Problem set 5 EE230 J. Bokor

1 Compound semiconductor alloys (such as $Al_xGa_{1-x}As$) with microscopic fluctuations in the alloy composition x.
Scattering rate for alloy scattering: (perturbation in the conduction and valence band edges)

$$S(p,p') = \frac{2\pi}{\hbar}\left(\frac{3\pi^2}{16}\right)\frac{|\Delta U|^2}{N\Omega}\,\delta(E'-E)$$

where N - atom concentration, Ω - normalization volume

and $\Delta U = x(1-x)(\chi_{GaAs} - \chi_{AlAs})$ (χ - electron affinity)

a) The alloy scattering rate vanishes at the compositional endpoints $x=0$ and $x=1$ because the points correspond to pure GaAs and AlAs, not alloys.

b) transition rate for alloy scattering

$$\frac{1}{\tau_m(p)} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') = \int N(\vec{p}')\,S(\vec{p},\vec{p}')\,d\vec{p}' = \int N(\vec{E}')\,S(E,E')\,dE'$$

$$= \int \frac{\Omega}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E'}\;\frac{2\pi}{\hbar}\left(\frac{3\pi^2}{16}\right)\frac{|\Delta U|^2}{N\Omega}\,\delta(E-E')\,dE' \qquad N(E) = \frac{\Omega}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}$$

$$= \frac{3\cdot 2\sqrt{2}\,m^{3/2}\,2\pi}{4\cdot 16\hbar^4 N}|\Delta U|^2\sqrt{E} = \frac{3\sqrt{2}\,\pi m^{3/2}|\Delta U^2|}{16\hbar^4 N}\frac{p}{\sqrt{2m}} \qquad E = \frac{p^2}{2m}$$

$$= \frac{3\pi m|\Delta U|^2}{16\hbar^4 N}\,p$$

$$\boxed{\tau_m(p) = \frac{16\hbar^4 N}{3\pi m|\Delta U|^2}\,p^{-1}} \qquad \frac{1}{\tau_m(E)} = \frac{3\pi(2m)^{3/2}}{32\hbar^4 N}|\Delta U|^2 E^{1/2}$$

2 Momentum relaxation times due to ionized impurity scattering.

a) $1/\tau_m$ for electrons with thermal average energy $E = \frac{3}{2}kT_L$ in GaAs doped at $N_D = 10^{18}\ cm^{-3}$ ($T_L = 300K$).
Compare to L_D and b_{max}:

$$L_D = \sqrt{\frac{k_s \epsilon_0 kT}{q^2 n}} = \left(\frac{(13.13 \times 8.854 \times 10^{-12}\ F/m)(1.38 \times 10^{-23}\ J/K)(300K)}{(1.602 \times 10^{-19}\ C)^2 (10^{18}\ cm^{-3})(10^6\ \frac{cm^3}{m^3})} \right)^{1/2}$$

$$L_D = 4.33 \times 10^{-9}\ m = \boxed{43.3\ Å}$$

$$b_{max} = \frac{1}{2}(N_D)^{-1/3} = \frac{1}{2}(10^{18})^{-1/3} = \boxed{50\ Å}$$

Since $L_D \approx b_{max}$ it is unclear whether to use the screened (Brooks-Herring) or the unscreened (Conwell-Weisskopf) theory, but since $L_D < b_{max}$, the screened theory is somewhat better.

BH: $\gamma^2 = 8m^* E L_D^2/\hbar^2 =$

$$= \frac{8(0.067 \times 9.11 \times 10^{-31}\ kg)(3/2 \times 1.38 \times 10^{-23}\ J/K)(300K)(4.33 \times 10^{-9}\ m)^2}{(1.055 \times 10^{-34}\ J.s)^2}$$

$$\gamma^2 = 5.11$$

$$\tau_m = \frac{16\sqrt{2m^*}\,\pi k_s^2 \epsilon_0^2}{N_D q^4} \left(\ln(1+\gamma^2) - \frac{\gamma^2}{1+\gamma^2} \right)^{-1} (E)^{3/2}$$

$$= \frac{16\pi\sqrt{2 \times 0.067 \times 9.11 \times 10^{-31}\ kg}\,(13.13 \times 8.854 \times 10^{-12}\ F/m)^2}{(10^{24}\ m^{-3})(1.602 \times 10^{-19}\ C)^4 \left(\ln(1+5.11) - \frac{5.11}{1+5.11}\right)} \left(\frac{3}{2} 1.38 \times 10^{-23}\ J/K \times 300K\right)^{3/2}$$

$$\boxed{\tau_m = 0.18\ ps} \qquad \boxed{1/\tau_m = 5.6 \times 10^{12}\ s^{-1}}$$

b) $E = 0.3\ eV$ (injection across a heterojunction barrier)
$\gamma^2 = 39.5$

BH: $\boxed{\tau_m = 1.4\ ps}$ $\boxed{1/\tau_m = 7.1 \times 10^{12}\ s^{-1}}$

3 Γ-L and L-Γ intervalley scattering processes

a) scattering rates for electrons in GaAs as a function of energy at 77K and 300K

$\boxed{\Gamma\text{-L scattering}}$ $$\frac{1}{\tau_{\Gamma L}} = \frac{\pi D_{\Gamma L}^2 Z_L}{2\rho\omega_{\Gamma L}}\left\{N_i\, g_L(E+\hbar\omega_{\Gamma L}-\Delta E_{L\Gamma}) + (N_i+1)\, g_L(E-\hbar\omega_{\Gamma L}-\Delta E_{L\Gamma})\right\}$$

$D_{\Gamma L} = 10\times10^8$ eV/cm

Z_L = # of final valleys (L valleys) = 4

$\hbar\omega_{\Gamma L} = 0.0278$ eV

ρ = mass density = 5.36 g/cm^3

$N_i(77K) = 0.0154$, $N_i(300K) = 0.518$

$$g_L(E) = \frac{\sqrt{2}(m_L^*)^{3/2}}{\pi^2\hbar^3}E^{1/2} = \frac{\sqrt{2}(0.222\times9.11\times10^{-31}\,kg)^{3/2}(1.6\times10^{-19}\,J/eV)^{1/2}}{\pi^2(1.055\times10^{-34}\,J\cdot s)^3}(E[eV])^{1/2}$$

$g_L(E) = (4.442\times10^{45}\sqrt{E})\frac{1}{Jm^3}$ (E is in eV) (E measured from bottom of Γ valley)

$\Delta E_{L\Gamma} = 0.29$ eV

77K: $$\frac{1}{\tau_{\Gamma L}} = \frac{\pi(10^9\,eV/cm\times1.602\times10^{-19}\frac{J}{eV}\times100\,cm/m)^2(4)(4.442\times10^{45}/Jm^3)}{2(5.36\times1000\,kg/m^3)(0.0278\times1.6\times10^{-19}\,J/1.055\times10^{-34}\,J\cdot s)}$$
$$\times\left(0.0154(E+0.0278-0.29)^{1/2}+1.0154(E-0.0278-0.29)^{1/2}\right)$$

$$\boxed{\frac{1}{\tau_{\Gamma L}} = 3.166\times10^{13}\left[0.0154(E-0.2622)^{1/2}+1.0154(E-0.3178)^{1/2}\right]s^{-1}}$$

300K $$\boxed{\frac{1}{\tau_{\Gamma L}} = 3.166\times10^{13}\left[0.518(E-0.2622)^{1/2}+1.518(E-0.3178)^{1/2}\right]s^{-1}}$$

$\boxed{\text{L-}\Gamma\text{ scattering}}$ $Z_\Gamma = 1$, $D_{L\Gamma} = 10^9$ eV/cm , $\Delta E_{\Gamma L} = -0.29$ eV

$$g_\Gamma(E) = \frac{\sqrt{2}(0.067\times9.11\times10^{-31}\,kg)^{3/2}(1.6\times10^{-19}\,J/eV)^{1/2}}{\pi^2(1.055\times10^{-34}\,J\cdot s)^3}(E[eV])^{1/2}$$

$g_\Gamma(E) = (7.365\times10^{44}\sqrt{E})\frac{1}{Jm^3}$ (E measured from bottom of L valley)

77K $$\boxed{\frac{1}{\tau_{L\Gamma}} = 1.312\times10^{12}\left[0.0154(E+0.3178)^{1/2}+1.0154(E+0.2622)^{1/2}\right]s^{-1}}$$

300K $$\boxed{\frac{1}{\tau_{L\Gamma}} = 1.312\times10^{12}\left[0.518(E+0.3178)^{1/2}+1.518(E+0.2622)^{1/2}\right]s^{-1}}$$

b) Reference:

"Determination of the LO-phonon and Γ-L intervalley scattering time in GaAs from hot-electron luminescence spectroscopy", W. Hackenberg and G. Fasol, Solid State Electronics, vol. 32, no. 12, pages 1247-51, 1989.

In this paper the authors plotted τ_{PL} as a function of electron kinetic energy E (figure 4) at 77K. The calculated values from part a) agree with the measurement within the experimental error.

For example:

energy	measured value	calculated value
370meV	120 ±50 fs	130 fs
330meV	360 ± 150 fs	272 fs

4 results are shown below.
initially, the results are noisy and sensitive to the specific random numbers called, but after ≈ 110 selections, the error is consistently below 10% //

10	tc(ave)=	3.09578e-15	error=	-7.12651e-02
20	tc(ave)=	3.73009e-15	error=	0.119028
30	tc(ave)=	4.09417e-15	error=	0.228252
40	tc(ave)=	3.71832e-15	error=	0.115496
50	tc(ave)=	3.60026e-15	error=	8.00773e-02
60	tc(ave)=	3.60114e-15	error=	8.03421e-02
70	tc(ave)=	3.54526e-15	error=	6.35790e-02
80	tc(ave)=	3.81408e-15	error=	0.144224
90	tc(ave)=	3.86869e-15	error=	0.160607
100	tc(ave)=	3.67939e-15	error=	0.103818
110	tc(ave)=	3.61377e-15	error=	8.41306e-02
120	tc(ave)=	3.61539e-15	error=	8.46180e-02
130	tc(ave)=	3.53864e-15	error=	6.15906e-02
140	tc(ave)=	3.53717e-15	error=	6.11501e-02
150	tc(ave)=	3.48233e-15	error=	4.46979e-02
160	tc(ave)=	3.54259e-15	error=	6.27758e-02
170	tc(ave)=	3.49138e-15	error=	4.74153e-02
180	tc(ave)=	3.42495e-15	error=	2.74841e-02
190	tc(ave)=	3.43126e-15	error=	2.93787e-02
200	tc(ave)=	3.35721e-15	error=	7.16171e-03