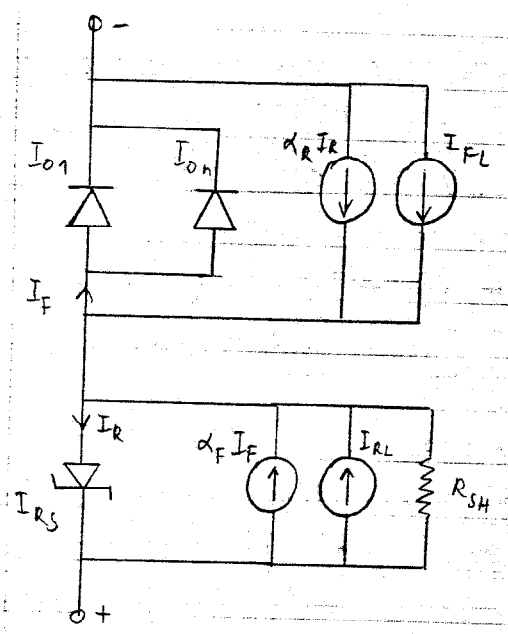
$n_i = 1.14 \times 10^{10}\ cm^{-3}$ at $T = 300\ K$

with an estimated uncertainty of about 11 %

(3) A third method of measuring n_i is described in the 3rd paper. It involves the measurement of the characteristics of semiconductor devices. The authors used a device based on the Passivated Emitter and Rear Cell (PERC), a silicon solar cell with energy conversion efficiency greater than 23 %.



The short-circuit current (I_{sc}) v.s. open-circuit voltage (V_{oc}) of the device was measured at varying light intensities.
In order to determine the intrinsic carrier concentration n_i from the measured $I_{sc} - V_{oc}$ values, the device was modeled as a bipolar transistor with the diffused top junction forming the emitter and the rear Schottky junction forming the collector. This modeling allowed them to draw the modified Ebers-Moll equivalent circuit for the device:



According to this model:

$$I_{sc} = I_{01}\left\{\exp\left[\frac{e}{kT}\left(V_{oc} + I_{sc} R'_{SH}\right)\right] - 1\right\} + I_{0n}\left\{\exp\left[\frac{e}{nkT}\left(V_{oc} + I_{sc} R'_{SH}\right)\right] - 1\right\} \quad (5)$$

The most important aspect of eq. (5) is that I_{01} is strongly dependent upon n_i.

The dependence of I_{01} on n_i in 1-D model of the cell is given by:

$$I_{01} = I_{0b} + I_{0e} \quad (6)$$

$$I_{0b} = \frac{A e D_n n_i^2}{W N_a^-}\left[\frac{W}{L_n}\coth\left(\frac{W}{L_n}\right)\right] \quad (7)$$

I_{0b} = saturation current associated with the p-region and near contact
I_{0e} = —— " —— —— " —— the front n-type surface diffusion and metalization
A = area of cell
e = electronic charge
D_n = diffusivity
L_n = diffusion length
} minority carrier electron.
N_a^- = ionized dopant concentration in the p-type substrate

Rearranging eq. (6) and (7) we get:

$$n_i = \left[\frac{W N_a^-}{A e D_n} \frac{(I_{01} - I_{0e})}{1-025} \frac{1}{[(W/L_n) \coth(W/L_n)]} \right]^{1/2} \quad \ldots\ldots (8)$$

Hence, by measuring the saturation current I_{01} and the other parameters in eq. (8), n_i can be determined. To obtain I_{01} from the experimental I_{sc} - V_{oc} - data, eq. (5) was fitted to the data with I_{01}, I_{0n}, n and R_{SH} being the fitting parameters.

In order to determine N_A^-, 4-point probe resistivity measurements were made. The diffusivity was determined via μ_n = mobility using the Einstein relation $D_n = \mu_n \frac{kT}{e}$; μ_n is determined by standard techniques described in the 1st paper.

The value of L_n does not have to be determined accurately since $W/L_n \coth(W/L_n) \approx 1$ provided that $L_n > W$.

In order to determine W, the depth of the junction (based on spreading resistance measurements) and the calculated depletion widths associated with the front and rear junction were subtracted from the substrate thickness.

Advantage:

Only a small number of parameters are needed. Among the data obtained are I_{sc} and V_{oc}, which are very easy to measure with high degree of accuracy, resulting in a low uncertainty in the calculated value of n_i.

Weakness:

It relies on a model for the device to extract the expression for n_i in terms of the device characteristics. If the model doesn't describe the performance of the device very well, then the value of n_i will be wrong.

Result :

The authors made measurements of n_i as a function of T for the temperature range $275\ K < T < 375\ K$.
The value at 300 K is :

$$n_i = 1.00 \times 10^{10}\ cm^{-3}$$

with an estimated uncertainty of only 3%

The best value to use for n_i at 300 K :

I would pick $n_i = 1.00 \times 10^{10}\ cm^{-3}$ at 300 K mainly because it has the smallest uncertainty.
It is also probably more suitable for device modeling since it was derived from the kind of model used in device modeling.

CHD