1. Effective mass mayhem:
   a) There are 6 “ellipsoidal” conduction band minima in silicon. At each minimum, there is a
two-fold symmetric transverse effective mass, \( m_t^* \), and a longitudinal effective mass, \( m_l^* \).

   Show that the acceleration, \( \frac{dv}{dt} \) is always along the direction of the force, \( \vec{F} \), and that the
“mobility” mass is given by:

   \[
   \frac{1}{m_\mu^*} = \frac{1}{3} \left( \frac{2}{m_t^*} + \frac{1}{m_l^*} \right)
   \]

   b) Show that the effective mass to use in the density of states expression is:

   \[
   m_N^* = 6^{2/3} \left[ m_l^* (m_t^*)^2 \right]^{1/3}
   \]

2. The conduction band of germanium consists of <111> valleys at the extreme of the Brillouin
zone, a “direct” <000> valley 0.15 eV above the conduction band minimum, and six <100>
valleys 0.18 eV above the conduction band minimum. The longitudinal and transverse
effective masses are:

   \[
   \begin{array}{lcc}
   \text{<111>} & m_l = 1.58m_0 & m_t = 0.082m_0 \\
   \text{<000>} & m_l = 0.036m_0 & m_t = 0.036m_0 \\
   \text{<100>} & m_l = 0.19m_0 & m_t = 0.97m_0 \\
   \end{array}
   \]

   The valence band consists of a set of two isotropic bands (“light hole” and “heavy hole”),
degenerate at \( k = 0 \), with effective masses \( m_{lh} = 0.044m_0 \), and \( m_{hh} = 0.28m_0 \). The band gap
\( E_g \) is 0.66 eV.

   a) Calculate \( n_l \) to an accuracy of better than 1% at \( T = 300K \), and \( T = 900K \), assuming that
the band edges do not shift with temperature.

   b) Find the position of \( E_F \) at \( T = 300K \), and \( T = 900K \), under the same assumption.

   c) For \( T = 0K \), suppose \( E_F \) lies at 0.25 eV above the conduction band minimum. Calculate
the electron density in each of these conduction band valleys.

   Be careful to consider population only within the first Brillouin zone when counting the
number of <111> valleys.
3. For a 3D, non-degenerate electron gas, the average kinetic energy per carrier is $3kT/2$. Derive the corresponding result for a 2D non-degenerate electron gas.

4. Show that for a Maxwell-Boltzmann distribution, the flux directed outward along one of the three coordinate axes is:

$$\frac{J}{(-q)} = n \sqrt{\frac{kT}{2\pi m^*}}$$

5. Literature search. The parameter $n_i$ is very important in device modelling, but is actually not that easy to measure accurately. There has been quite a bit of discussion about the correct value of this quantity for silicon in the relatively recent literature (90s). Values from the mid-1970’s are obsolete. Your assignment is to critically review this literature. Describe the various experimental methods for measuring $n_i$ and their strengths and weaknesses. Decide for yourself what value is the best one to use for $n_i$ at 300K in silicon and explain why.